Is ultra-high model resolution necessary to improve probabilistic predictions?

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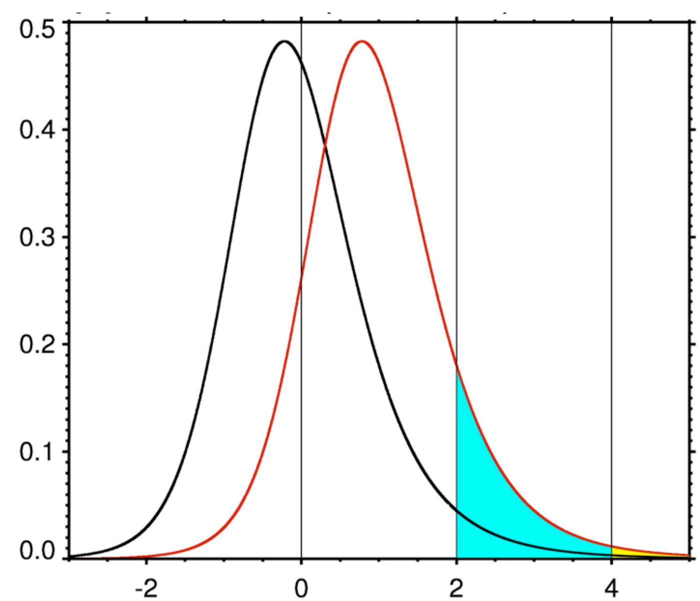
Beyond the synoptic time scale, weather and climate predictions are inherently probabilistic.

Does one need ultra-high resolution models to correctly represent the associated probability distribution functions (PDFs), which are generally non-Gaussian ?

Or can lower resolution models with a combination of *deterministic and stochastic* parameterizations of unresolved processes be adequate for this purpose ?



The PDFs of daily atmospheric variations are not Gaussian. They are generally skewed and heavy tailed, and in a distinctive way. This has large implications for extreme weather statistics.

Skewness $S = \langle x^3 \rangle / \sigma^3$ and

Kurtosis $K = \langle x^4 \rangle / \sigma^4 - 3$

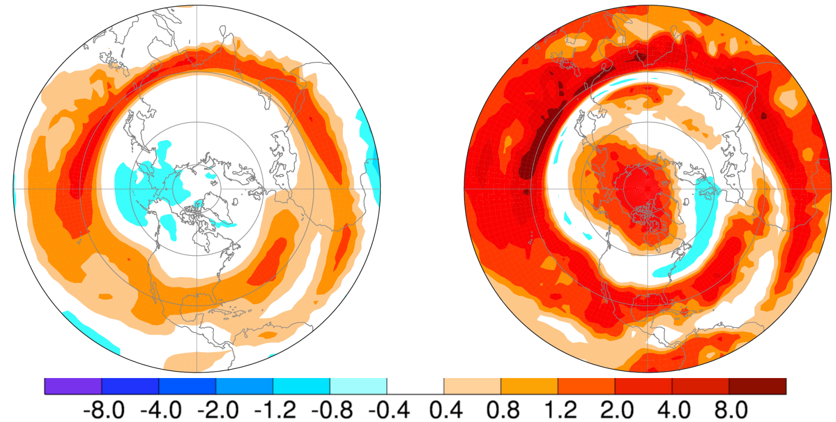
of wintertime daily anomalies x of **250 mb Vorticity** in the 140-yr 20th Century Reanalysis (Compo et al 2011)

These distinctive non-Gaussian properties are captured by a general class of so-called

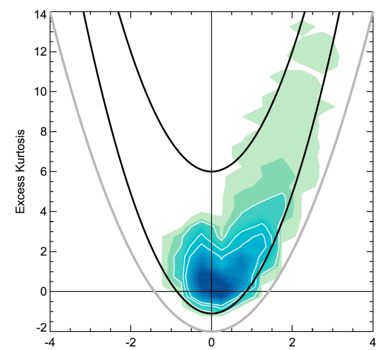
Stochastically Generated Skewed ("SGS") *probability distributions.*

Sardeshmukh, Compo, Penland (2015)

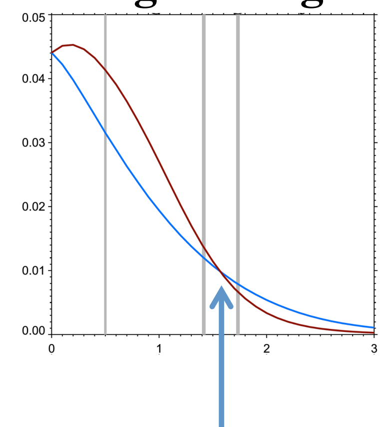
Skewness *S* **Kurtosis *K***



K* vs *S



Average Histograms



Note the parabolic inequality $K \geq 3/2\, S^2$

Note that the crossover point where $p(x) = p(-x)$ lies between 1.4σ and 1.7σ

A simple mechanism for generating skewed heavy-tailed probability distributions

("Stochastically Generated Skewed (SGS) distributions" *Sardeshmukh and Sura J. Clim 2009*)

$$\frac{dx}{dt} = -\left(\lambda + \frac{1}{2}E^2\right)x + b\,\eta_1 + (Ex + g)\eta_2 - \frac{1}{2}Eg$$

where b and g are amplitudes of "additive" noise, and E is amplitude of "multiplicative" noise

If $E \to 0$ *then pdf* $p(x)$ *of* $x \to$ *Gaussian pdf*

If $E \neq 0$ *then* $p(x)$ *is skewed and heavy-tailed, with* $K > \frac{3}{2}S^2$ *and* $p(x) = p(-x)$ *at* $\hat{x} \approx \sqrt{3}\sigma$

$$\text{Mean} \quad \mu = \langle x \rangle = 0$$

$$\text{Variance} \quad \sigma^2 = \langle x^2 \rangle = \frac{g^2 + b^2}{2\lambda(1-\alpha)} \qquad \text{where } \alpha \equiv \frac{E^2}{2\lambda} \quad \left(< \frac{1}{3} \text{ if Kurtosis exists}\right)$$

$$\text{Skewness} \quad S = \frac{\langle x^3 \rangle}{\sigma^3} = \frac{2E}{\lambda(1-2\alpha)}\frac{g}{\sigma}$$

$$\text{Kurtosis} \quad K = \frac{\langle x^4 \rangle}{\sigma^4} - 3 = \frac{3}{2}\left[\frac{1-2\alpha}{1-3\alpha}\right]S^2 + \frac{6\alpha}{[1-3\alpha]} > \frac{3}{2}S^2$$

How well do atmospheric models capture the PDFs of daily variability ?

Results from medium (T159) to high (T2047) resolution runs

from project ATHENA (Jung et al 2012, Kinter et al 2013)

+ additional T95 resolution runs

- 1989-2007 (1 run per year, 1 ensemble member)
- These are AMIP runs, with specified observed SSTs
- We focus here on the DJF season

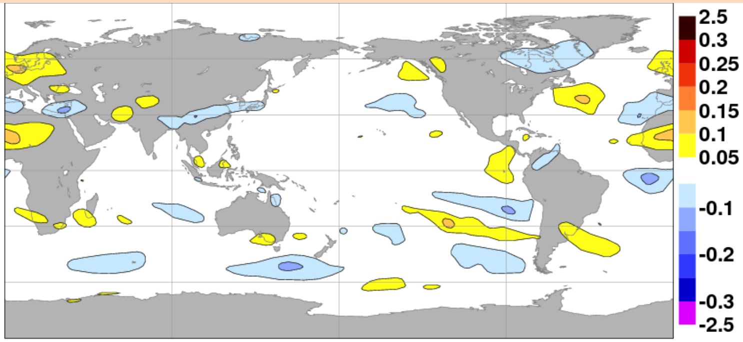
All validations are of T42-truncated fields against the ERA-Interim Reanalysis

Linus Magnusson ECMWF

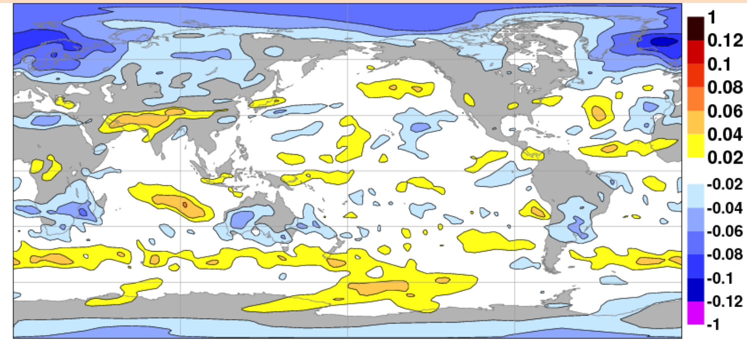
Biases in DJF mean and daily standard deviation of 200 mb Vorticity

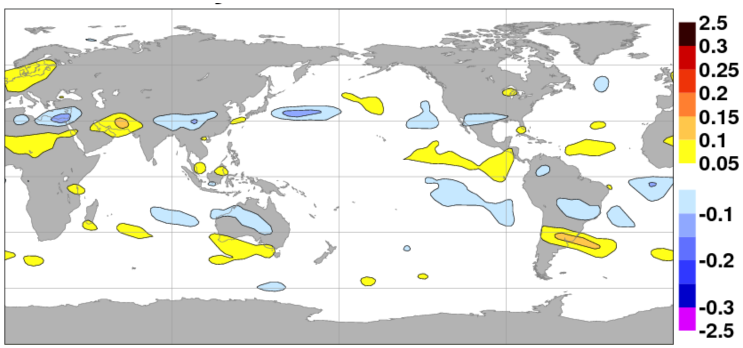
Biases in mean

Biases in Standard Deviation

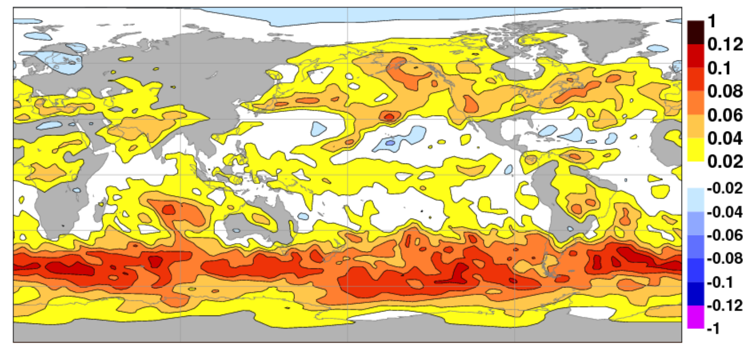


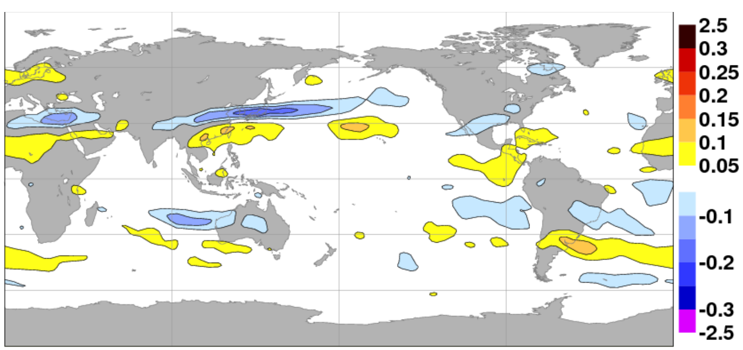
T95



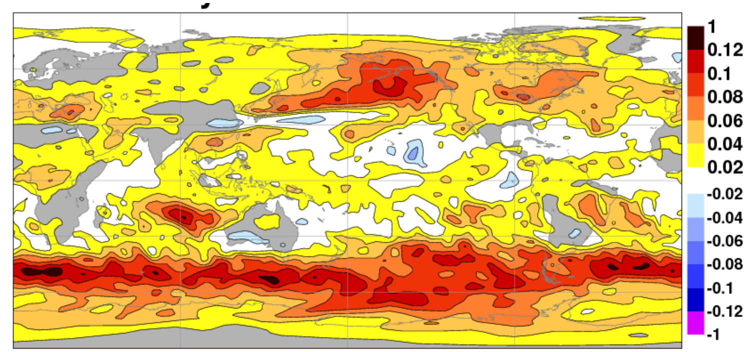


T511



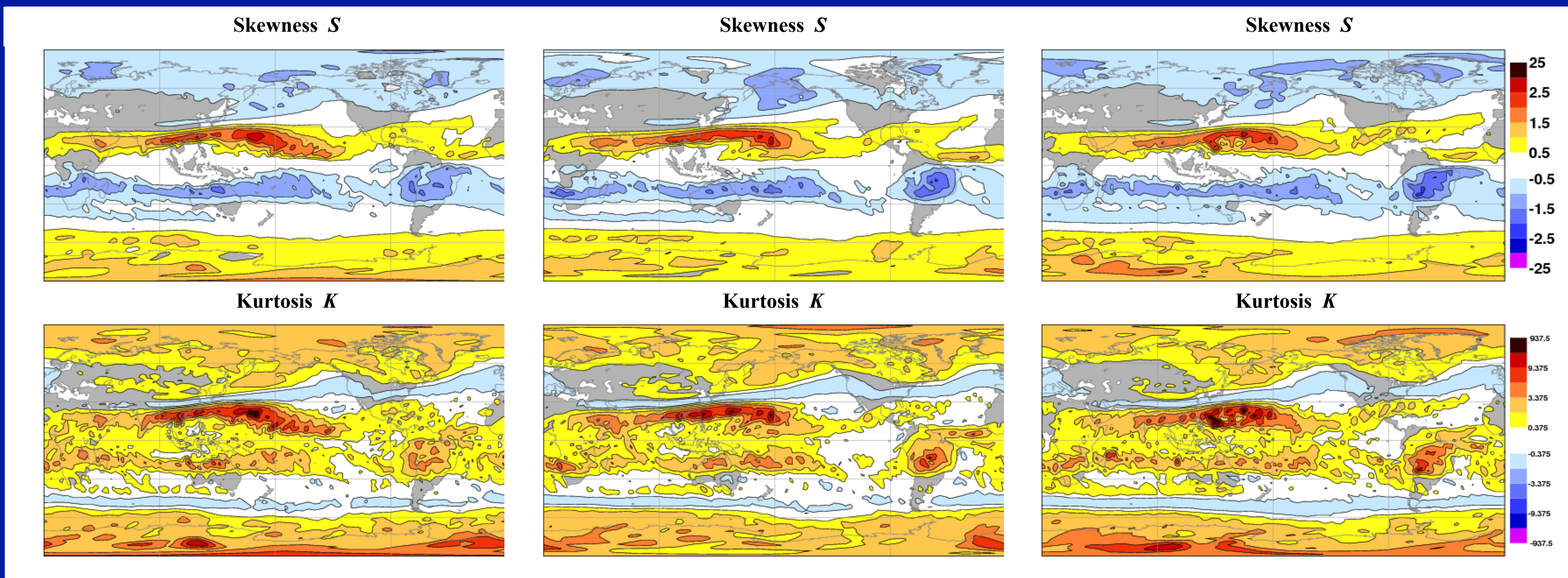


T2047



Skewness *S* and Kurtosis *K* of daily 200 mb Vorticity in DJF are very similar and realistic at both T95 (~ 120 km) and T2047 (~ 6 km) resolutions

ERA Interim | T95 | T2047



NOTE that the color scale for kurtosis is (1.5*x*x) the color scale for the skewness

to highlight the consistency of both the patterns and magnitudes of S and K
with the “SGS” probability distribution theory.

Why are Skewness and Kurtosis unaffected by increases of model resolution ?
Is this because increasing model resolution basically just adds <u>additive</u> noise ?

$$\frac{dx}{dt} = -\left(\lambda + \frac{1}{2}E^2\right)x + b\,\eta_1 + (Ex+g)\eta_2 - \frac{1}{2}Eg$$

where b and g are amplitudes of "additive" noise, and E is amplitude of "multiplicative" noise

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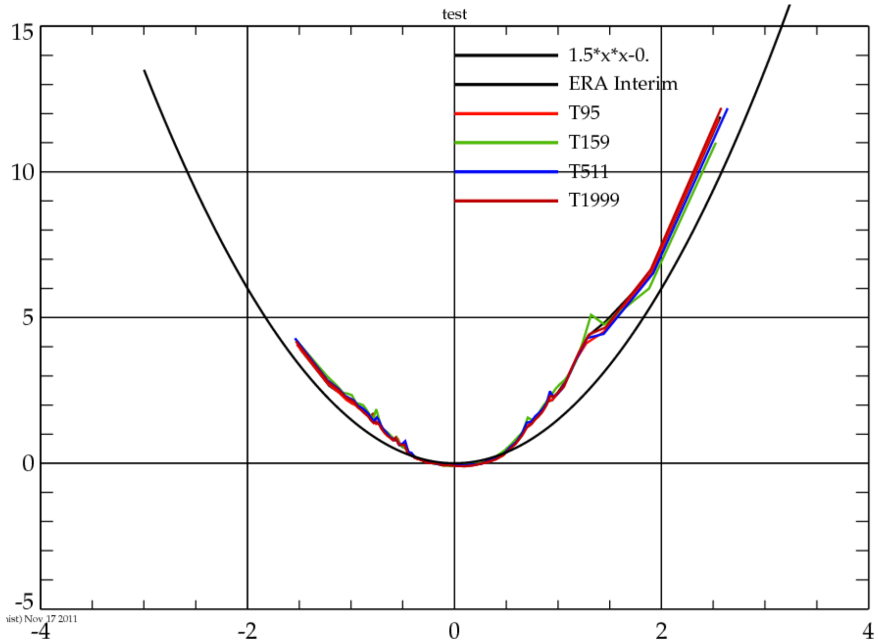
NOTE THAT if we change the additive noise forcing by a factor θ,
so that $b \to \theta b$ *and* $g \to \theta g$, *then* $\sigma \to \theta\sigma$, *but S and K are unchanged !*

Consistent with the simple theory of "SGS" probability distributions, the K-S curve is indeed almost identical for different model resolutions !

Kurtosis



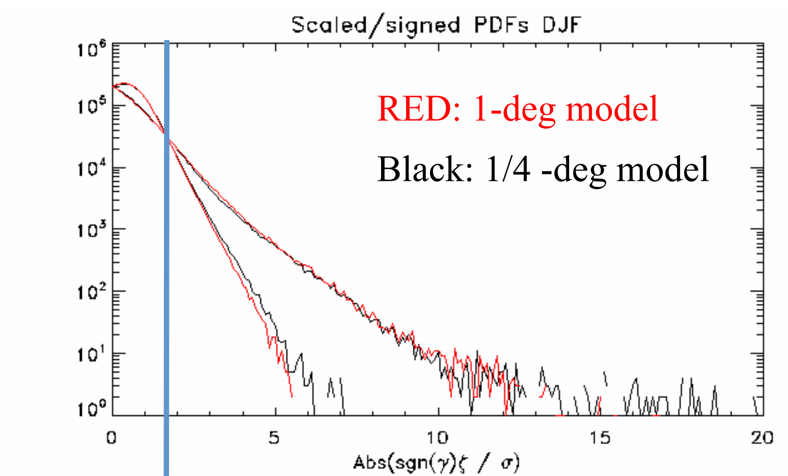
Skewness

This result raises the important issue of whether one needs ultra-high resolution models to represent the *shapes* of the observed non-Gaussian PDFs, given that even a low-resolution T95 (~100 km) model can already capture them,

and that even in the T95 model the non-Gaussianity is effectively due to "SGS" dynamics.

Essentially similar results are obtained from NCAR/CAM5-SE AMIP runs for 1979-2012 at ~ 1-degree and ~ 1/4-degree resolutions

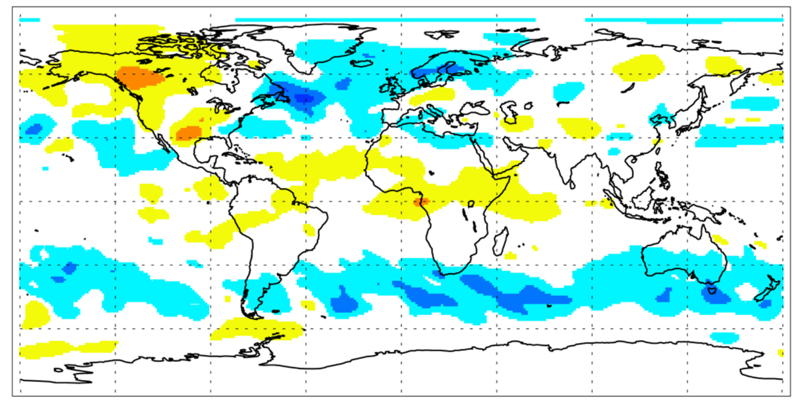
Globally averaged histograms of standardized positive and negative 200 mb Vorticity anomalies



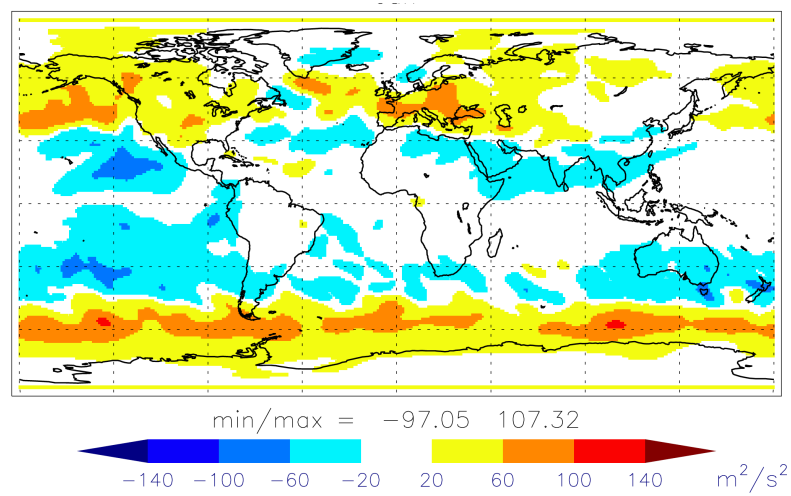
Consistent with SGS distribution theory, the point where p(x) = p(-x) is near 1.7σ

200 mb Transient Kinetic Energy error (Jan)

1-degree (~ 100 km) model

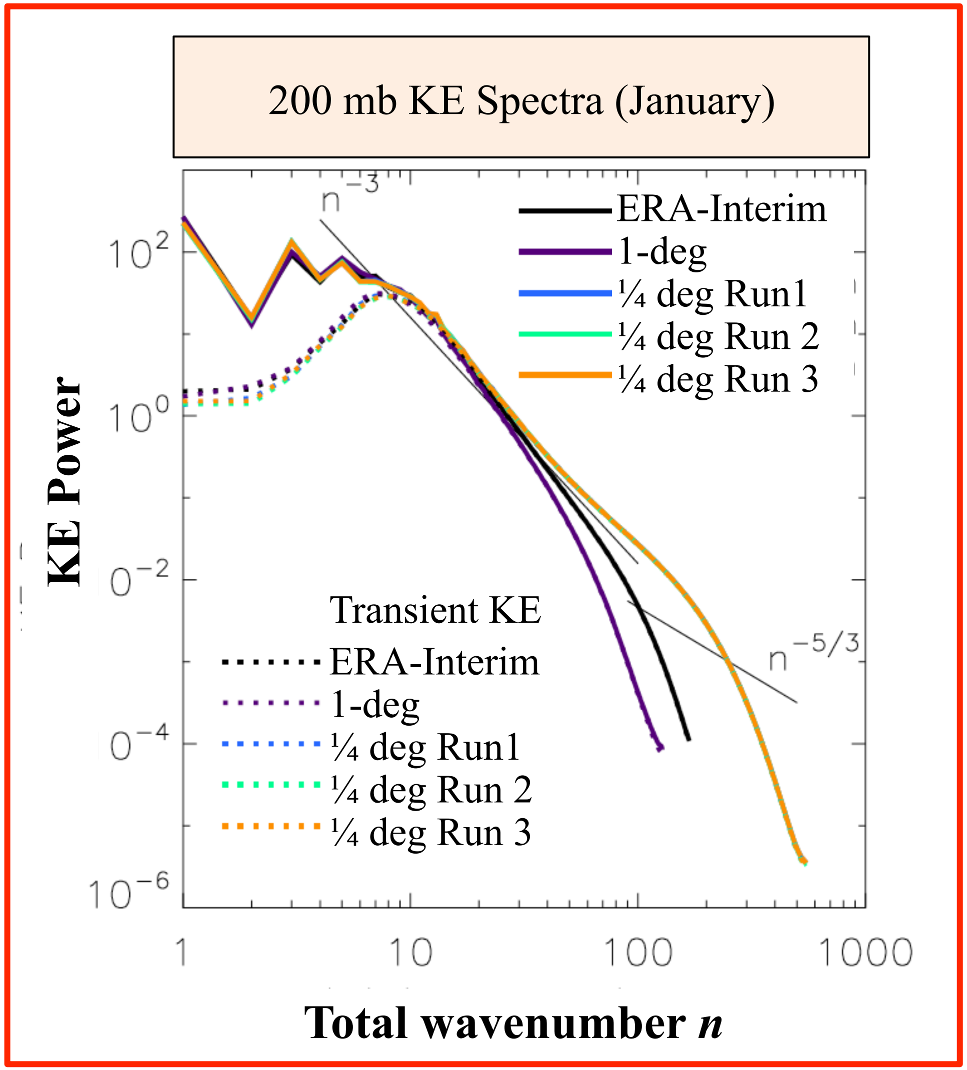


1/4-degree (~ 25 km) model



Julio Bacmeister NCAR

The 200 mb Kinetic Energy spectra are very different at subsynoptic scales in the NCAR ~ 1 deg and ~ ¼ deg models and in the ERA-Interim (~0.5 deg) reanalyses



The -3 power-law decay of the spectra is well understood, but deviations from it are not

2 - D Turbulence :

$$\frac{\partial \xi}{\partial t} = -\nabla \cdot (\mathbf{v}\xi) \qquad \xi = \nabla^2 \psi, \ \mathbf{v} = \hat{e} \times \nabla \psi$$

$$\rightarrow E_n \propto n^{-3} \qquad \text{for large } n$$

Diffusive, Stochastically forced approximation :

$$\frac{\partial \xi}{\partial t} \approx \nu \nabla^2 \xi + S\eta$$

$$\rightarrow E_n \approx \left(\frac{a^4}{\nu} S^2\right) n^{-3} \quad \text{for large } n$$

Impact of other possible stochastic forcings :

If $S^2 = S_0^2 + n(n+1)S_1^2$ is a mixture of forcing that is white in the vorticity and energy norms, then

$$E_n \approx \left(\frac{a^4}{\nu} S_0^2\right) n^{-3} + \left(\frac{a^4}{\nu} S_1^2\right) n^{-1} \quad \text{for large } n$$

will be shallower than an n^{-3} spectrum.

A minimal model of the KE spectrum (Leith 1996) and scale-dependent predictability

$$\frac{\partial q}{\partial t} = \nu\nabla^2 q - \alpha q + S\eta$$

diffusion — damping — stochastic forcing

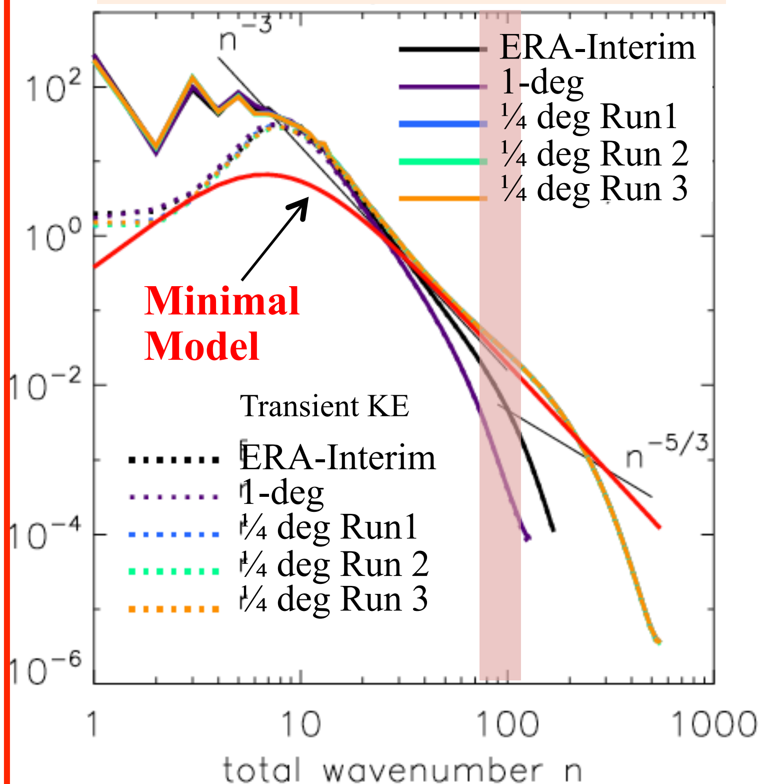
$q = \nabla^2\psi - (1/L_R^2)\psi, \quad |\mathbf{v}|^2 = |\nabla\psi|^2$

ν = diffusion coefficient (2-hour damping at $n = 100$)

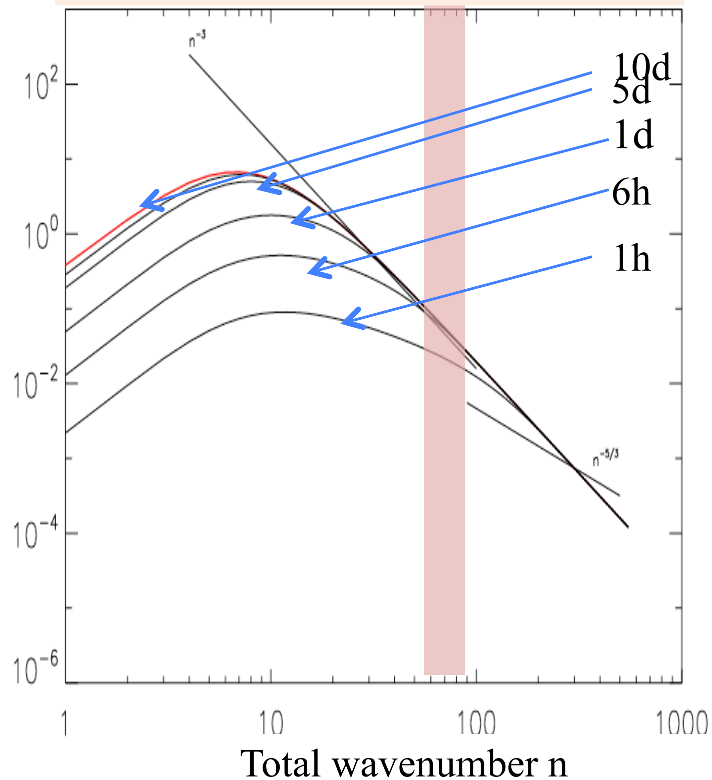
α = 15-day damping

L_R = Rossby deformation radius = 1000 km

200 mb KE Spectra (January)



Scale-dependent Predictability



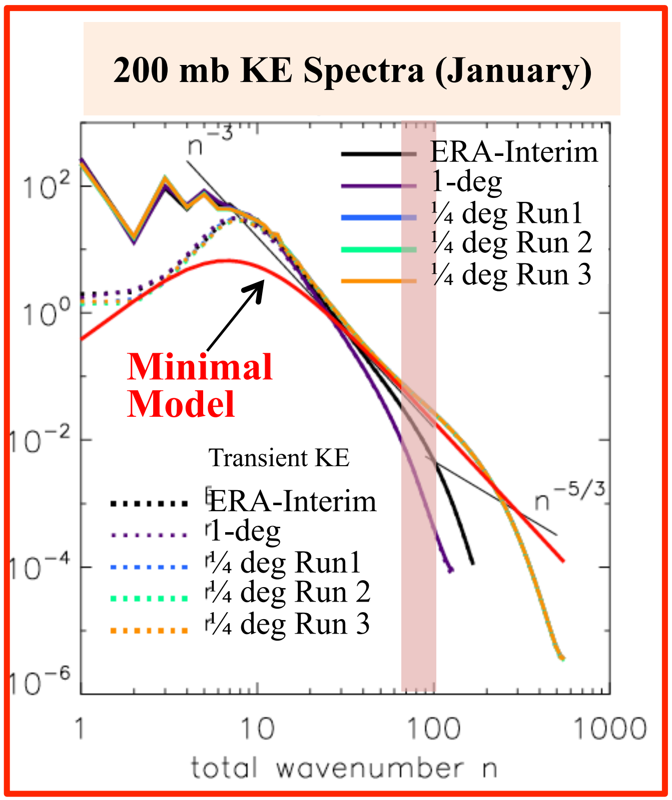
Evolution of spectrum from a 0 initial condition.

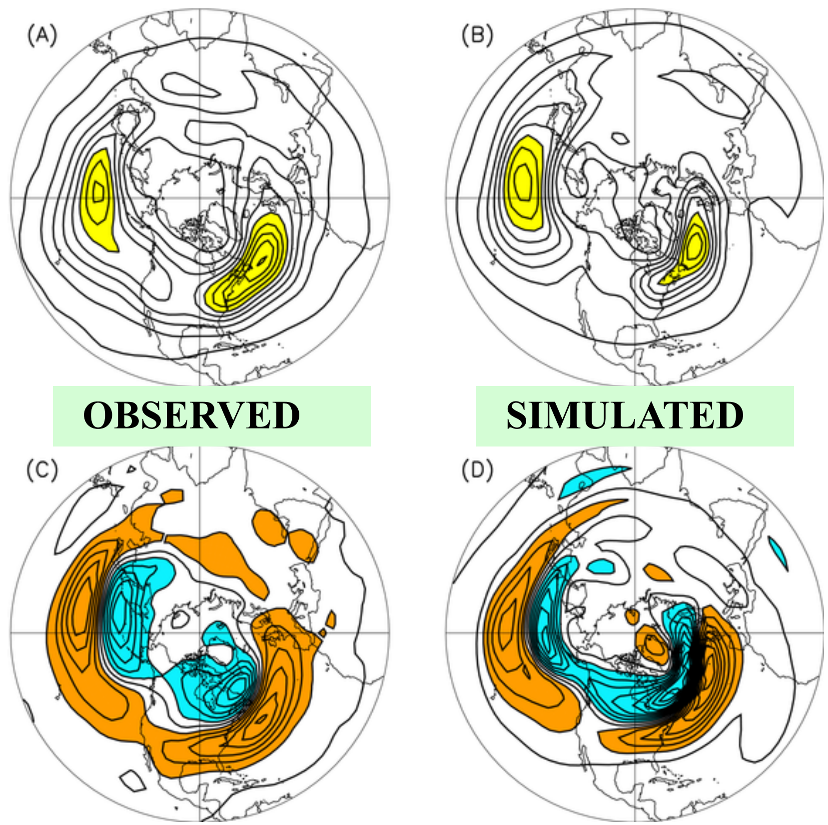
The rapid saturation at small scales is consistent with Lorenz (1969)

Understanding the deviations of the KE spectrum from the minimal model

Atmospheric variability is not homogeneous and isotropic !

On synoptic and larger scales, interactions with the mean state are obviously important. However, the essence of these interactions is captured by even a very low-resolution (T31) linear stochastically forced 2-level QG model linearized about the observed mean state.





Synoptic eddy variance

Synoptic eddy feedback on mean flow

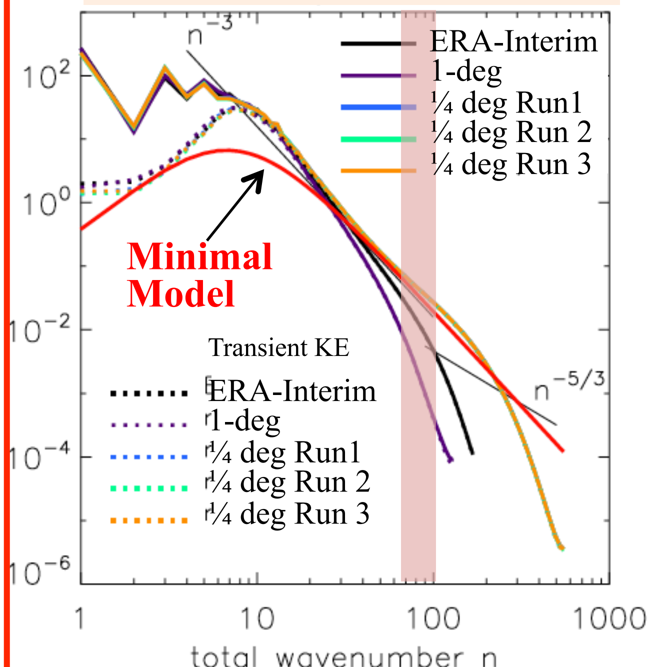
Whitaker and Sardeshmukh 1998

Understanding the deviations of the KE spectrum from the minimal model

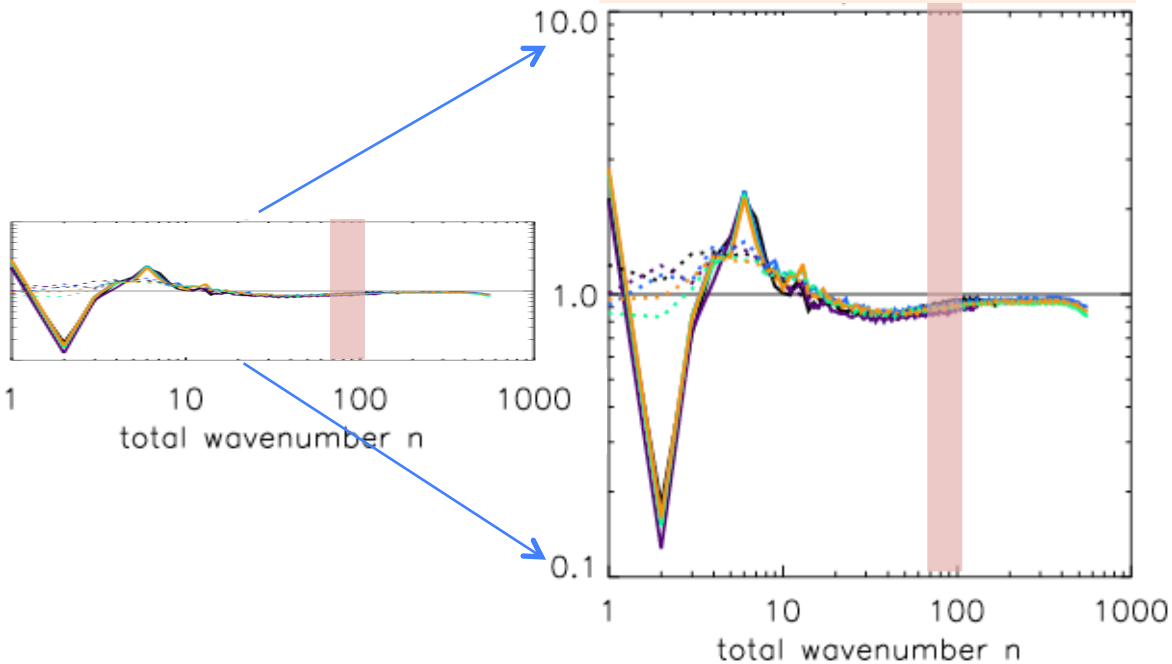
Atmospheric variability is not homogeneous and isotropic !

On synoptic and larger scales, interactions with the mean state are obviously important. Not surprisingly, the January/July spectral ratios differ from 1 on these scales. More surprisingly, the ratios on sub-synoptic scales are not constant as one might expect from turbulence cascade theory.

200 mb KE Spectra (January)



Ratio January/July

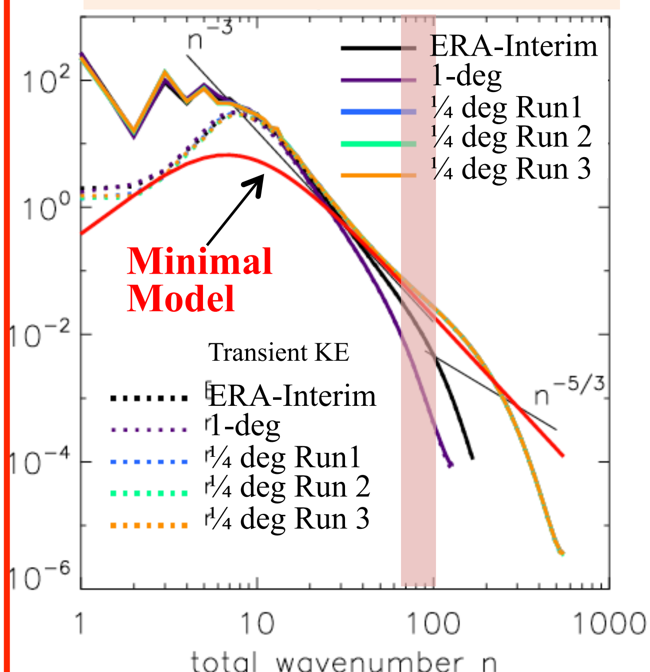


Understanding the deviations of the KE spectrum from the minimal model

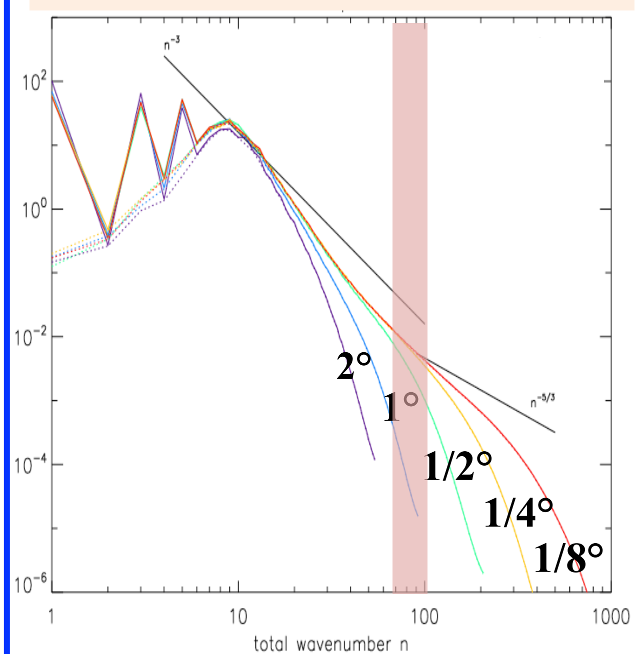
Atmospheric variability is not homogeneous and isotropic !

On sub-synoptic scales, a fully nonlinear but simple dry adiabatic GCM (Held and Suarez 1994) relaxed to a prescribed zonally symmetric temperature field, and run at progressively higher resolutions, is unable to capture a flatter than -3 slope even at the highest 1/8-degree resolution.

200 mb KE Spectra (January)



200 mb KE Spectra (simple GCM)



Note that the 1/4 and 1/8 degree models have nearly identical spectra for *n* < 100

Understanding the deviations of the KE spectrum from the minimal model

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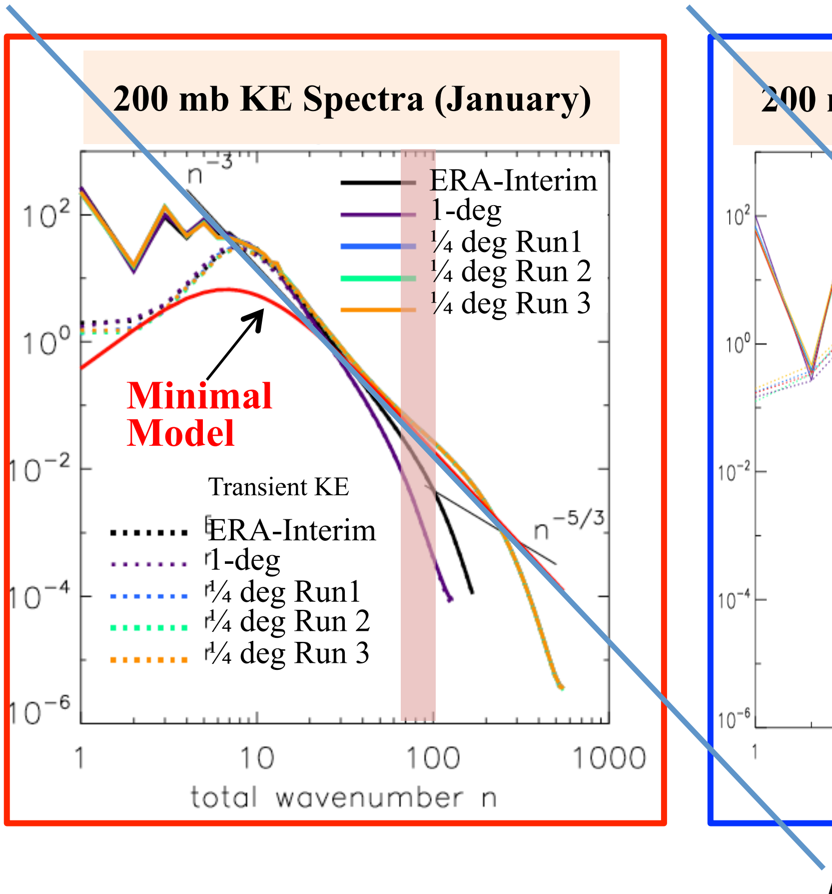
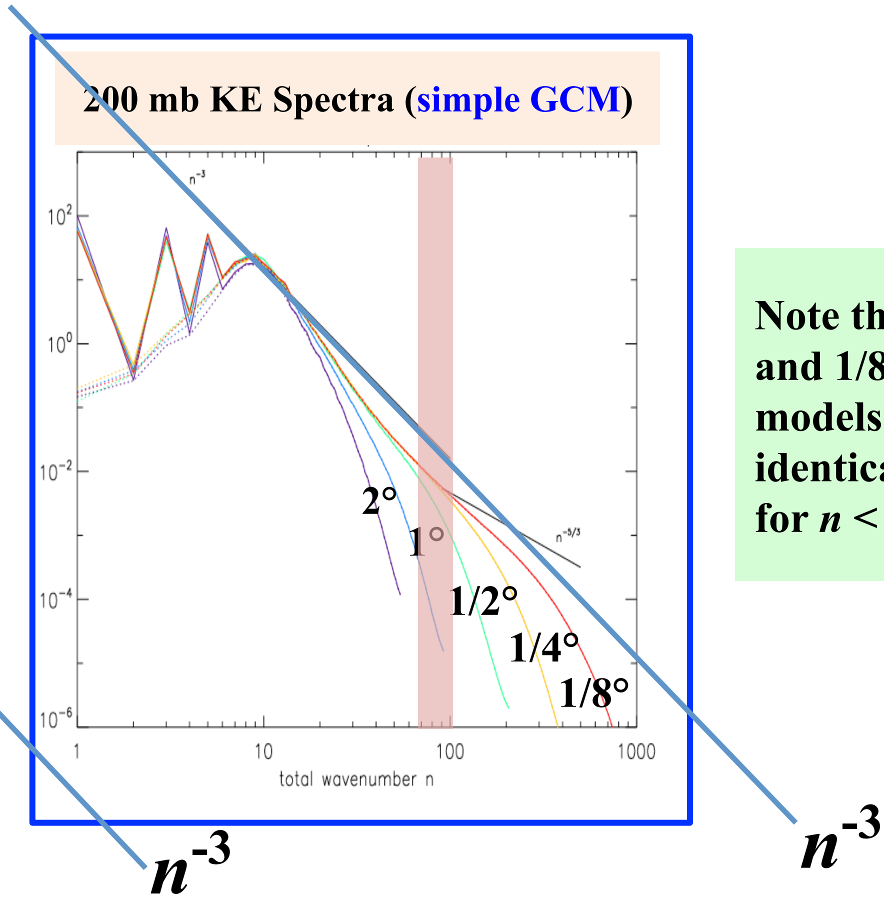
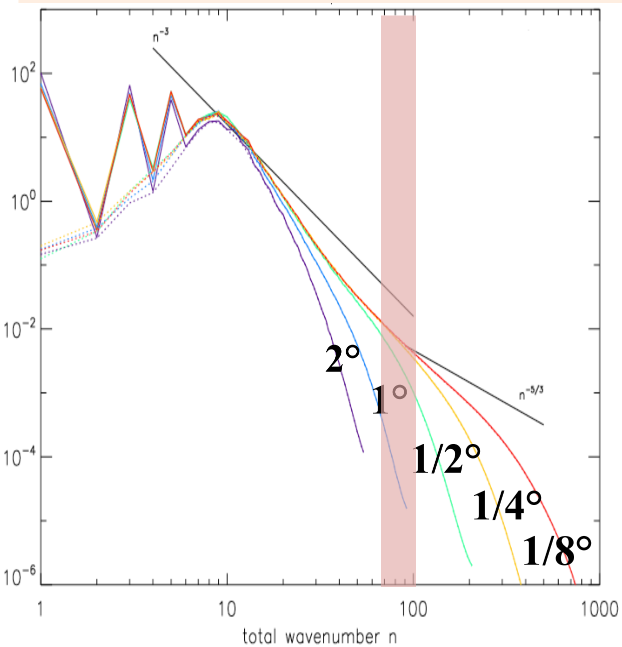




Note that the 1/4 and 1/8 degree models have nearly identical spectra for $n < 100$

In the simple GCM, the small scale variability has a similar character to large scale variability, with maxima in middle latitudes and minima in the tropics

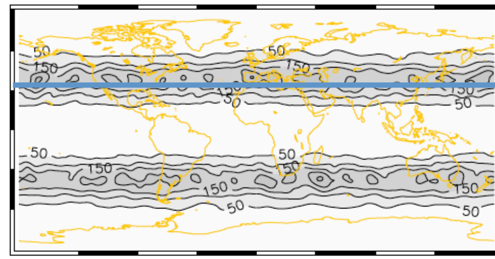
200 mb KE Spectra (simple GCM)

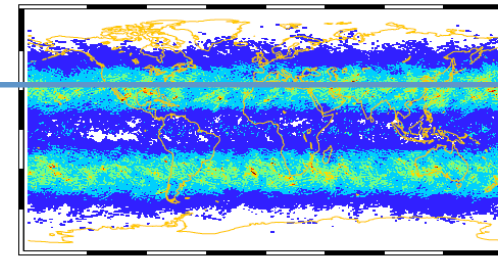


KE of large-scale transients
(1 < *n* < 63)

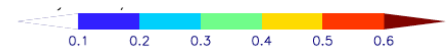
KE of smaller-scale transients
(64 < *n* < 100)

1/4-degree (~ 25 km) simple GCM



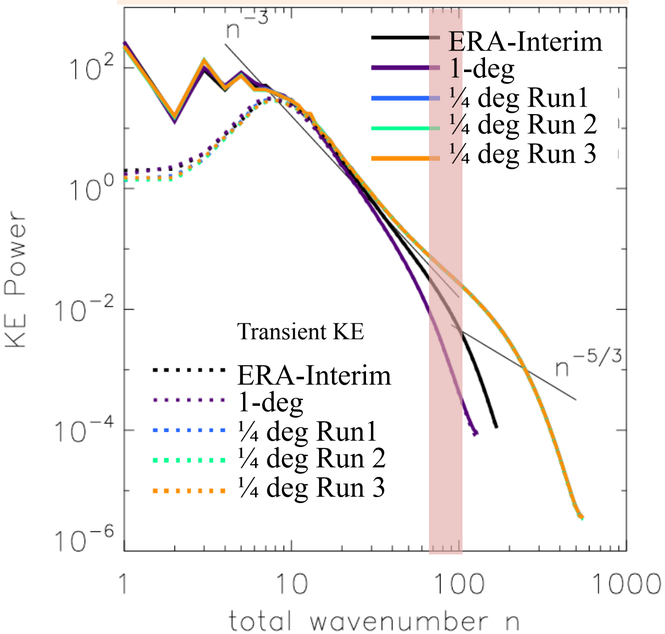


Contour Interval 50 m^2/s^2



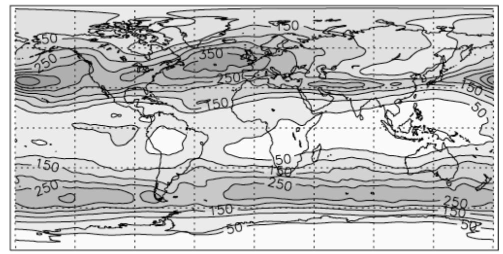
In the full GCM, the small scale variability has a different character from large scale variability, and is strongly associated with small scale diabatic processes missing in the simple GCM

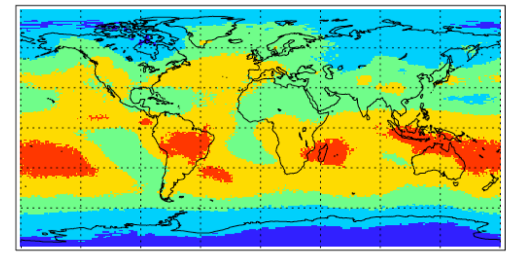
200 mb KE Spectra (Jan)



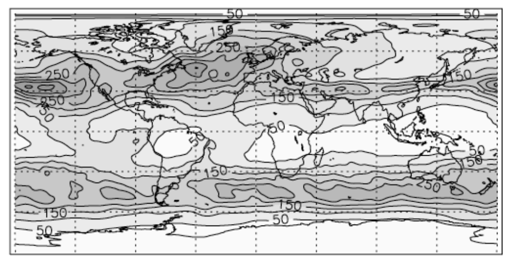
KE of large-scale transients (1 < *n* < 63)	KE of smaller-scale transients (64 < *n* < 100)

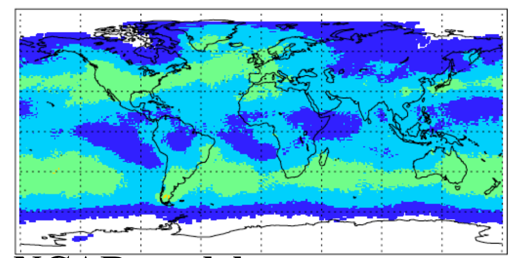
1/4-degree (~ 25 km) NCAR model



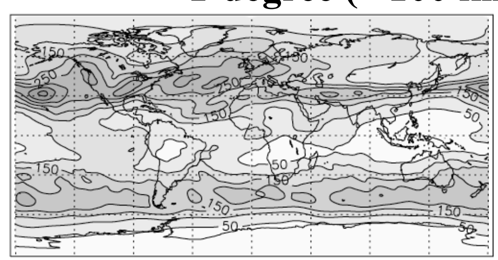


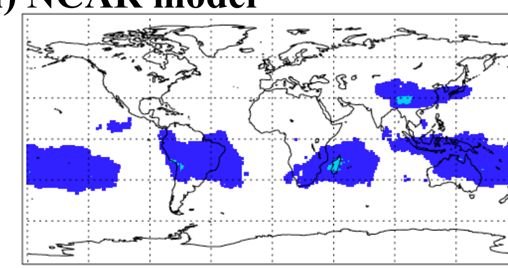
1/2-degree (~ 50 km) ERA-Interim Reanalysis





1-degree (~ 100 km) NCAR model





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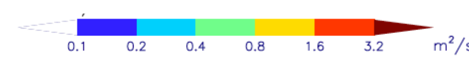
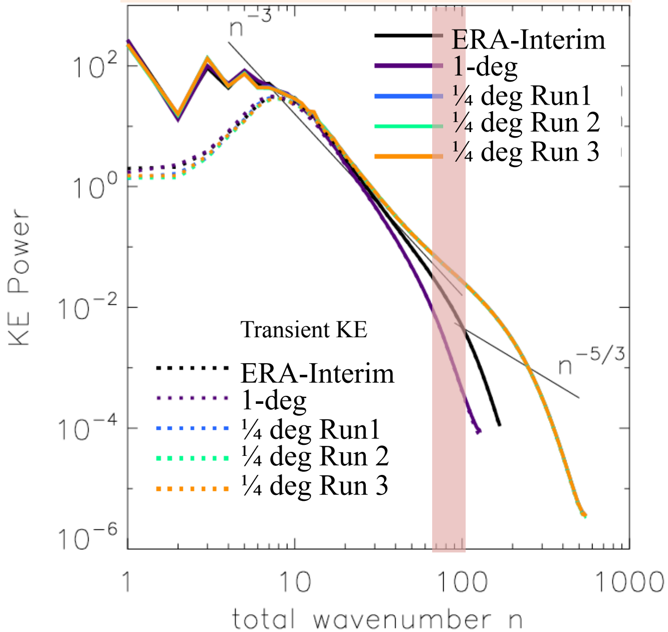


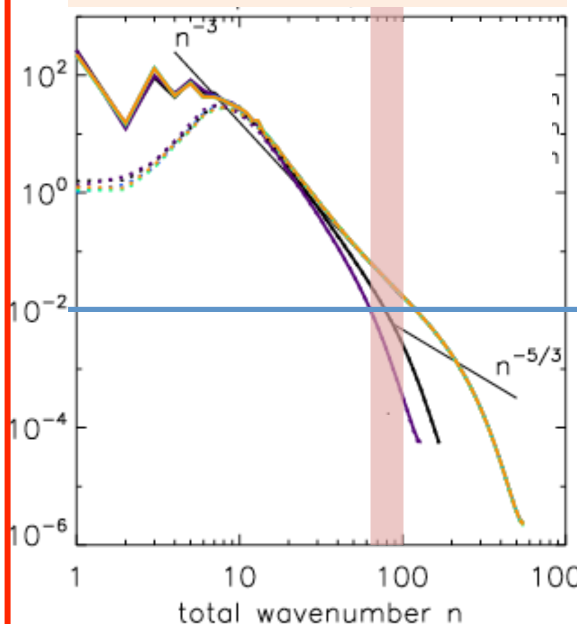
In the full GCM, the small scale variability has a different character from large scale variability, and is strongly associated with small scale diabatic processes missing in the simple GCM

In the full GCM, the diabatic processes contribute to the flatter sub-synoptic spectrum through an enhanced divergent KE contribution

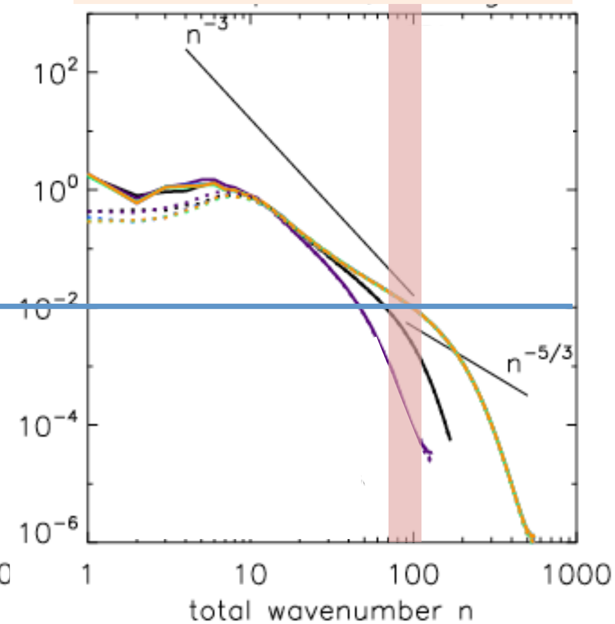
200 mb KE Spectra (Jan)



Rotational KE Spectra



Divergent KE Spectra



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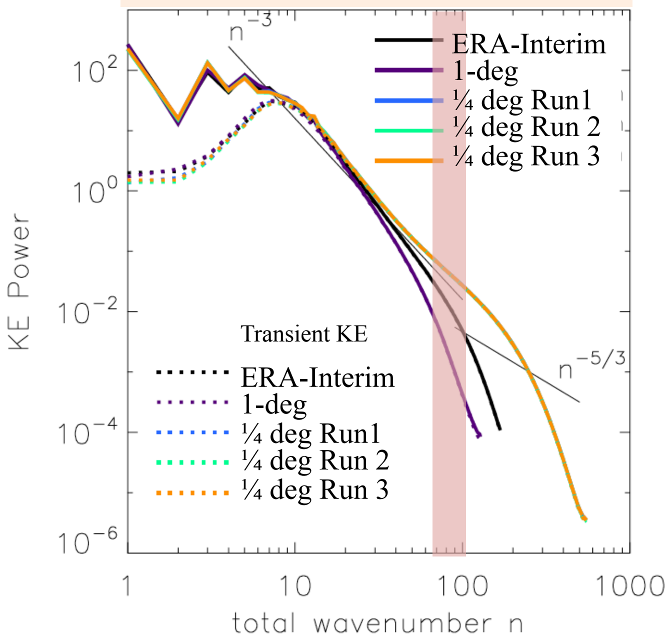
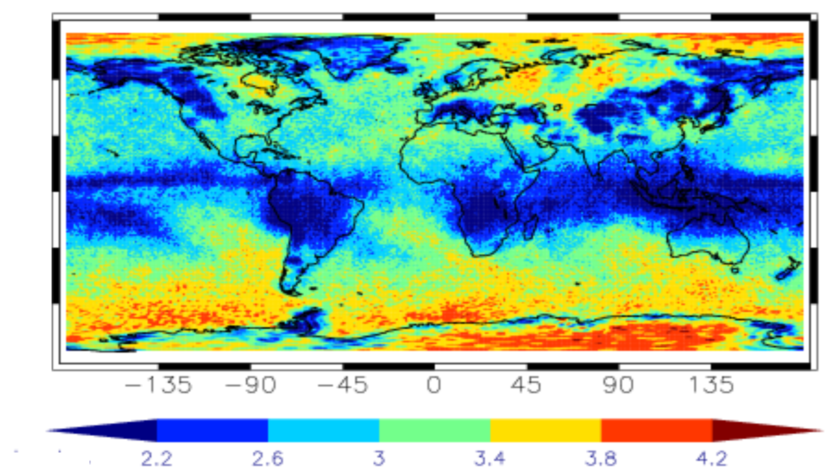
In the 1/4-degree full GCM, the impact of diabatic and (and to some extent, orographic) processes on the spectrum is evident in the smaller slopes α of the local spectra $n^{-\alpha}$

200 mb KE Spectra (Jan)



Local slopes α at $n = 100$

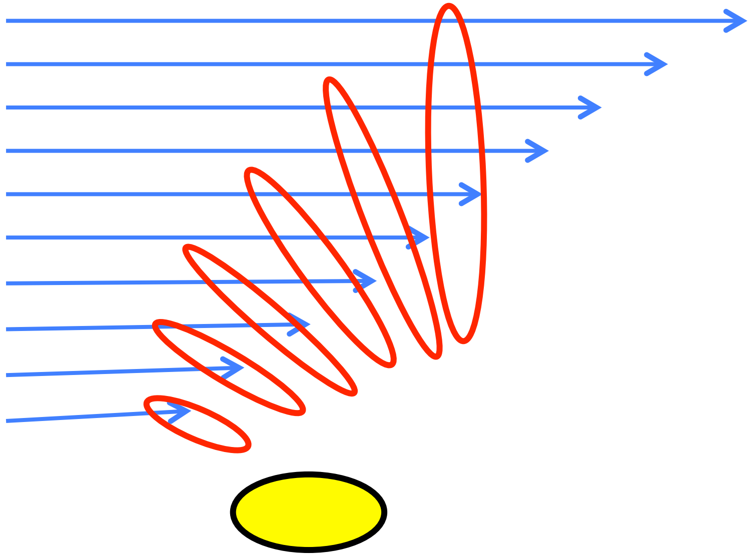


The relative flatness of the global sub-synoptic spectrum results from averaging the disparate local spectra, and not from a single scale-interaction process

Question : To what extent is the "Nastrom-Gage Spectrum" an average of such disparate spectra ?

An important LINEAR mechanism of generating a flatter sub-synoptic spectrum :

Upscale non-modal eddy energy growth by energy extraction from background shears



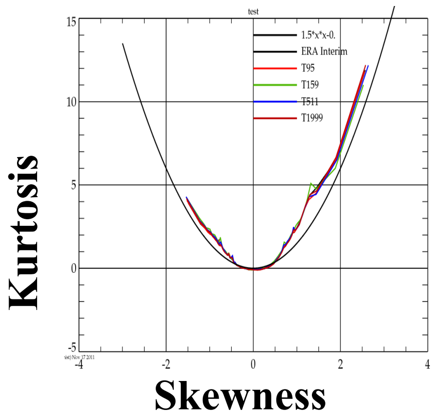
1. This process can be efficiently excited by diabatic forcing

2. It is sensitive to diffusion

These two facts help us understand most of the sensitivity of sub-synoptic variance to model resolution shown in this talk

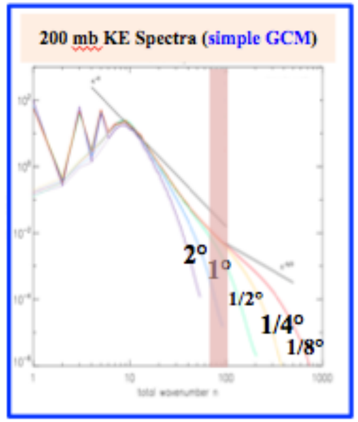
Orr 1907, Buizza and Palmer 1995, Sardeshmukh et al 1997

In summary, a moderately high model resolution of ~T500 (~ ¼ degree) may be sufficient for probabilistic predictions of large-scale anomalies (n < 100) because . . .



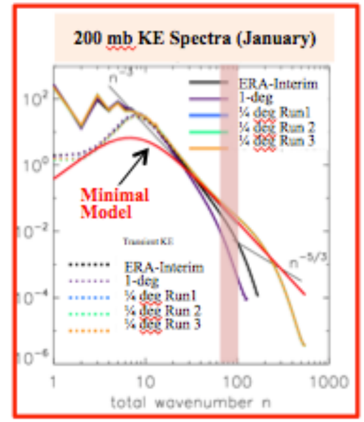
1

The *shapes* of the pdfs are highly insensitive to changes of resolution.



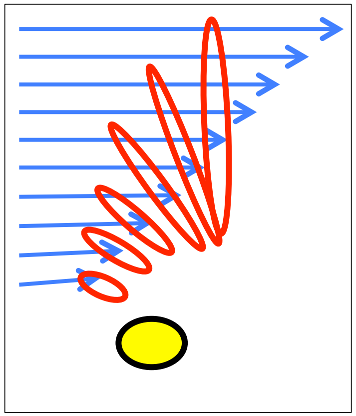
2

The *widths* of the pdfs due to adiabatic processes are insensitive to higher resolution.



3

The *widths* of the pdfs due to small scale diabatic processes are more sensitive, but are basically stochastically generated.



4

Feedbacks from both small scale adiabatic and diabatic processes are sensitive to diffusion.

It comes down to having the correct balance between the diffusive and diabatic heating tendencies of small-scale eddies. There is no basic reason why such a balance cannot be achieved, at least statistically, in lower resolution models.

Additional Slides

Summary

1) Since predictions beyond synoptic time scales are inherently probabilistic, models need to represent the mean, width, and shape of the changing PDFs, which are generally not Gaussian. **But is ultra-high resolution necessary for this purpose**?

3) Intercomparison of ECMWF atmospheric models at resolutions ranging from 120 km to 6 km shows that **the distinctive *shapes* of the observed PDFs are remarkably well captured at all resolutions**, **but their *widths* are not**. Similar results are obtained for NCAR models at 100 and 25 km resolutions.

5) This behavior is explained by the simple theory of Stochastically Generated Skewed "SGS" PDFs, which suggests that **higher model resolution simply enhances the effectively stochastic forcing (thereby increasing variance and shallowing the KE spectrum) without changing the PDF shape.** This additional stochastic forcing is apparently mostly associated with small scale diabatic processes.

6) It may thus not be necessary to use ultra-high resolution models to predict changes of PDFs. Lower resolution models with a suitably "scale aware" combination of deterministic and stochastic parameterizations may suffice.

Can one reproduce not only the *stationary PDFs* with lower resolution models, but also the *conditional PDFs* given an initial condition or a change in forcing ?

The conditional PDF $p(\mathbf{x},t\,|\,\mathbf{x}_0,0)$ of a multivariate nonlinear stochastically driven system

$\frac{d\mathbf{x}}{dt} = \mathbf{A}(\mathbf{x},t) + \mathbf{B}(\mathbf{x},t)\eta$ is governed by the Fokker-Planck equation

$$\frac{\partial p}{\partial t} = -\sum_i \frac{\partial}{\partial x_i}\left[\left(A_i + \frac{1}{2}\sum_j\sum_m \frac{\partial B_{im}}{\partial x_j}B_{jm}\right)p\right] + \frac{1}{2}\sum_i\sum_j\sum_m \frac{\partial^2}{\partial x_i \partial x_j}(B_{im}B_{jm}p).$$

Multivariate linear case: $\frac{d\mathbf{x}}{dt} = \mathbf{A}\mathbf{x} + \mathbf{B}\eta$ $\quad p$ is multivariate Gaussian

Univariate linear case: $\frac{dx}{dt} = -\lambda x + b\eta$ $\quad p$ is univariate Gaussian

stationary pdf: $$p(x) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{x^2}{2\sigma^2}\right) \qquad \sigma^2 = \frac{b^2}{2\lambda}$$

conditional pdf: $$p(x,t\,|\,x_0,0) = \frac{1}{\sigma_t\sqrt{2\pi}}\exp\left(-\frac{\left(x - x_0 e^{-\lambda t}\right)^2}{2\sigma_t^2}\right) \qquad \sigma_t^2 = \sigma^2\left(1 - e^{-2\lambda t}\right)$$

There are infinite combinations of A and B that will reproduce any desired *stationary PDF*. However, only one combination will reproduce the *conditional PDFs*.

High model resolution may not be essential: climate change example

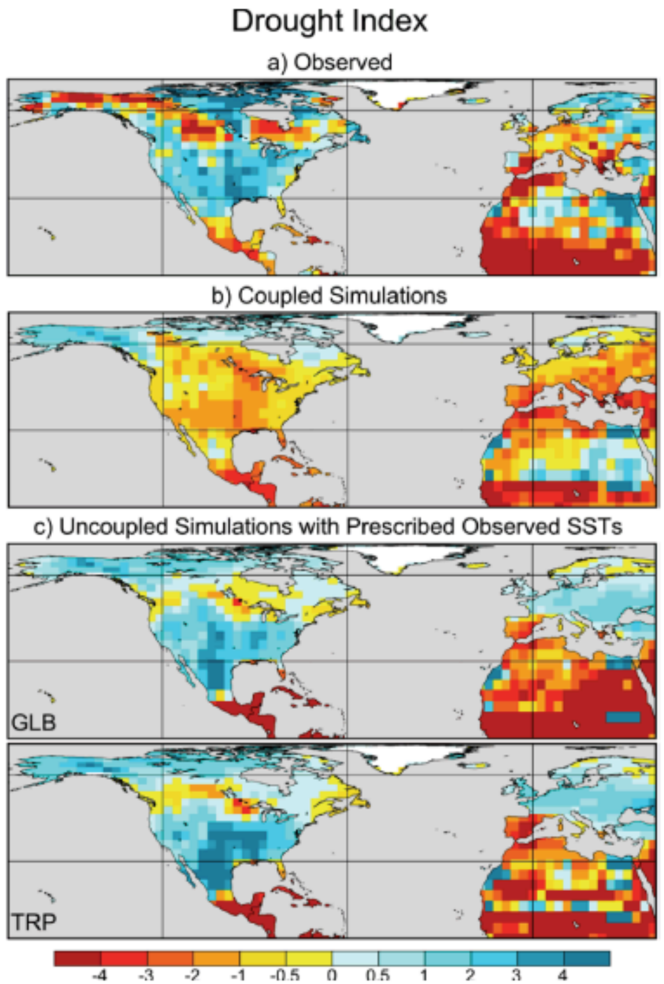
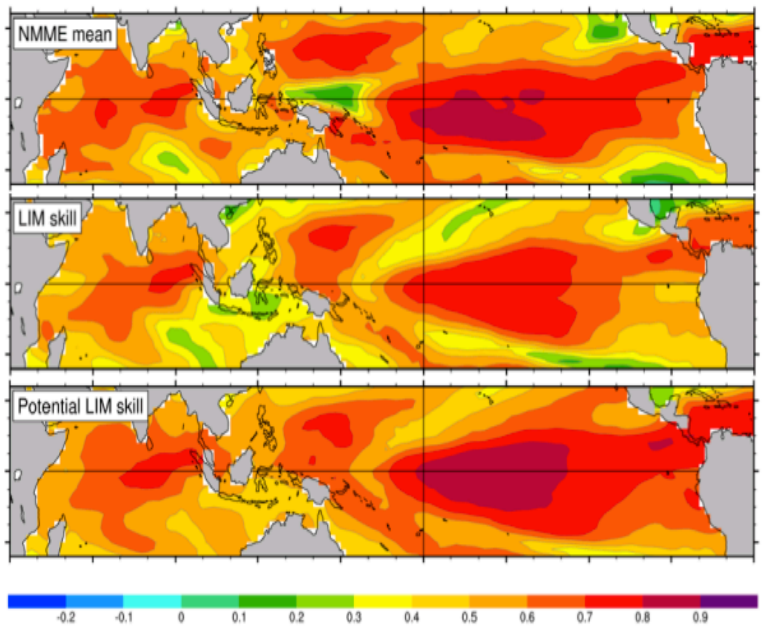
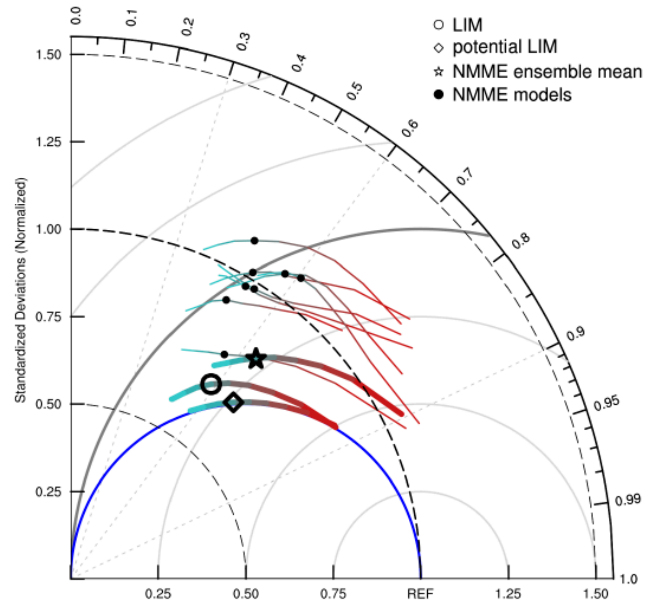


Figure 1. Trends of the annual-mean Palmer Drought Severity Index (PDSI) in non-glaciated regions over 1951–1999, derived from (a) observational datasets, (b) multi-model ensemble mean of coupled simulations forced by observed greenhouse gas and other radiative forcing variations from CMIP3, and (c) multi-model ensemble mean of prescribed SST simulations, with the SST changes prescribed globally (GLB) and only in the tropics (TRP). Warm and cold colors (negative and positive values) indicate a trend towards stronger and weaker droughts, respectively, over this period. (from Shin and Sardeshmukh 2011).

High model resolution may not be essential: seasonal scale example

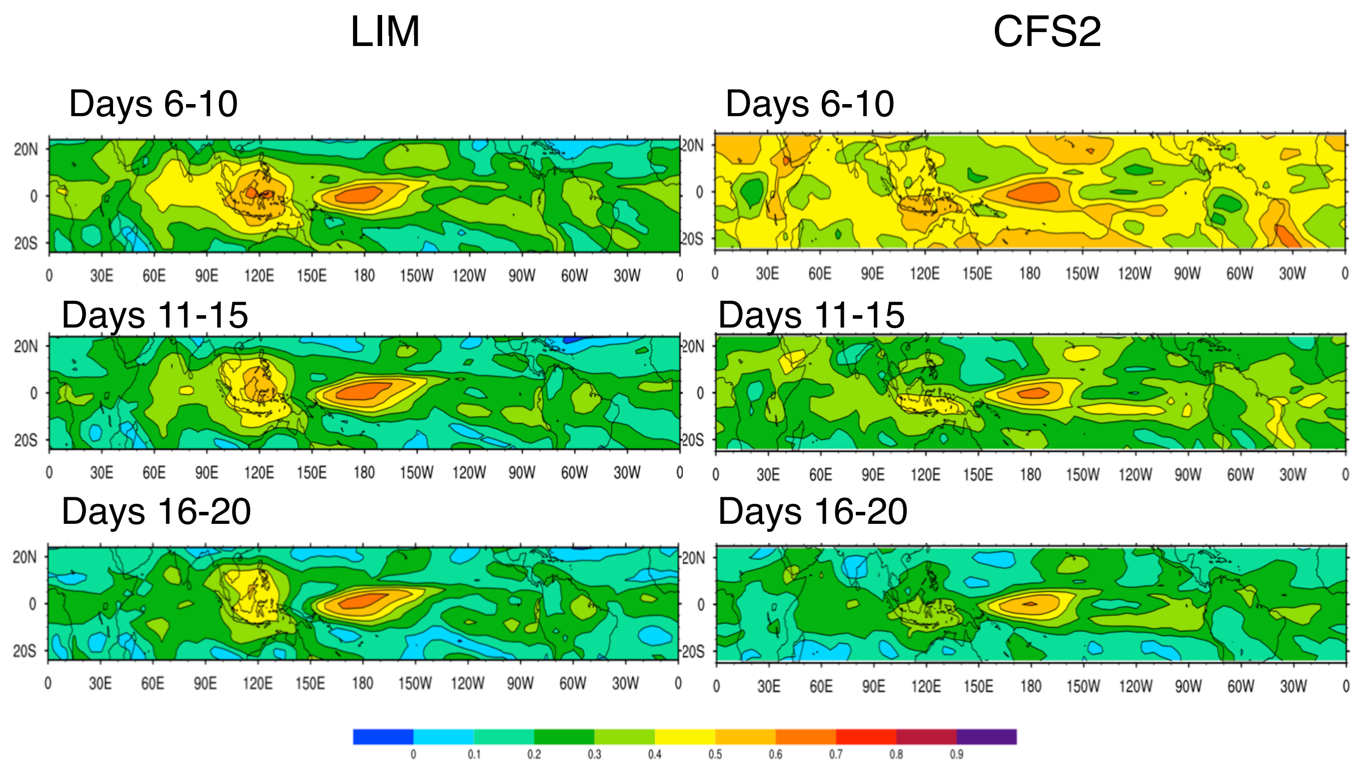




High resolution may not be essential: subseasonal scale example

The subseasonal skill of a low-resolution LIM is comparable to that of the NCEP/CFS2 model.

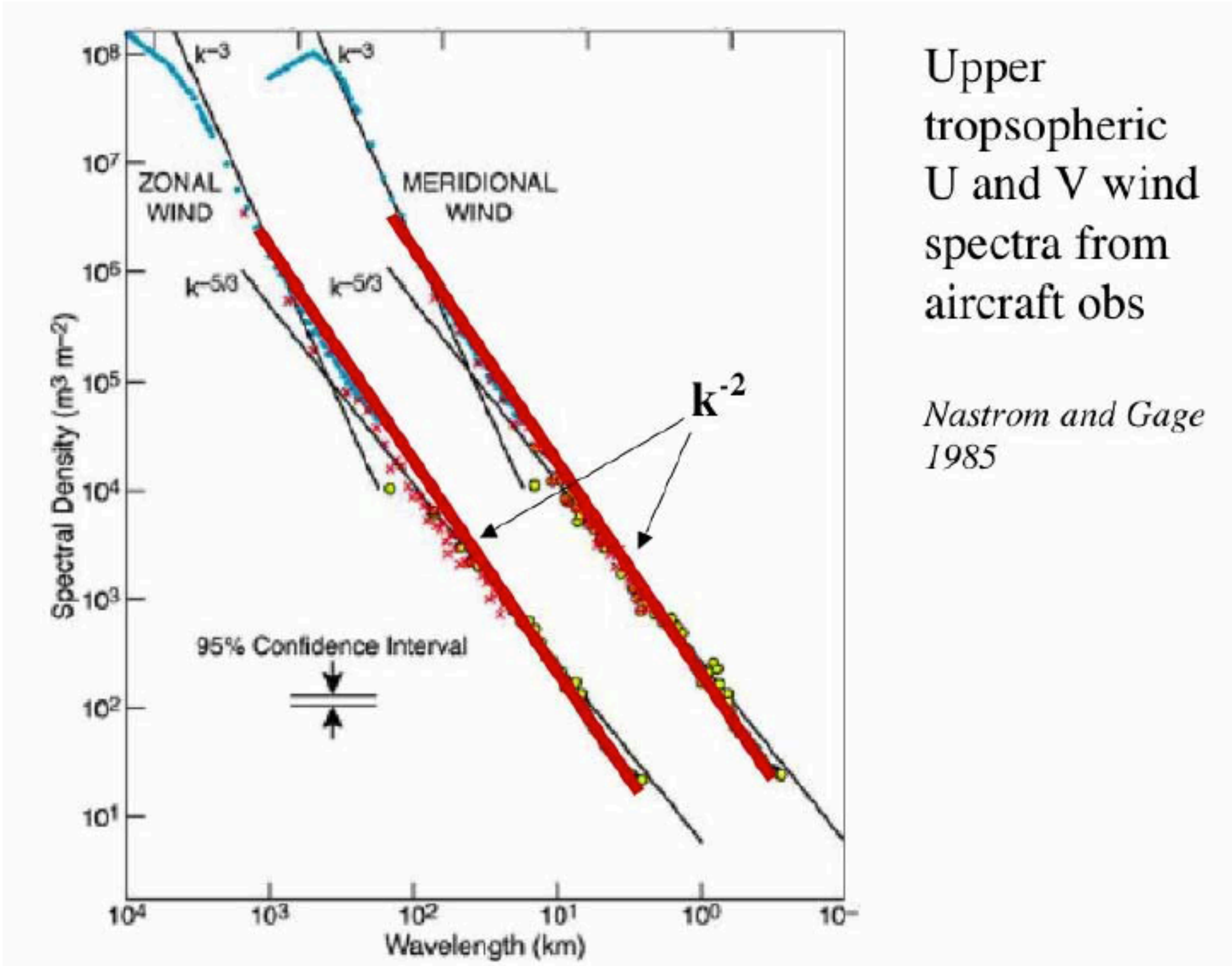
OLR Anomaly Correlation forecast skill, 1999-2009 (Daily start dates)



Note the similarity of the skill patterns

Newman, Sardeshmukh, et al 2017

Nastrom – Gage spectrum



Upper tropsopheric U and V wind spectra from aircraft obs

Nastrom and Gage 1985