Empirical evaluated SDE modelling and predictability estimates: applications to chaotic dynamical systems and climate low-frequency variability

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Abstracts

This paper develops and validates a method of empirical modelling for a dimensionality-reduced system of a non-linear dynamical system based on the framework of the stochastic differential equation (SDE). Following the mathematical theorem corresponding to some inverse problem of the probability theory, we derive empirically evaluating formulae for the drift vector and the diffusion matrix of the SDE. Focusing on a low-dimensional dynamical system of the Lorenz system, we empirically reconstruct an SDE that approximates the original time-series on the projected 2-dimensional plane. To assess the availability of the obtained SDE, we compare the ensemble variance of solutions generated by the numerical SDE to that of the trajectories of the projected orbit of the time-series. The results have good agreements in the distribution of values of the ensemble variance and show the predictive ability of the SDE modelling. This framework can be applied to empirical estimate of ensemble spread of an operational climate forecast system. Applying the present methodology to dimensionality-reduced climate reanalysis datasets, we can evaluate a magnitude of error growth, corresponding to the ensemble spread, as diffusion of solution paths of the estimated SDE in the low-dimensional phase space. A linearised nonstationary fluctuation–dissipation relation (NFDR) derived by the empirically constructed SDE provides a good representation in the phase space of the inhomogeneous distribution of the forecast spread of the operational ensemble forecast conducted by the Japan Meteorological Agency. In particular, the linearised NFDR captures the local maximum of the forecast spread during the onset period of the major stratospheric sudden warming events. The present study gives theoretical fundamentals of the predictability estimates for the ensemble forecasts.

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