Semi-Lagrangian advection, Shape-Preserving and Mass-Conservation

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OUTLINE

- Shape-Preserving and Mass-Conservative semi-Lagrangian advections (Monique)
- Application in the context of the Yin-Yang grid (Abdessamad)
- Application in the context of multi-year simulations with chemistry (Jean)

Semi-Lagrangian advection of Tracer mixing ratio ϕ

$$\phi = \frac{\textbf{Tracer density}}{\textbf{Air density}} = \frac{\rho\phi}{\rho}$$

$$\frac{d\phi}{dt} = \frac{\partial\phi}{\partial t} + u\frac{\partial\phi}{\partial x} = 0$$

Advection equation

$$\frac{x - x_d}{\Delta t} = u\left(x_m, t - \frac{\Delta t}{2}\right)$$

Estimate departure points

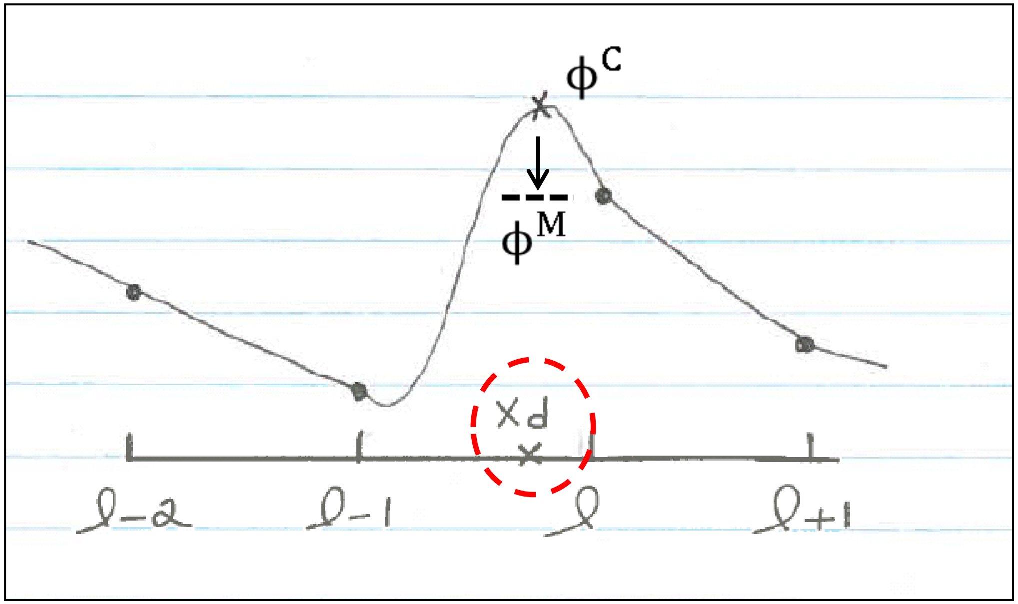
$$\phi(x, t) = \phi(x_d, t - \Delta t)$$

Interpolation at departure points

$$\phi_i = \phi_{\bar{d}}$$

Cubic interpolation and Shape-Preserving for Tracer mixing ratio ϕ

Cubic Interpolation ϕ^C



$$\phi_i^C = \Sigma_l \, \omega_{i\,l} \, \phi_l^-$$

$$\Sigma_l \, \omega_{i\,l} = 1$$

Mono ϕ^M = ϕ^C + Shape-Preserving

$$\phi^M = \max[\phi_l^-, \min(\phi_l^-, \phi^C)]$$

Semi-Lagrangian advection of Tracer density $\rho\phi$

ρ = Air density

$$\frac{d\rho}{dt} + \rho\frac{\partial u}{\partial x} = 0 \quad + \quad \frac{d\phi}{dt} = 0 \quad \Rightarrow \quad \frac{d(\rho\phi)}{dt} + (\rho\phi)\frac{\partial u}{\partial x} = 0$$

Divergence term

Flux form

$$\frac{\partial(\rho\phi)}{\partial t} + \frac{\partial(\rho\phi u)}{\partial x} = 0$$

Local Mass Conservation of Tracer density $\rho\phi$

$$\int_{a(x,t)}^{b(x,t)} \left[\frac{\partial(\rho\phi)}{\partial t} + \frac{\partial(\rho\phi u)}{\partial x}\right] dx = 0$$

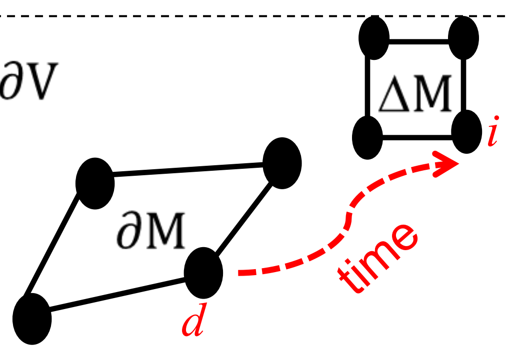
$$\frac{d}{dt}\int_{a(x,t)}^{b(x,t)} \rho\phi \, dx = 0$$

Leibniz integral rule

$$(\rho\phi)\,\Delta V = (\rho\phi)^{-}\,\partial V$$

$$\phi\,\Delta M = \phi^{-}\,\partial M$$

with $\partial M = \rho\partial V$



Global Mass Conservation of Tracer density $\rho\phi$

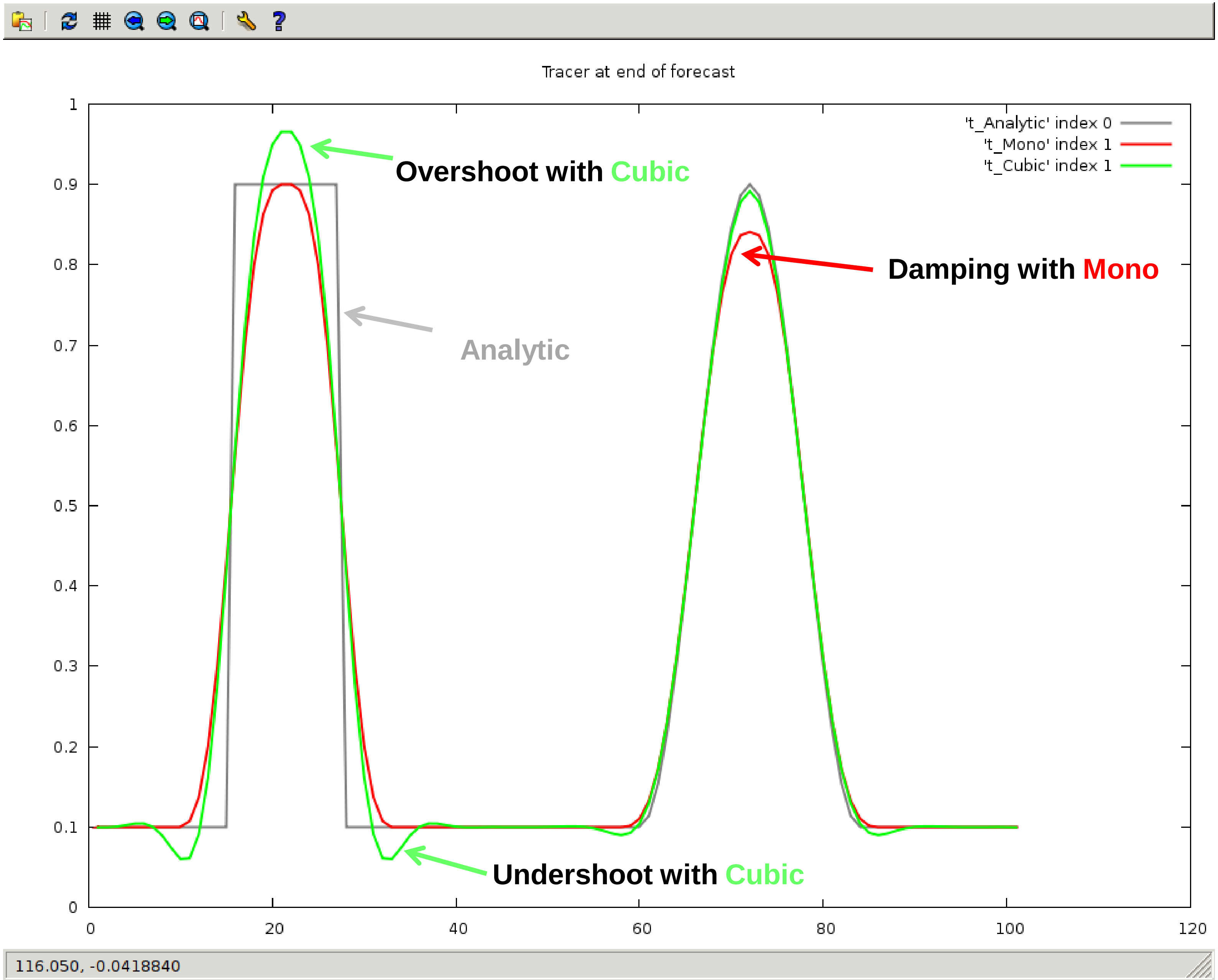
$$\int_{0}^{L} \left[\frac{\partial(\rho\phi)}{\partial t} + \frac{\partial(\rho\phi u)}{\partial x}\right] dx = 0$$

$$\frac{\partial}{\partial t}\int_{0}^{L} \rho\phi \, dx = 0$$

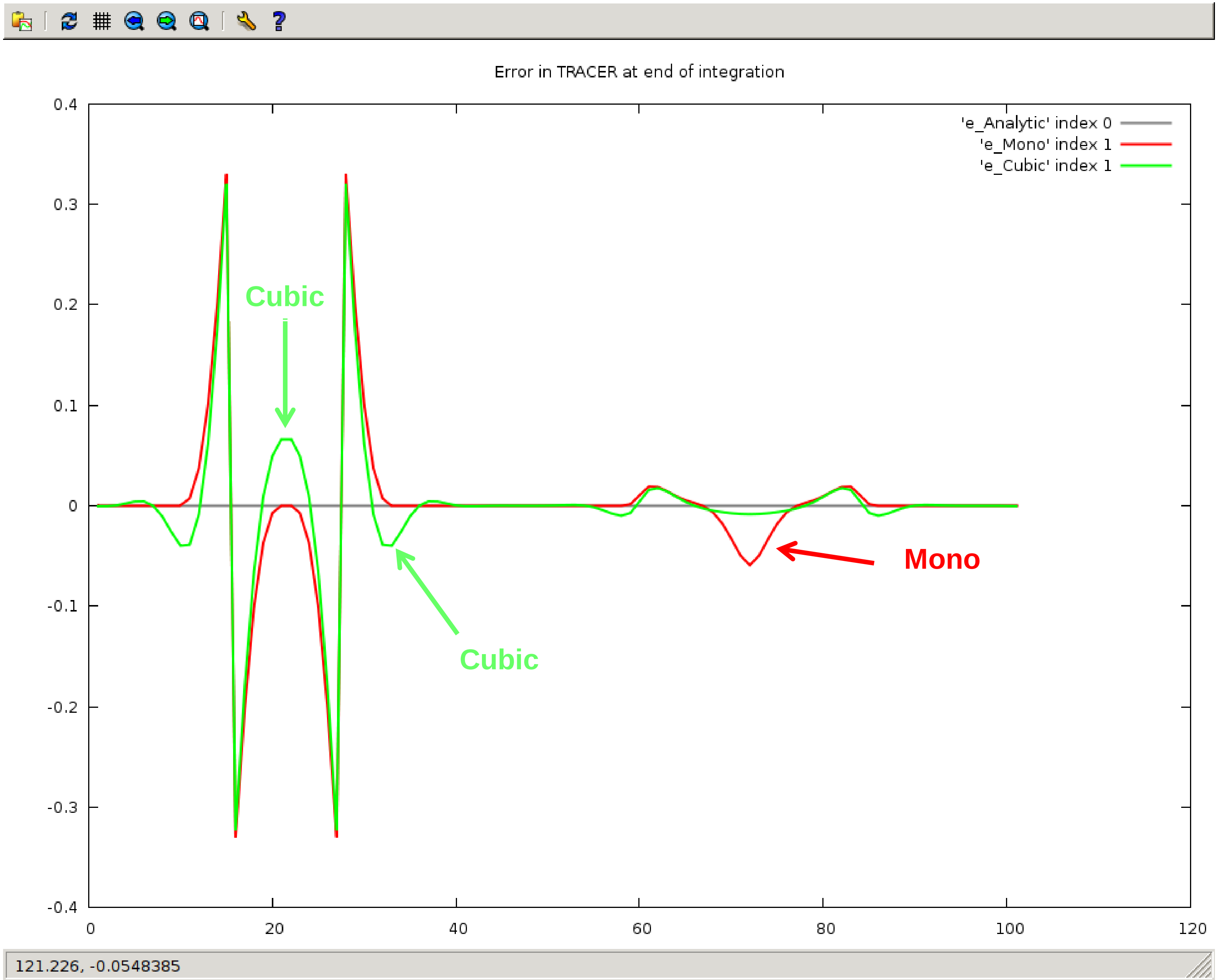
$$\sum_i (\rho\phi)_i \, dV_i = \sum_i (\rho\phi)_i^{-} \, dV_i$$

$$\sum_i \phi_i dM_i = \sum_i \phi_i^{-} dM_i^{-}$$ **with** $dM = \rho dV$

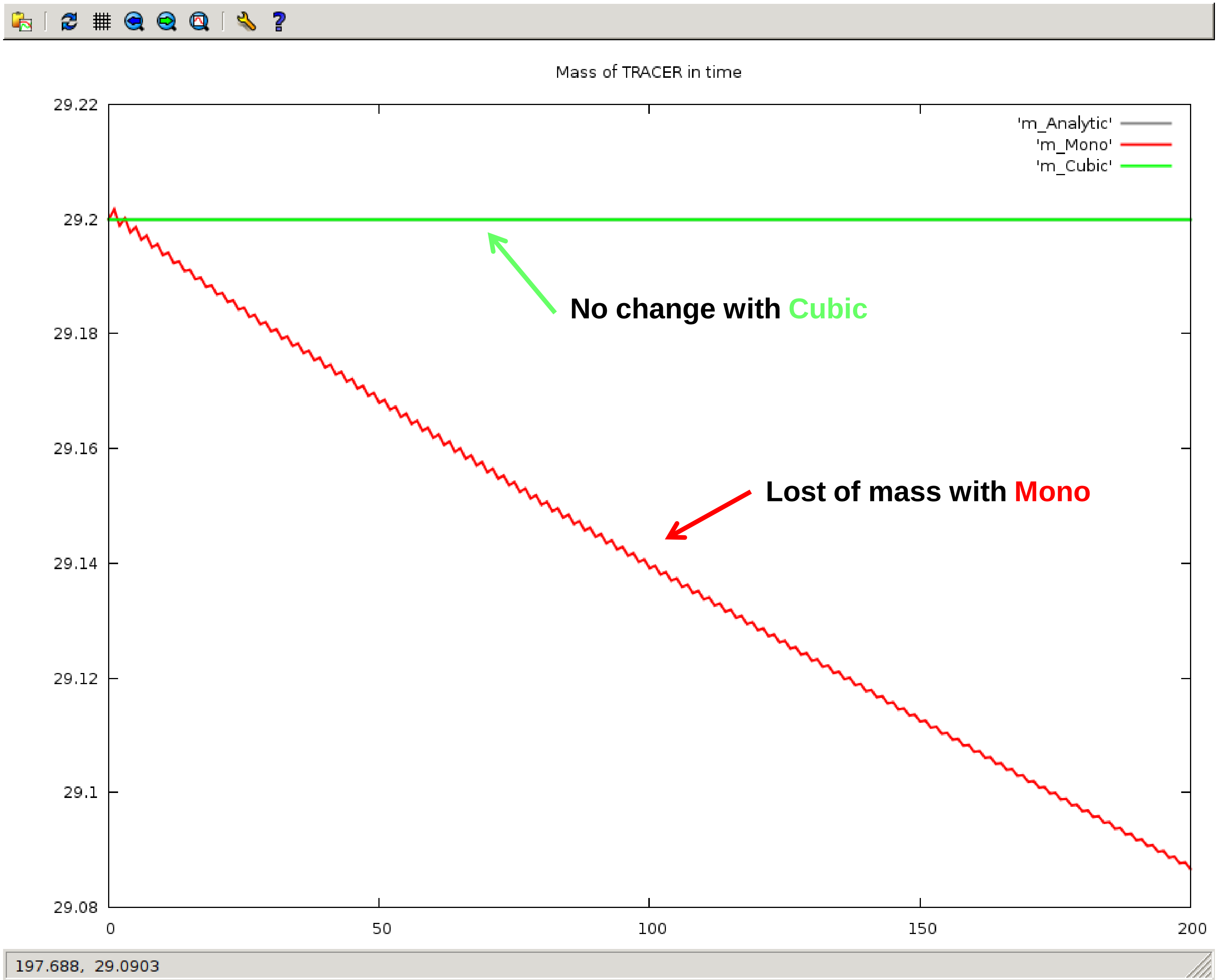
Advection 1D with U=constant: Forecasts after one revolution CFL = 1/2



Advection 1D with U=constant: Errors after one revolution

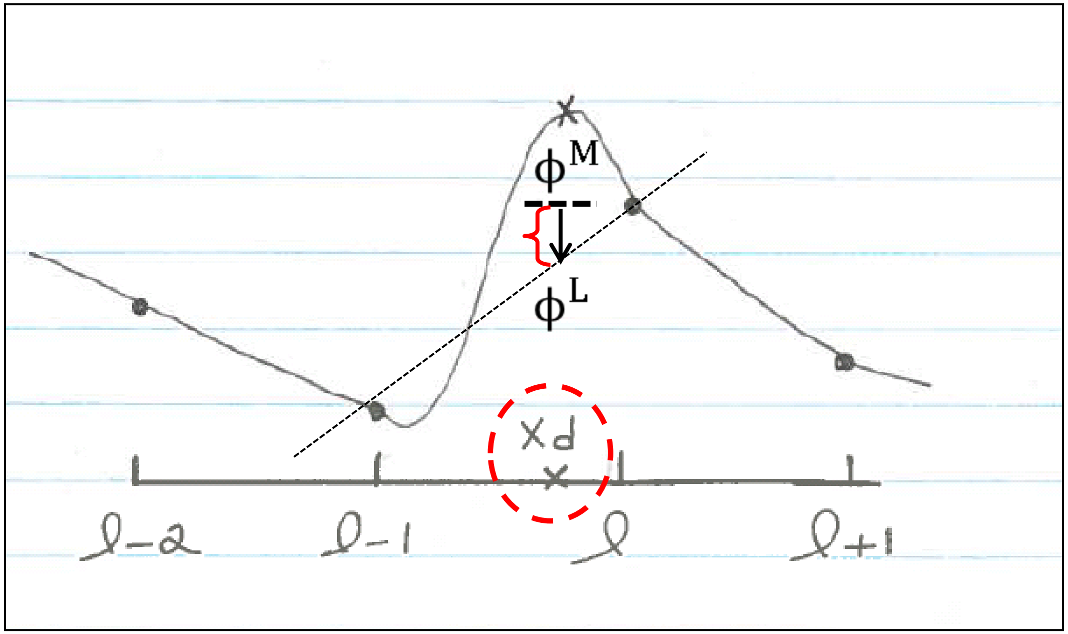


Advection 1D with U=constant: Masses in time



Bermejo-Conde (2002): ϕ^{M}+ Global Mass Conservation

Subtract Mass at all i with $\omega_i > 0$ to correct excess of mass dM in Global Mass



$$\phi_i^{B-C} = \phi_i^M - \lambda\, \omega_i$$

$$\lambda = \frac{dM}{\sum_{\bar{i}} \omega_{\bar{i}}\, (\rho\, V)_{\bar{i}}}$$

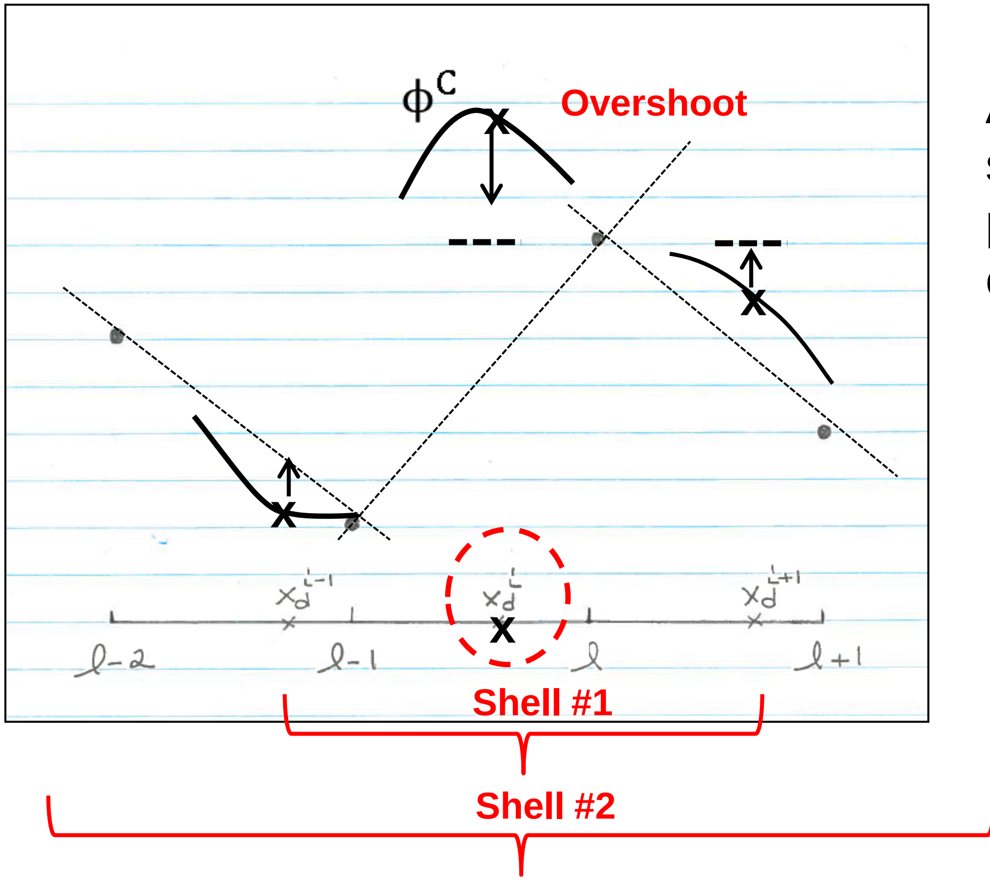
$$\omega_i = \max(0, \mathrm{sgn}(dM)\, , \mathrm{sgn}\,(\phi_i^M - \phi_i^L\,)\,|\,\phi_i^M - \phi_i^L|^p) \quad \text{with } \phi^L \text{ Linear interpolator}$$

The corrections are related to the smoothness of the tracer

See also Gravel and Staniforth (1994)

ILMC in Sorensen et al (2013): ϕ^C + Shape-Preserving + Global Mass Conservation

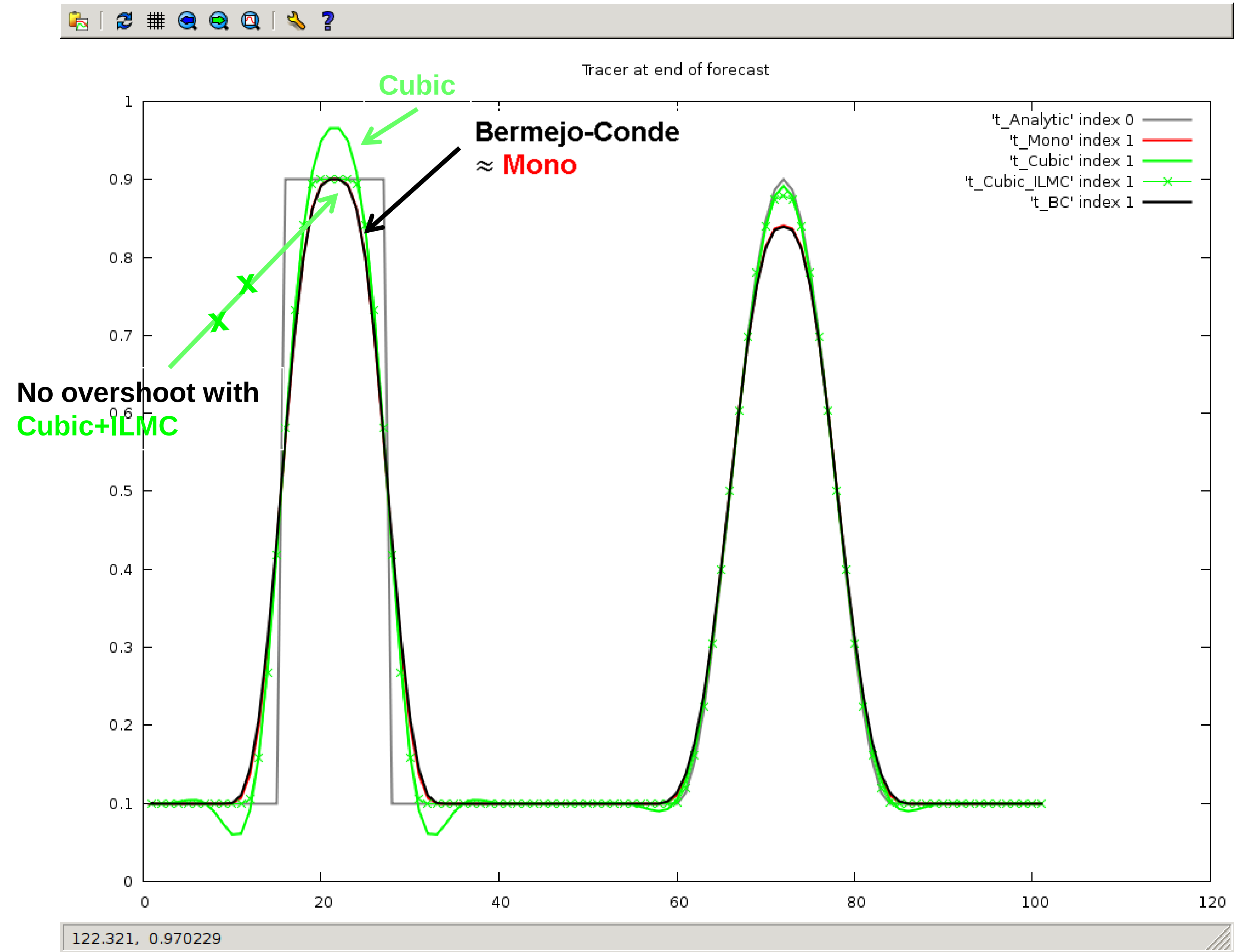
ILMC = Iterative Locally Mass Conserving



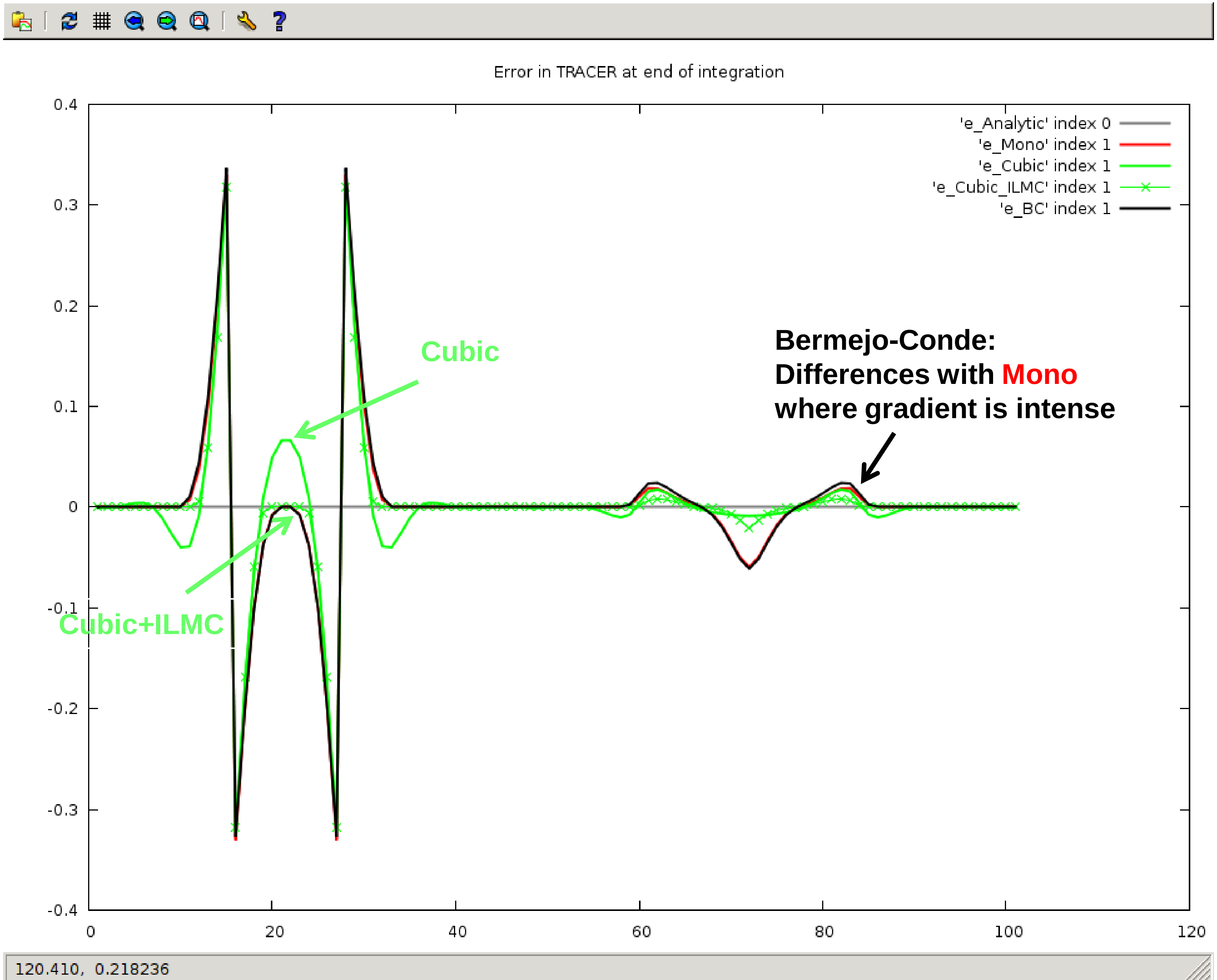
At violation (i.e. Overshoot), subtract iteratively the necessary percentage of mass in the surrounding cells

Iterative filter:
Look into Shell #1, Shell #2, Shell #3 …

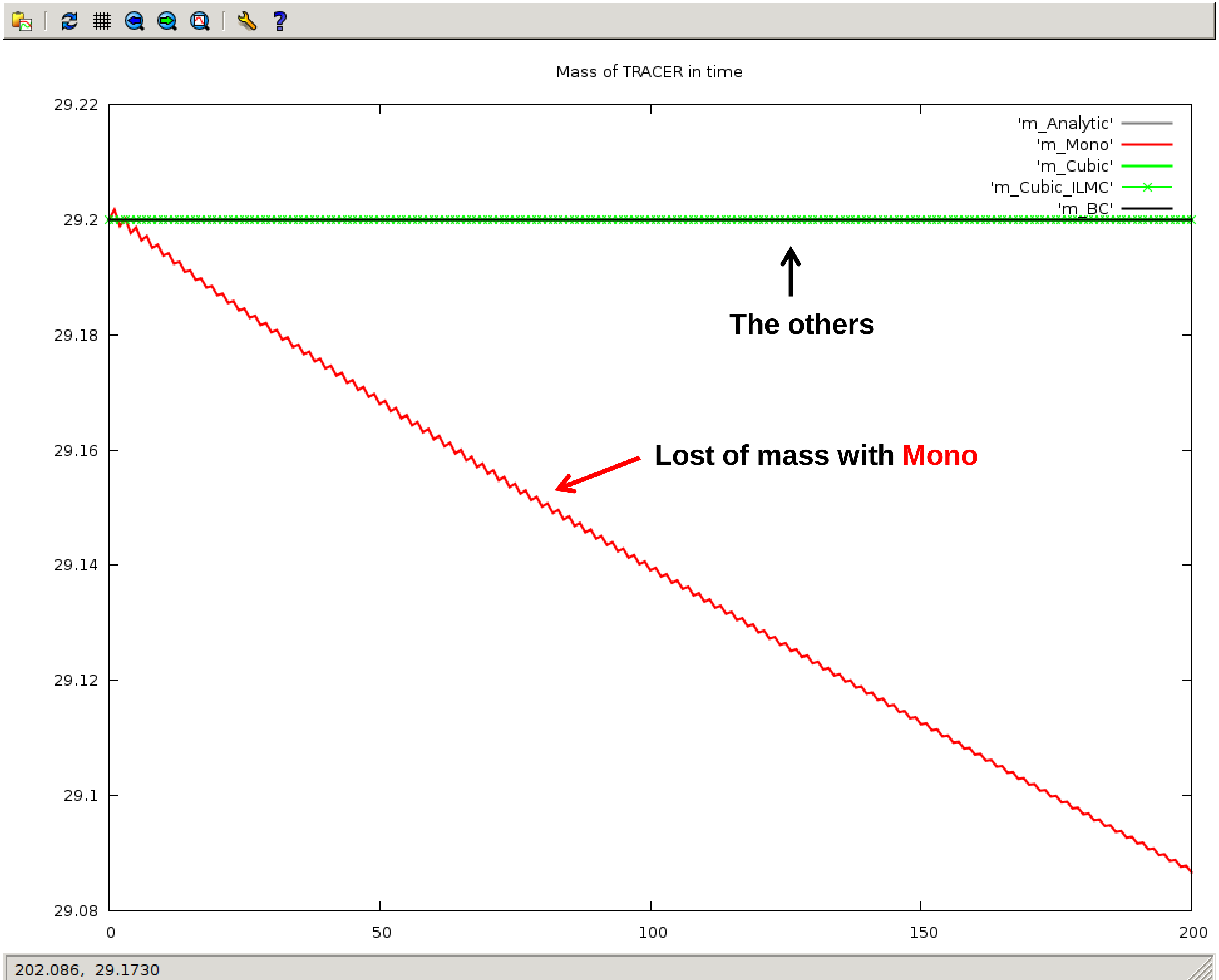
Advection 1D with U=constant: Forecasts after one revolution



Advection 1D with U=constant: Errors after one revolution



Advection 1D with U=constant: Masses in time



Kaas (2008): Local Mass Conservation (#1)

$$\frac{d(\rho\phi)}{dt} + (\rho\phi)\frac{\partial u}{\partial x} = 0$$

Semi-Lagrangian treatment

$$\frac{(\rho\phi)_i - (\rho\phi)_d^-}{\Delta t} + \frac{\left[(\rho\phi)\frac{\partial u}{\partial x}\right]_i + \left[(\rho\phi)\frac{\partial u}{\partial x}\right]_d^-}{2} = 0$$

$$(\rho\phi)_i = \frac{\left[1 - \frac{\Delta t}{2}\left(\frac{\partial u}{\partial x}\right)_d^-\right]}{\left[1 + \frac{\Delta t}{2}\left(\frac{\partial u}{\partial x}\right)_i\right]}(\rho\phi)_d^-$$

Divergence contribution

$$(\rho\phi)_i = \beta_i \sum_l \omega_{i\,l}\,(\rho\phi)_l^- \qquad \sum_l \omega_{i\,l} = 1$$

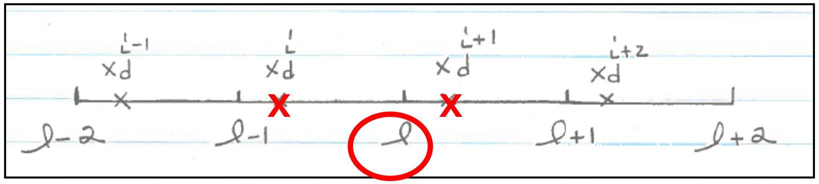
$$(\rho\phi)_i = \sum_l \widehat{\omega}_{i\,l}\,(\rho\phi)_l^- \quad \text{with} \quad \widehat{\omega}_{i\,l} = \frac{V_l}{V_i}\frac{\omega_{i\,l}}{\sum_{\bar{l}} \omega_{\bar{l}\,l}}$$

Local Correction of the interpolated weights $\omega_{i\,l}$ that takes Divergence into account and ensures Local Mass conservation

Kaas (2008): Local Mass Conservation (#2)

$$\hat{\omega}_{i\,l} = \frac{V_l}{V_i} \frac{\omega_{i\,l}}{\sum_{\bar{i}} \omega_{\bar{i}\,l}}$$

No or little divergence: The spatial density of upstream departure points is equal or close to the density of the Eulerian grid points

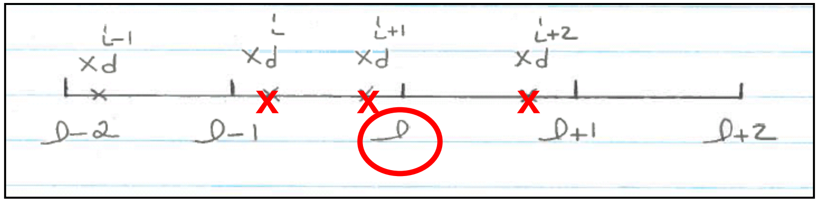


$$\sum_{\bar{i}} \omega_{\bar{i}\,l} \sim \frac{1}{4} + \frac{3}{4} = 1$$

$$(\rho\phi)_i \sim \omega_{i\,l-1}\,(\rho\phi)^-_{l-1} + \omega_{i\,l}\,(\rho\phi)^-_l$$

Forecasted Tracer Density similar to non-divergent situation

In a situation with positive divergence in a region, the spatial density of upstream departure points is greater than the density of the Eulerian grid points



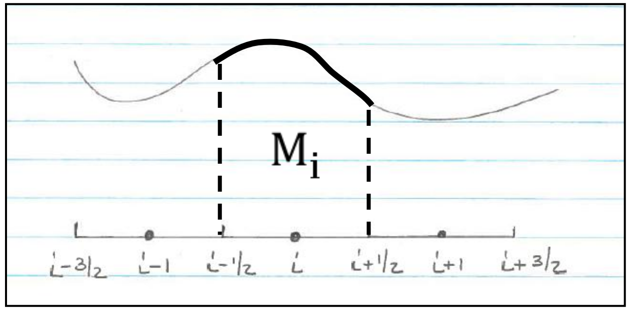
$$\sum_{\bar{i}} \omega_{\bar{i}\,l} \sim \frac{1}{4} + \frac{3}{4} + \frac{1}{4} = \frac{5}{4} > 1$$

$$(\rho\phi)_i \sim \omega_{i\,l-1}\,(\rho\phi)^-_{l-1} + \frac{4}{5}\,\omega_{i\,l}\,(\rho\phi)^-_l$$

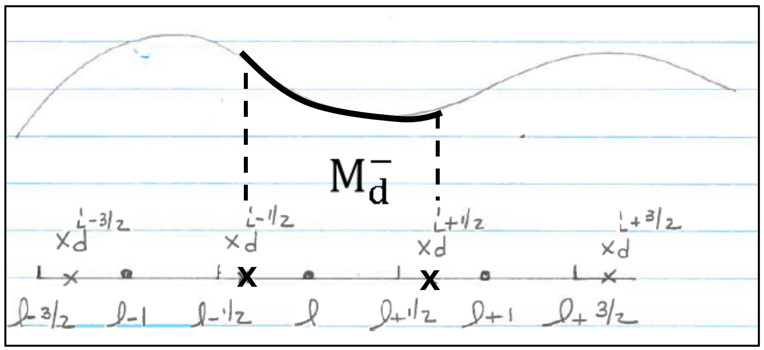
Forecasted Tracer Density smaller than non-divergent situation

Zerroukat (2012): Local Mass Conservation (#1)

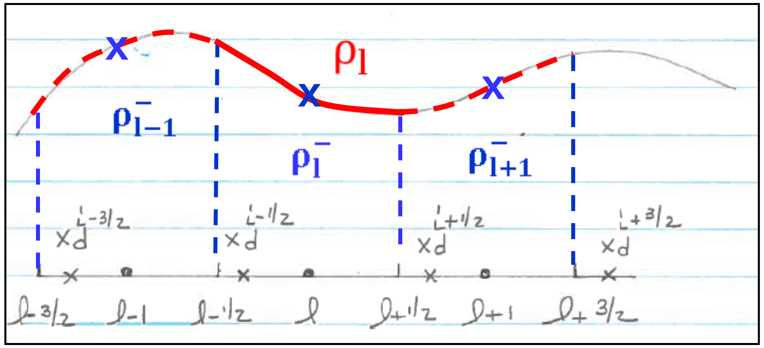
$$M_i = M_d^-$$



Slice-1D



time



Evaluate the integral M_d^-

using the parabolas $\rho_l(x) = a_l + b_l\, x + c_l x^2$

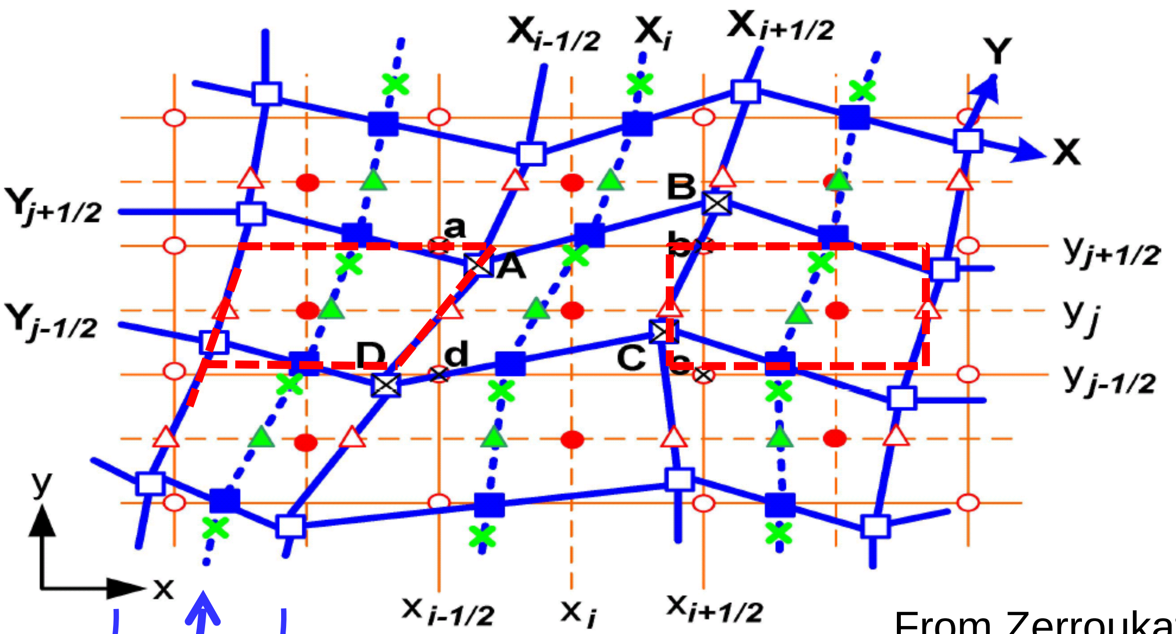
$$\int_{x_{l-3/2}}^{x_{l-1/2}} \rho_l(x)\, dx = \rho_{l-1}^-\left(x_{l-\frac{1}{2}} - x_{l-\frac{3}{2}}\right)$$

$$\int_{x_{l-1/2}}^{x_{l+1/2}} \rho_l(x)\, dx = \rho_l^-\left(x_{l+\frac{1}{2}} - x_{l-\frac{1}{2}}\right)$$

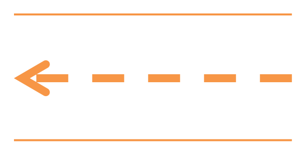
$$\int_{x_{l+1/2}}^{x_{l+3/2}} \rho_l(x)\, dx = \rho_{l+1}^-\left(x_{l+\frac{3}{2}} - x_{l+\frac{1}{2}}\right)$$

As in Laprise and Plante (1995)

See also Mahidjiba et al. (2008)

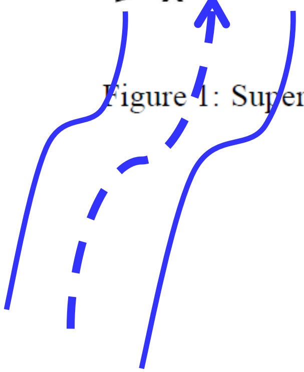


Slice-2D

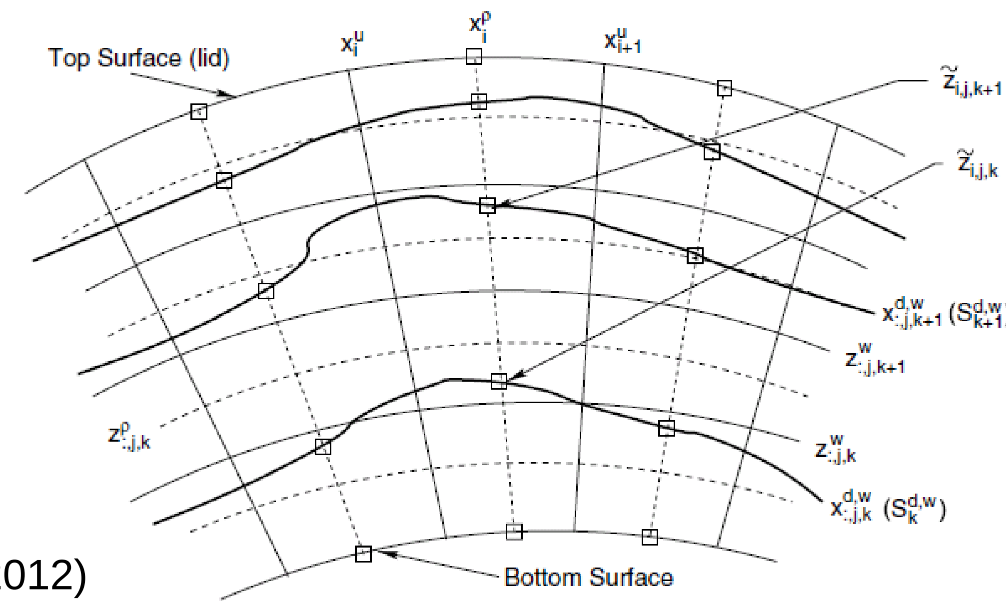


From Zerroukat (SRNWP-NT, Zagreb, 2006)

Figure 1: Superposition of Lagrangian and Eulerian grids away from poles.



Slice-3D



From Zerroukat and Allen (2012)

SUMMARY

Schemes	Shape-Preserving	Mass Conservation
Cubic ϕ^C		
Mono ϕ^M	YES (ϕ^M)	
Mono Bermejo-Conde	YES (ϕ^M)	Global
ILMC Bermejo-Conde	YES (ILMC)	Global
Kaas		Local
Slice		Local
Cubic + ILMC	YES (ILMC)	**No change with respect to Base**
Kaas + ILMC	YES (ILMC)	**No change with respect to Base**
Slice + ILMC	YES (ILMC)	**No change with respect to Base**

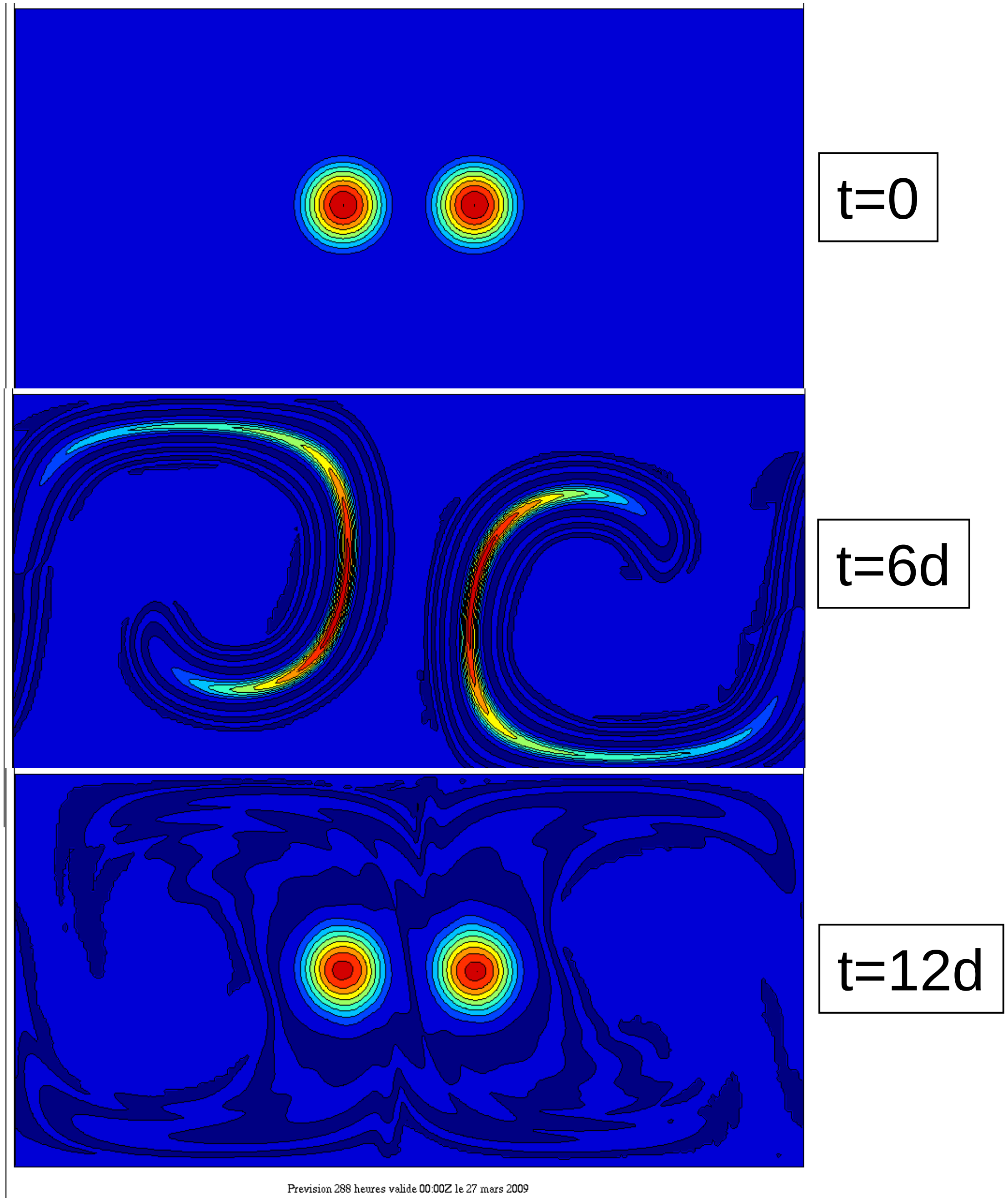
Advection 2D: Lauritzen and Thuburn (2012)

Resolution 360x180 DT=3600 s

Cubic interpolation

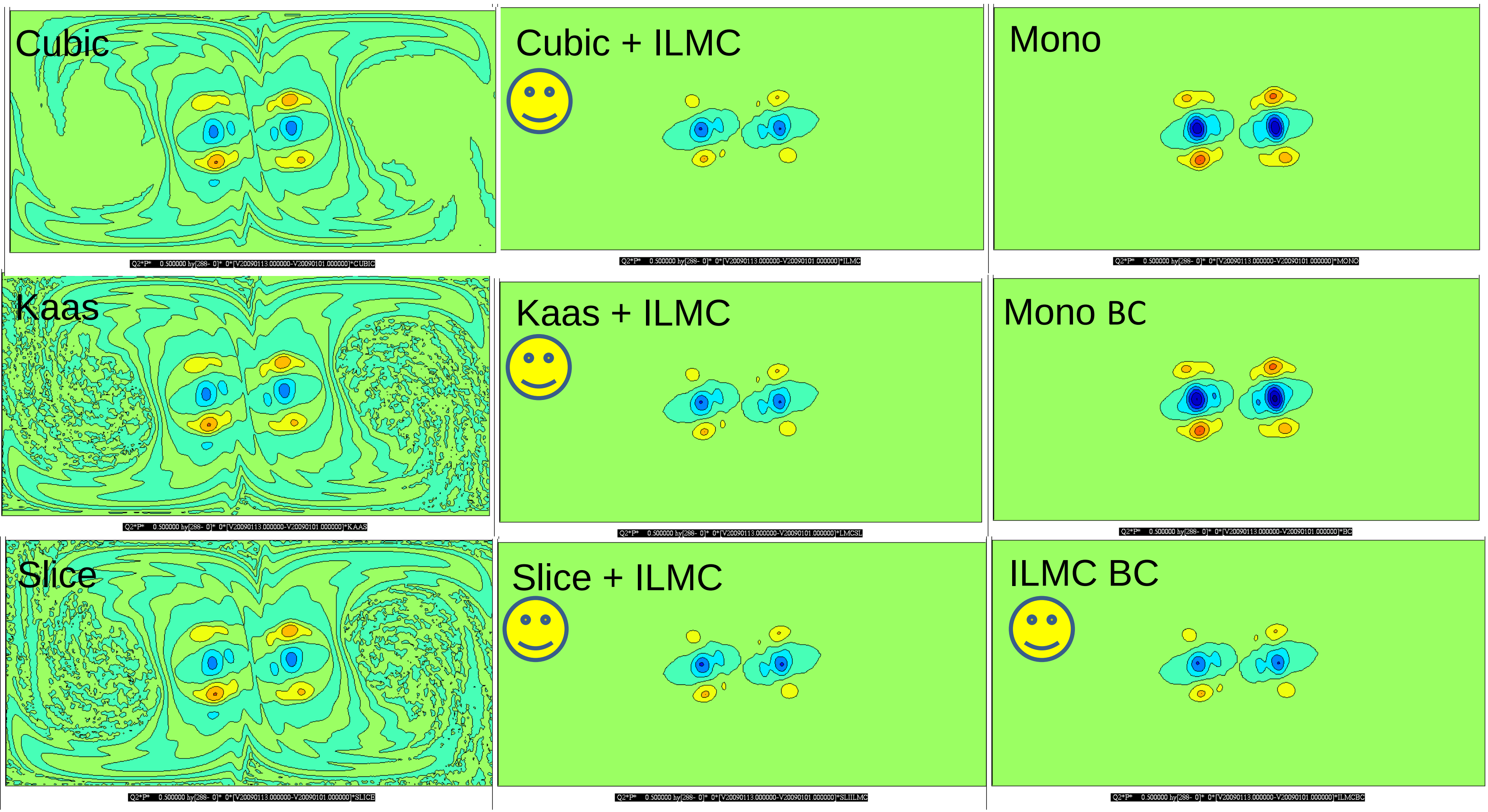
Tracer Q2

Non-divergent reversal winds

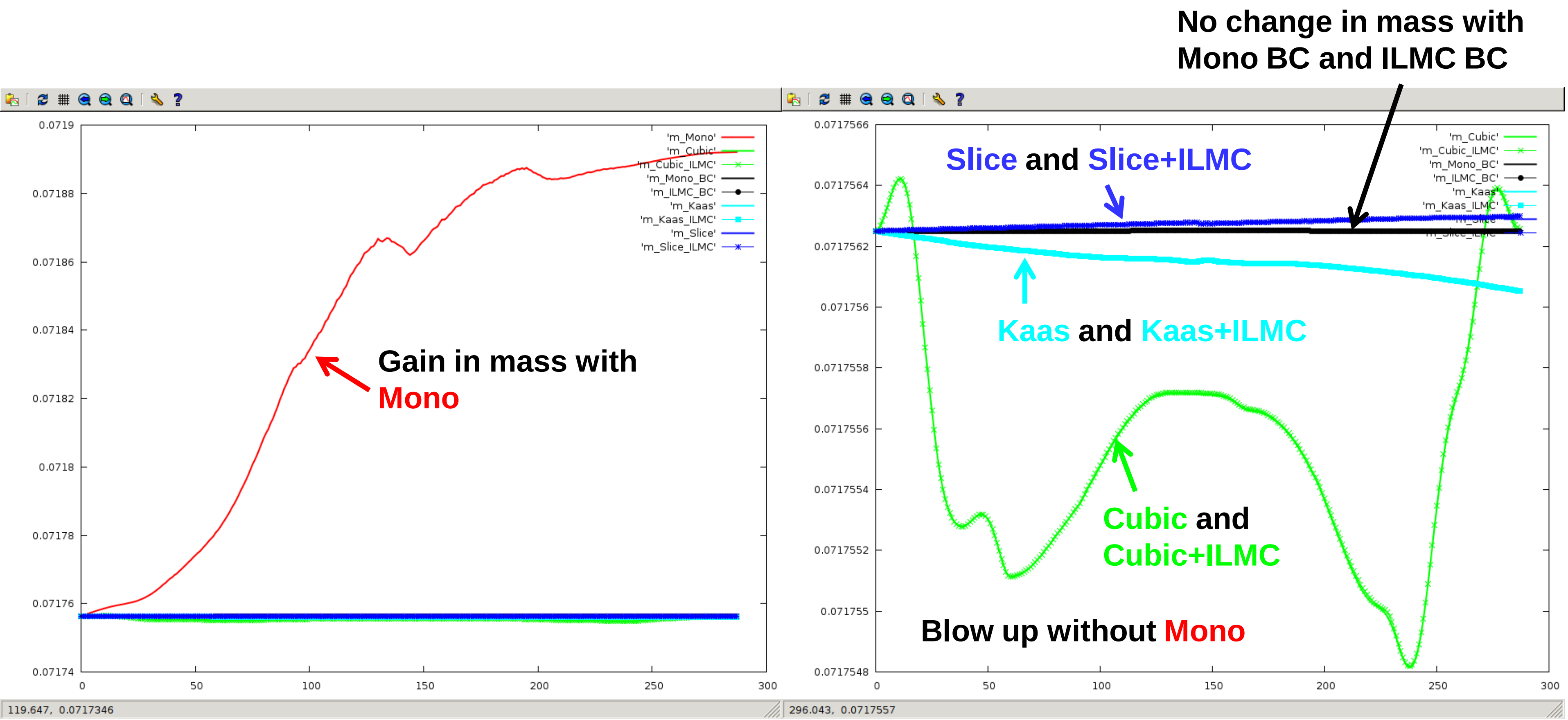


Advection 2D: Tracer Q2: Errors

t=12d



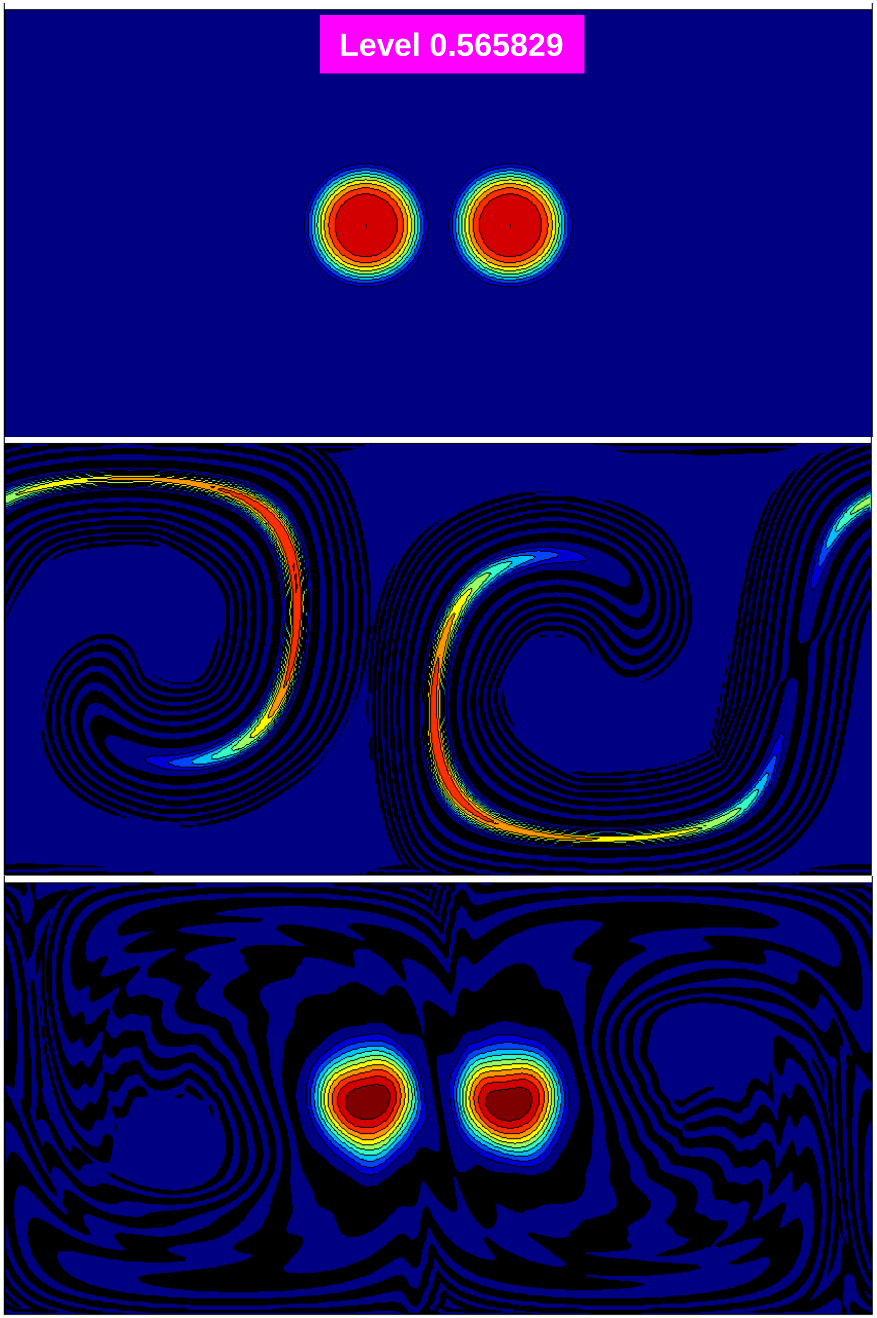
Advection 2D: Tracer Q2: Masses in time



Small change in mass in Kaas and Slice due to use of Cubic interpolation at Poles

Advection 3D: DCMIP 2012: Tracer Q1

Resolution 360x180x60 DT=3600 s



Prevision 288 heures valide 00:00Z le 13 janvier 2009

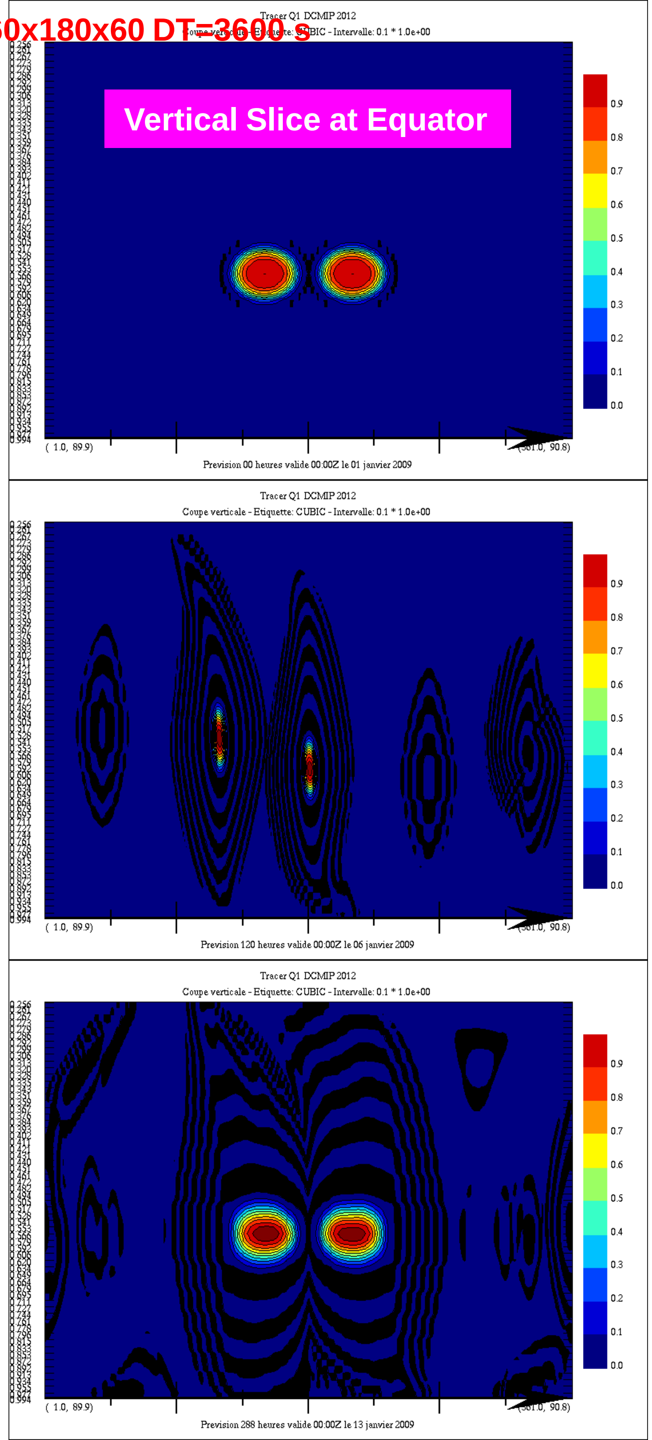
Cubic

Divergent reversal winds

t=0

t=5d

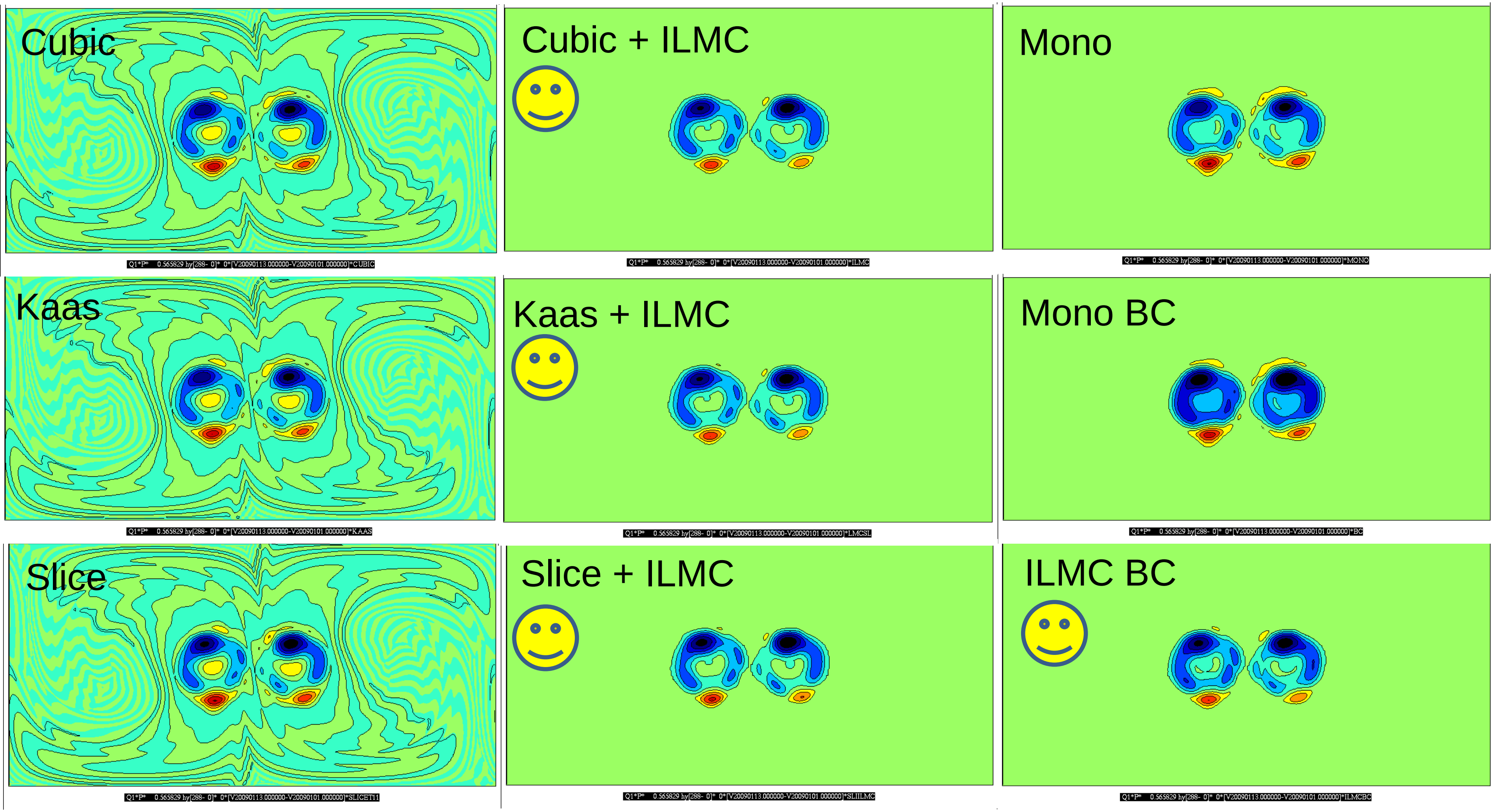
t=12d



Advection 3D: Tracer Q1: Errors

t=12d

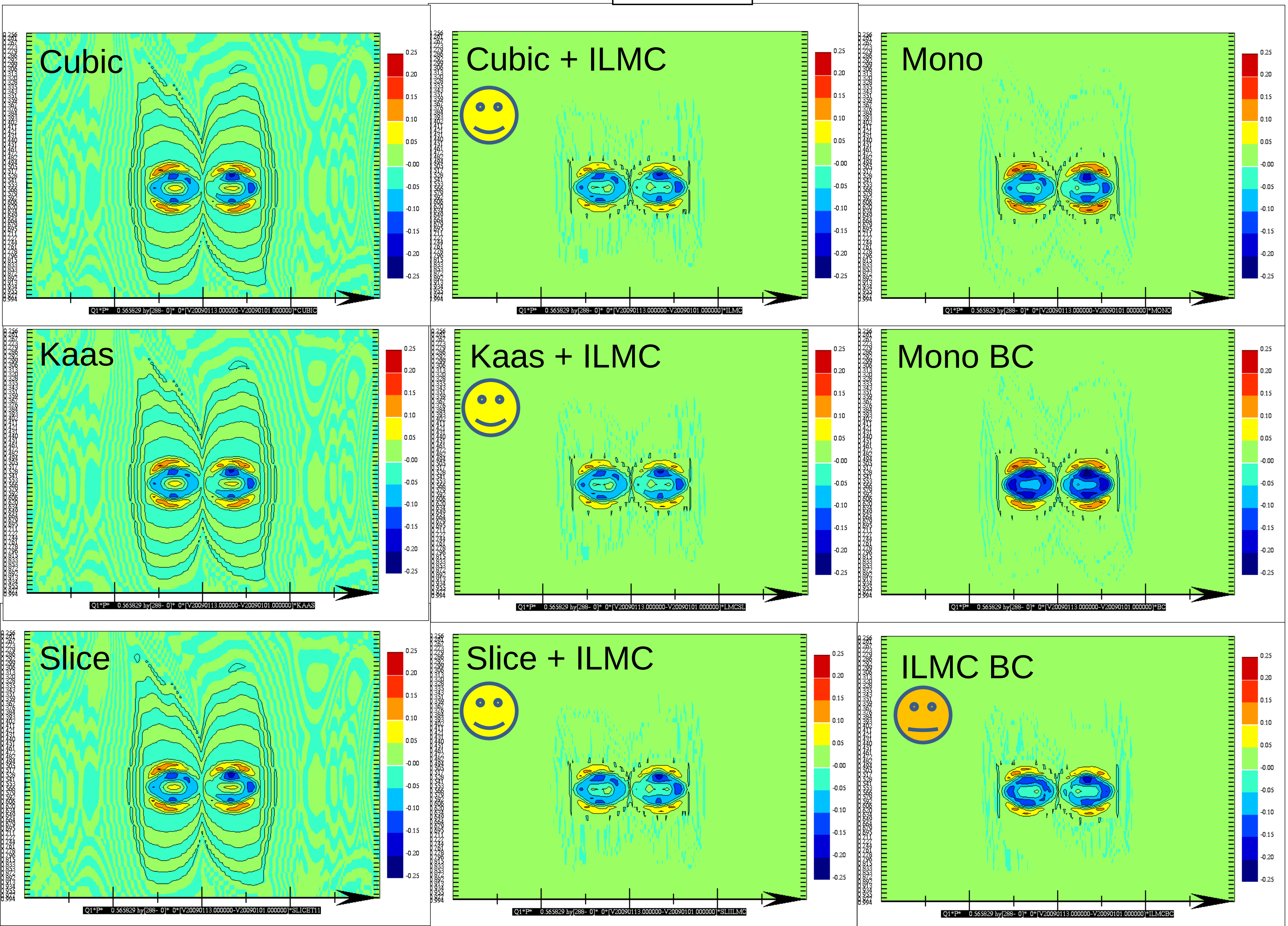
Level 0.565829



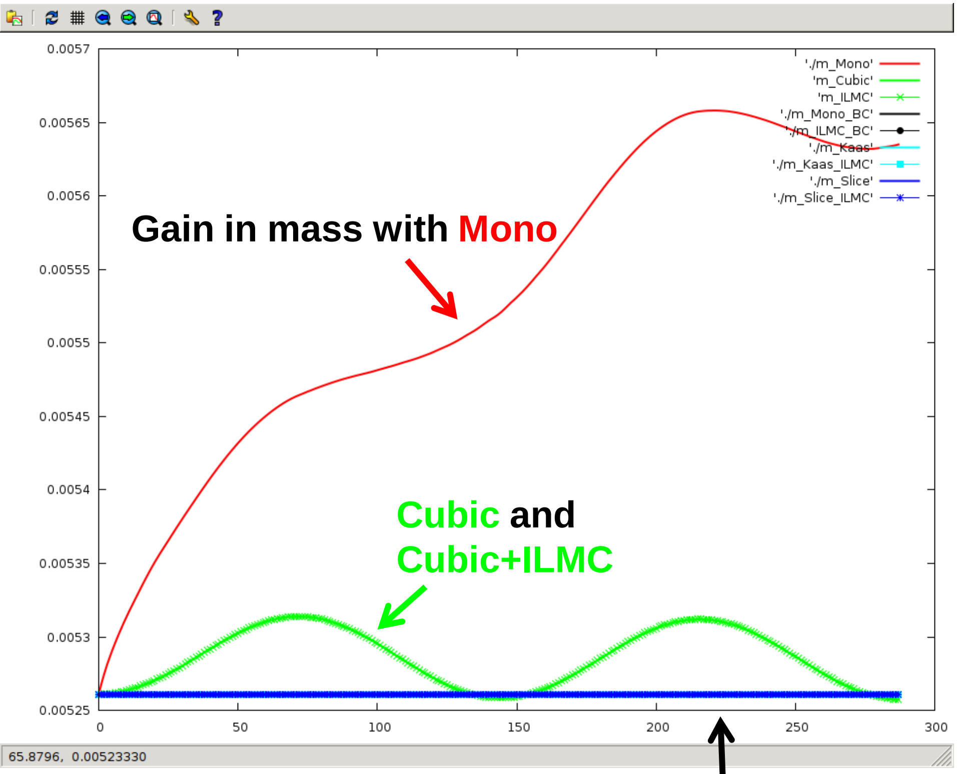
Advection 3D: Tracer Q1: Errors

t=12d

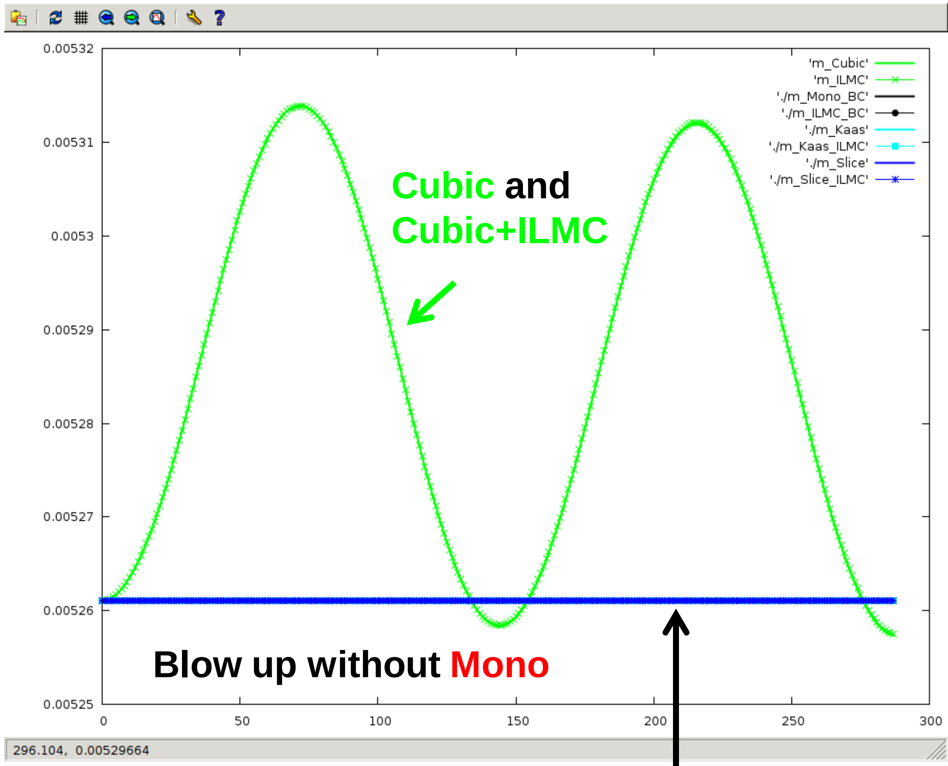
Vertical Slice at Equator



Advection 3D: Tracer Q1: Masses in time

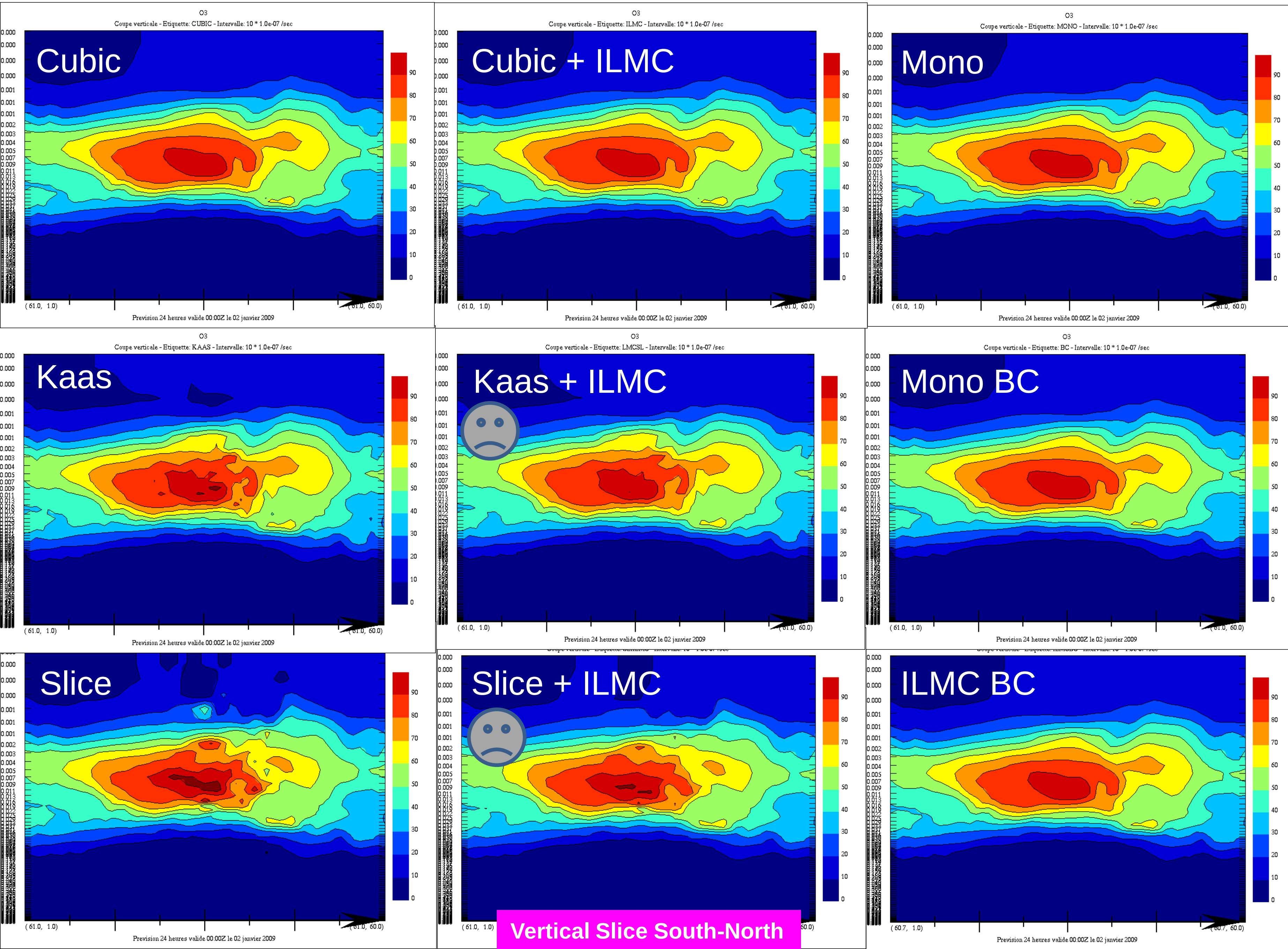


The others

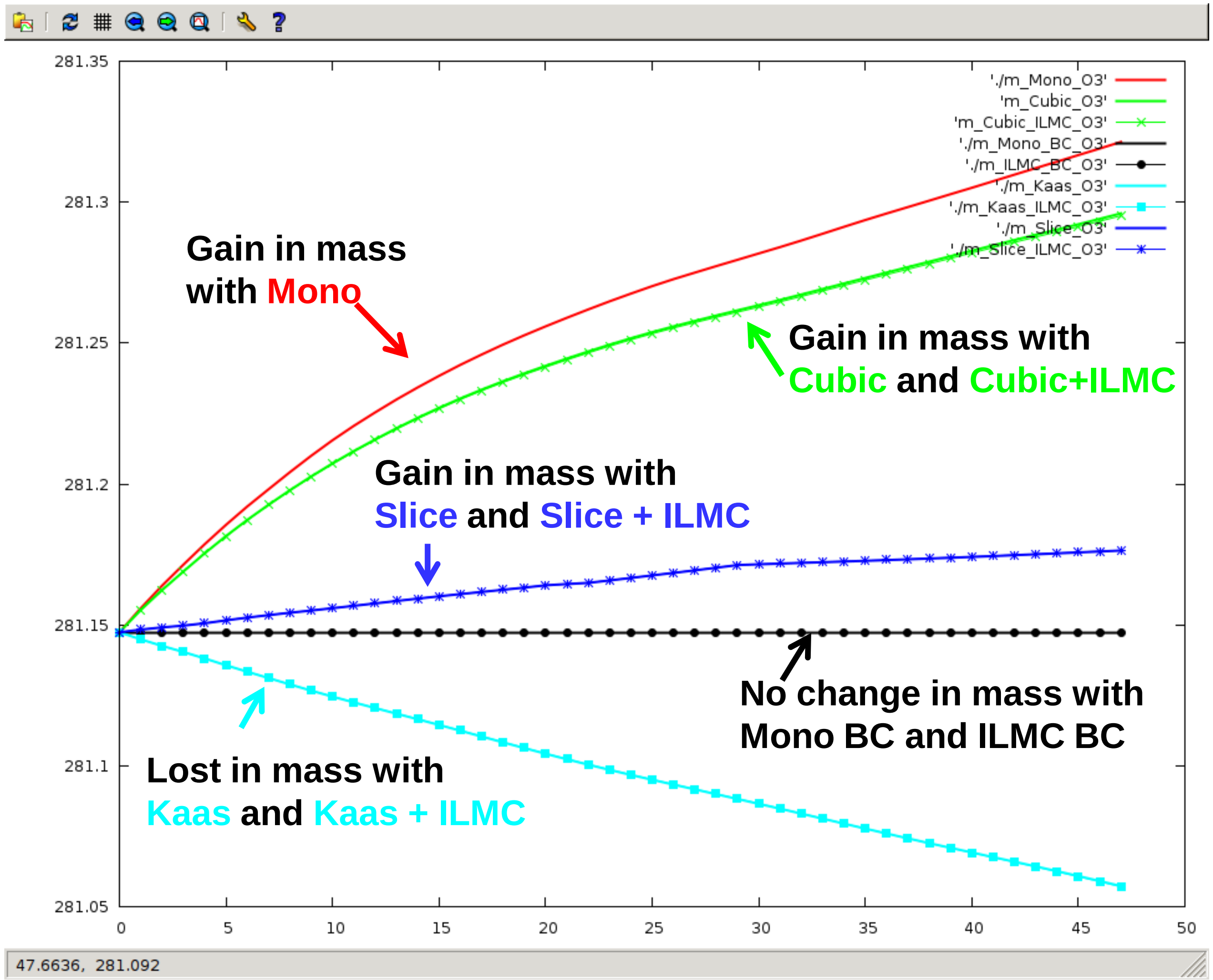


No change in mass with
The others

1 day Forecast with Physics (no Chemistry): Tracer O3: **Resolution 120x60x80 DT=1800 s**



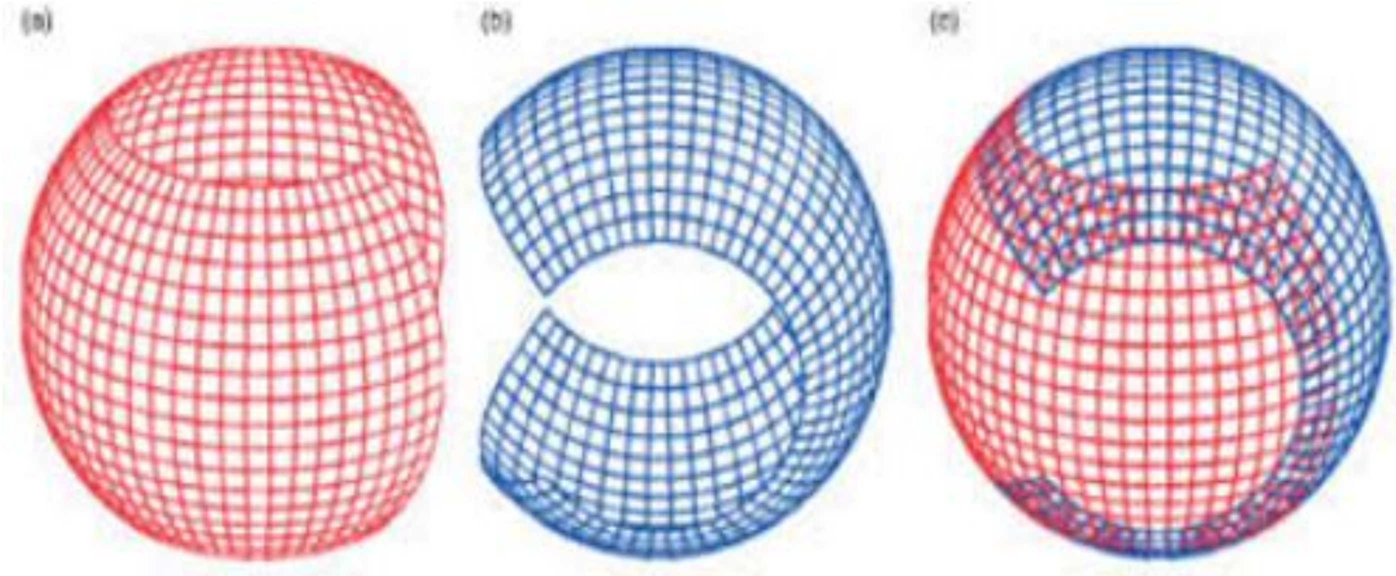
1 day Forecast with Physics (no Chemistry): Tracer_O3: Masses in time



- **More Testcases with vertical velocity w/o mountains required to consolidate Kaas and Slice**
- **Need to regularize the divergence that is implied by the departure points in Kaas and Slice**

Yin-Yang Grid for global Forecast

YIN YAN Yin-Yang

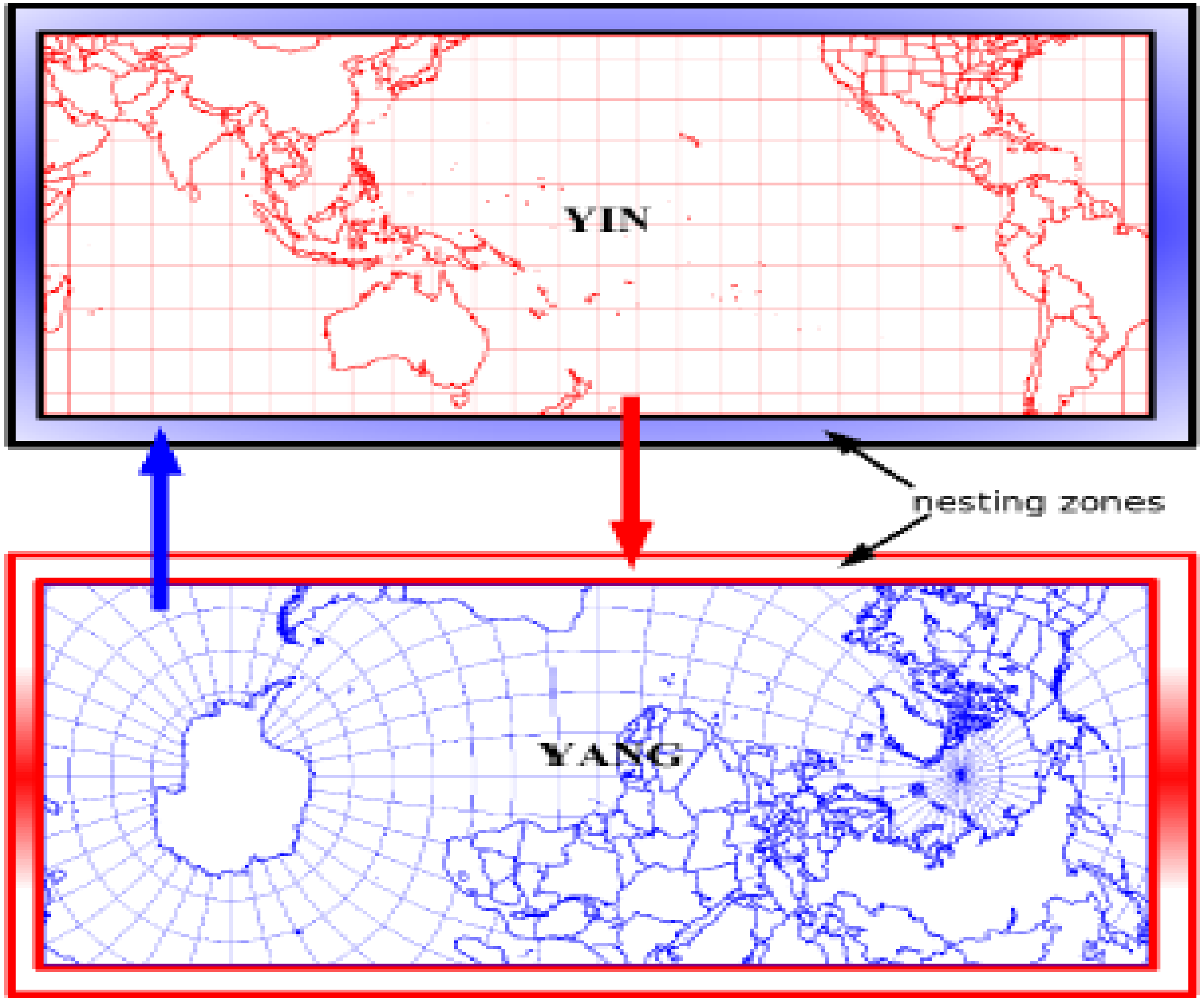


A two-way nesting method between two-limited area models; Qaddouri and Lee QJRMS 2011

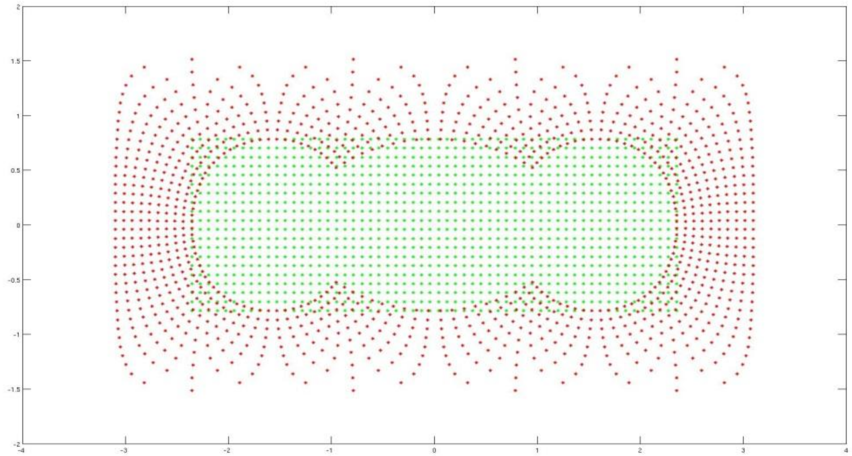
Semi-Lagrangian on Yin-Yang grid

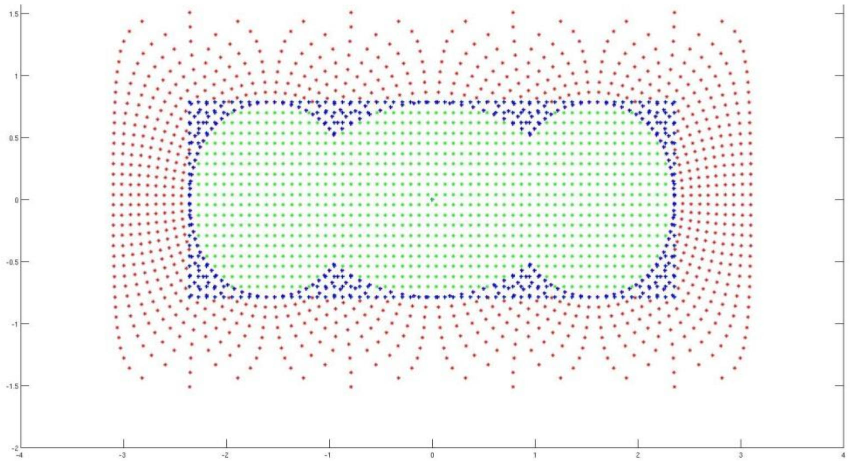
- 1-Extend each panel (Yin, Yang) by a halo (size depends on CFL_max),
- 2-Interpolate from other panel to the halos the fields and winds from previous time-step,
- 3-Do Semi-Lagrangian as usual in each panel. Goto 2

Data Exchange between Yin and Yang subgrids



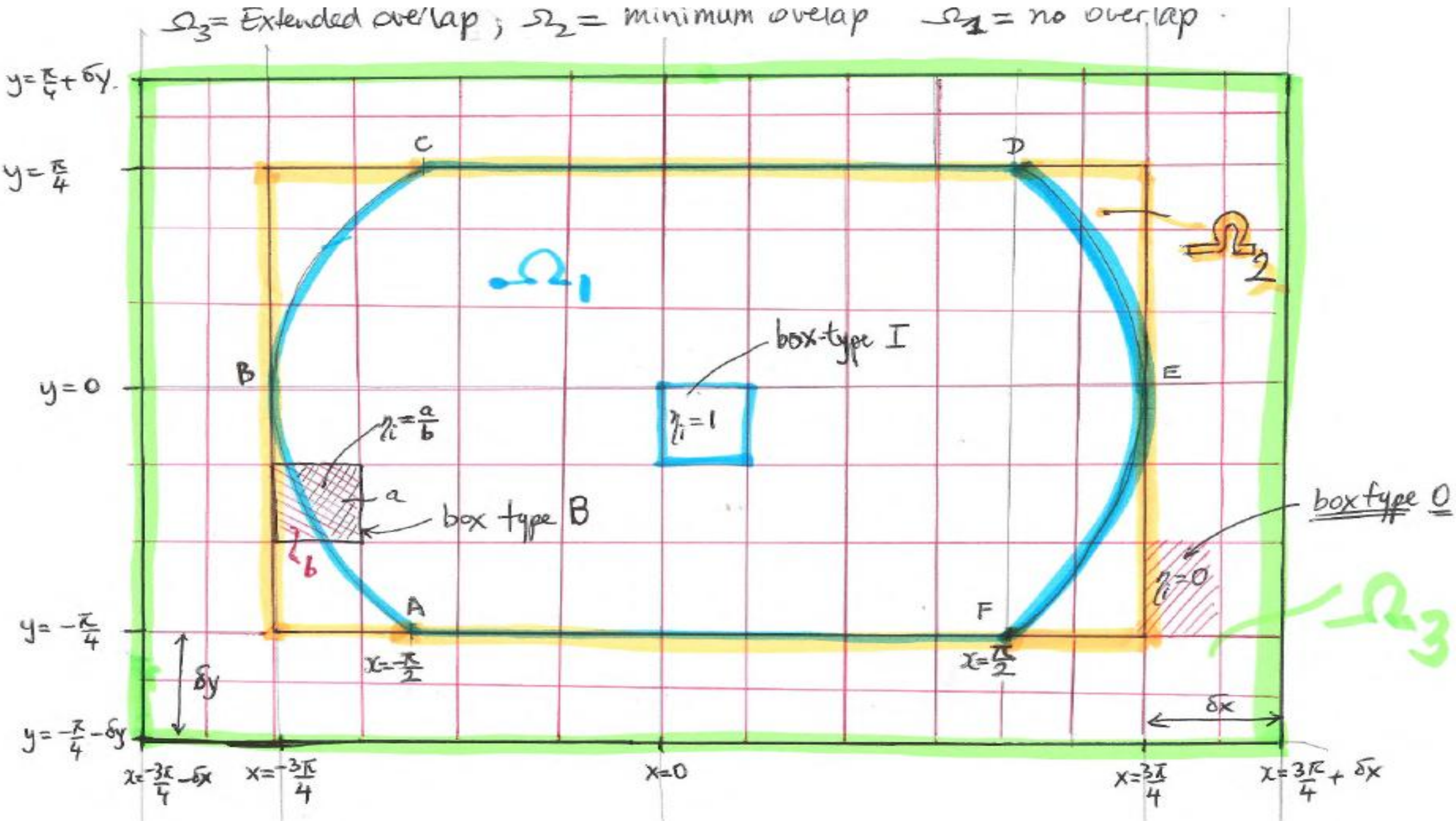
zero minimal-overlap





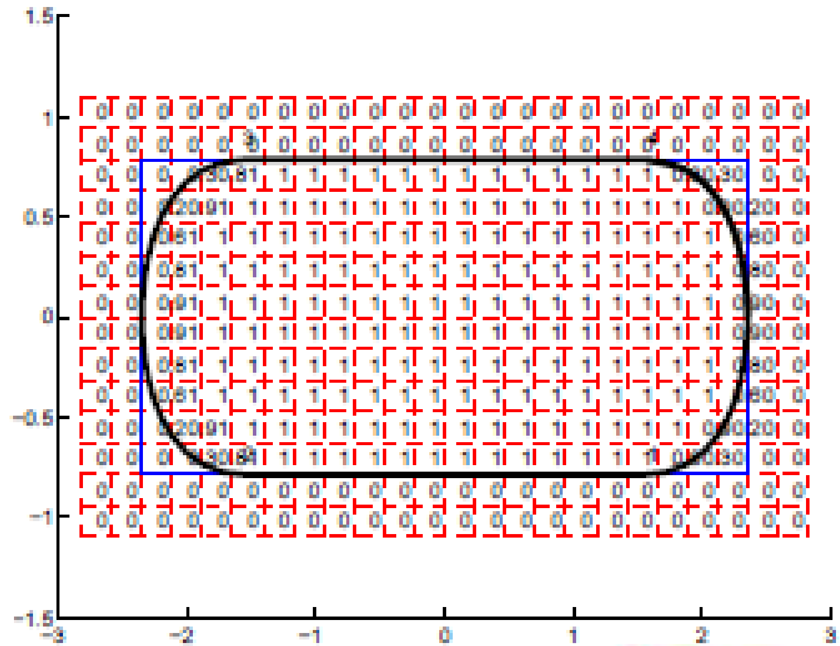
computing mass in the overlap

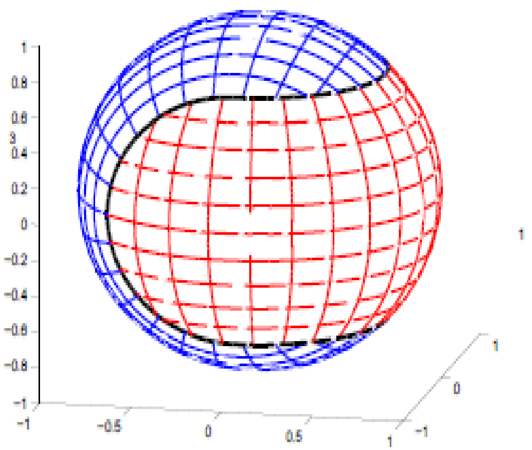
Zerroukat (UKMetO)

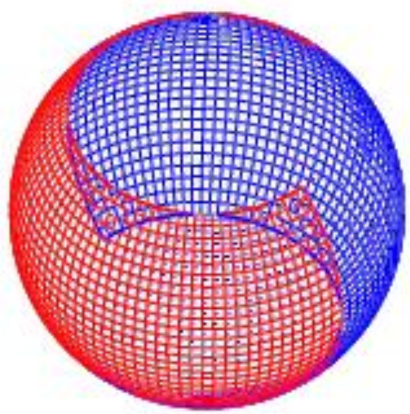


Weight computation

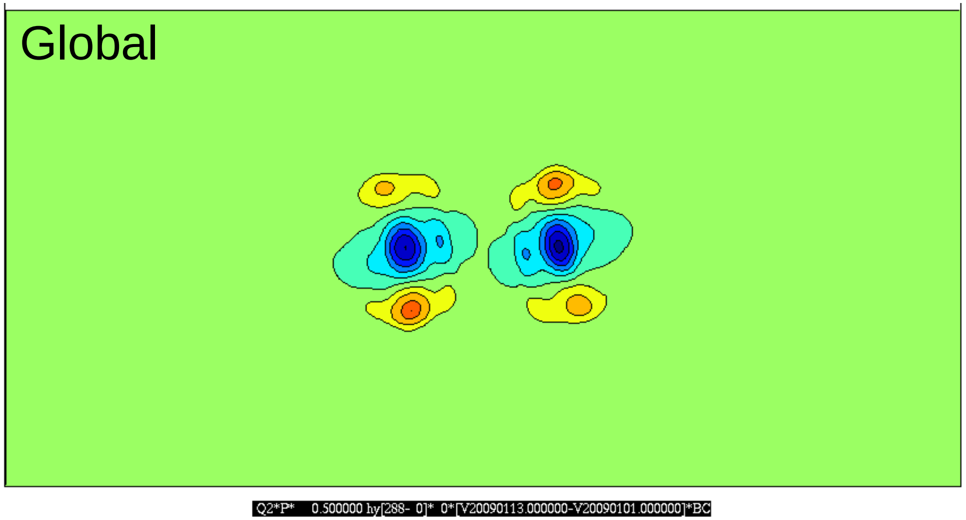
Zerroukat

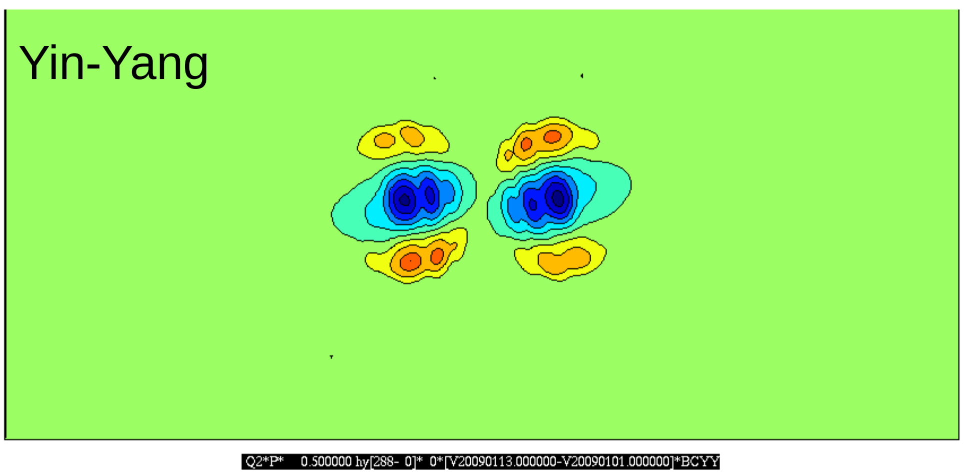




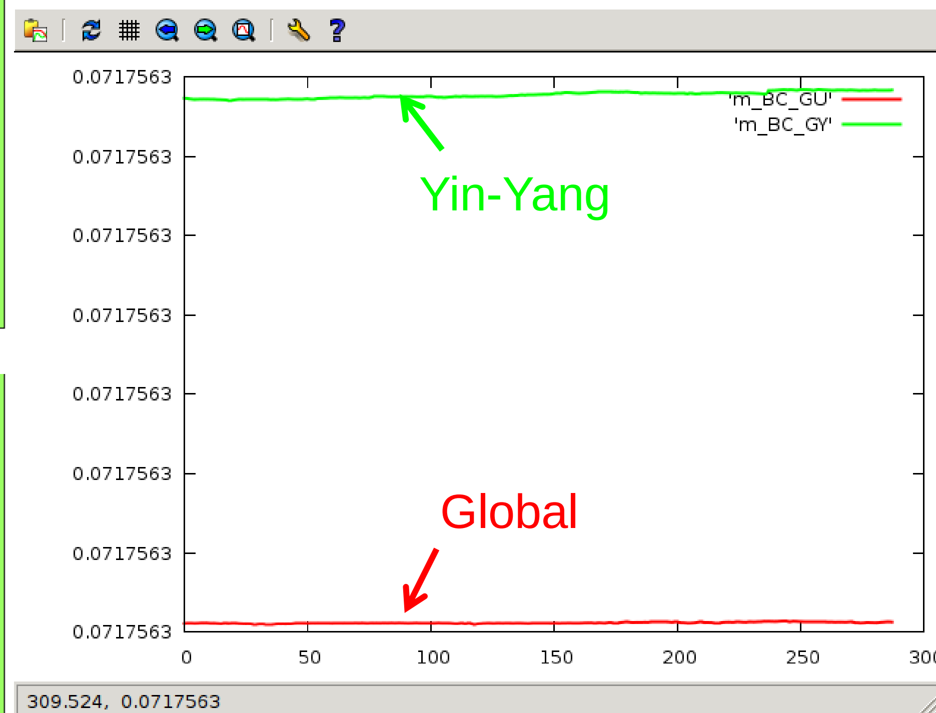


Advection 2D: Mono Bermejo-Conde: Global *versus* Yin-Yang





Masses in time



Errors Q2

t=12d

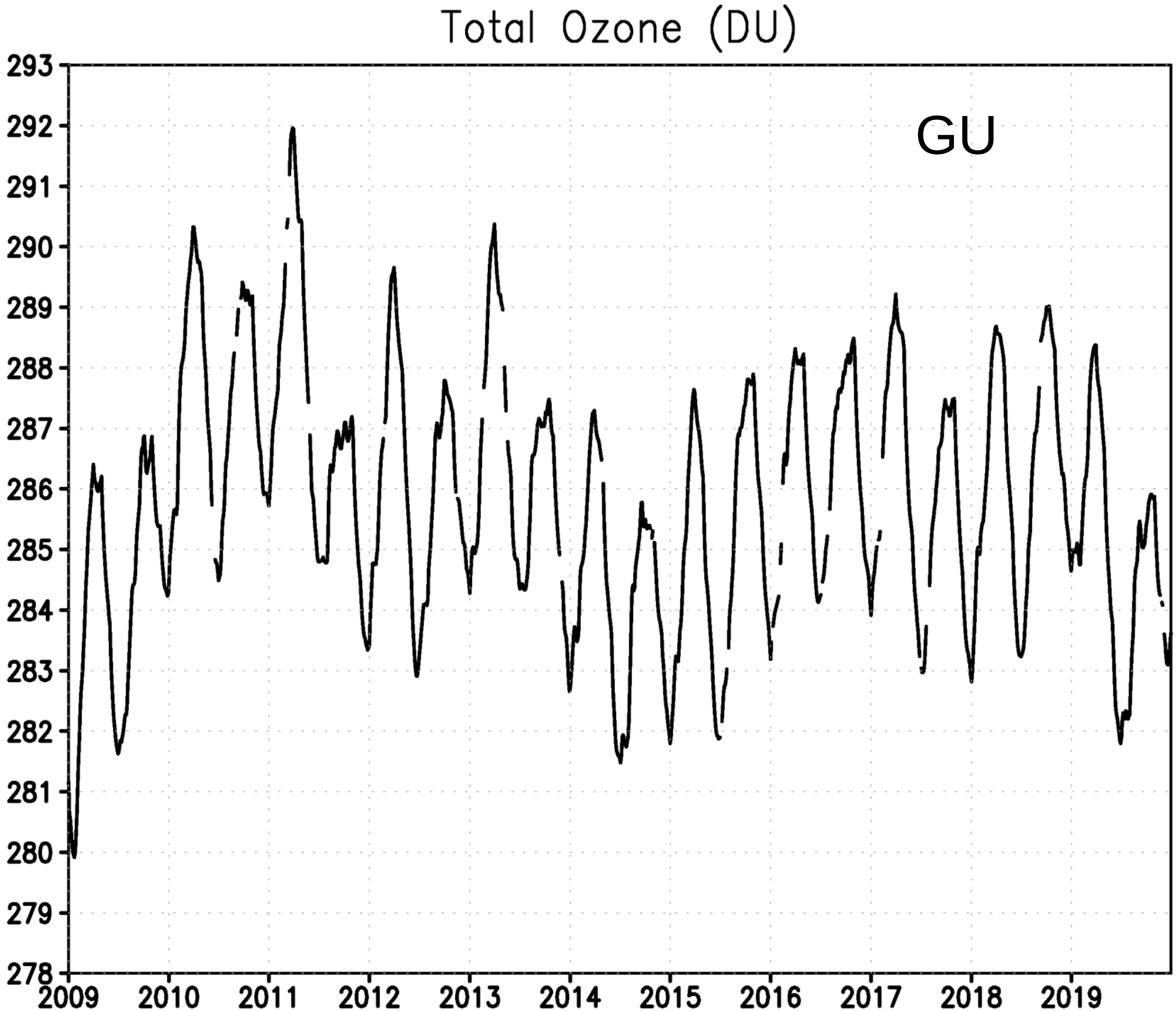
Experiments

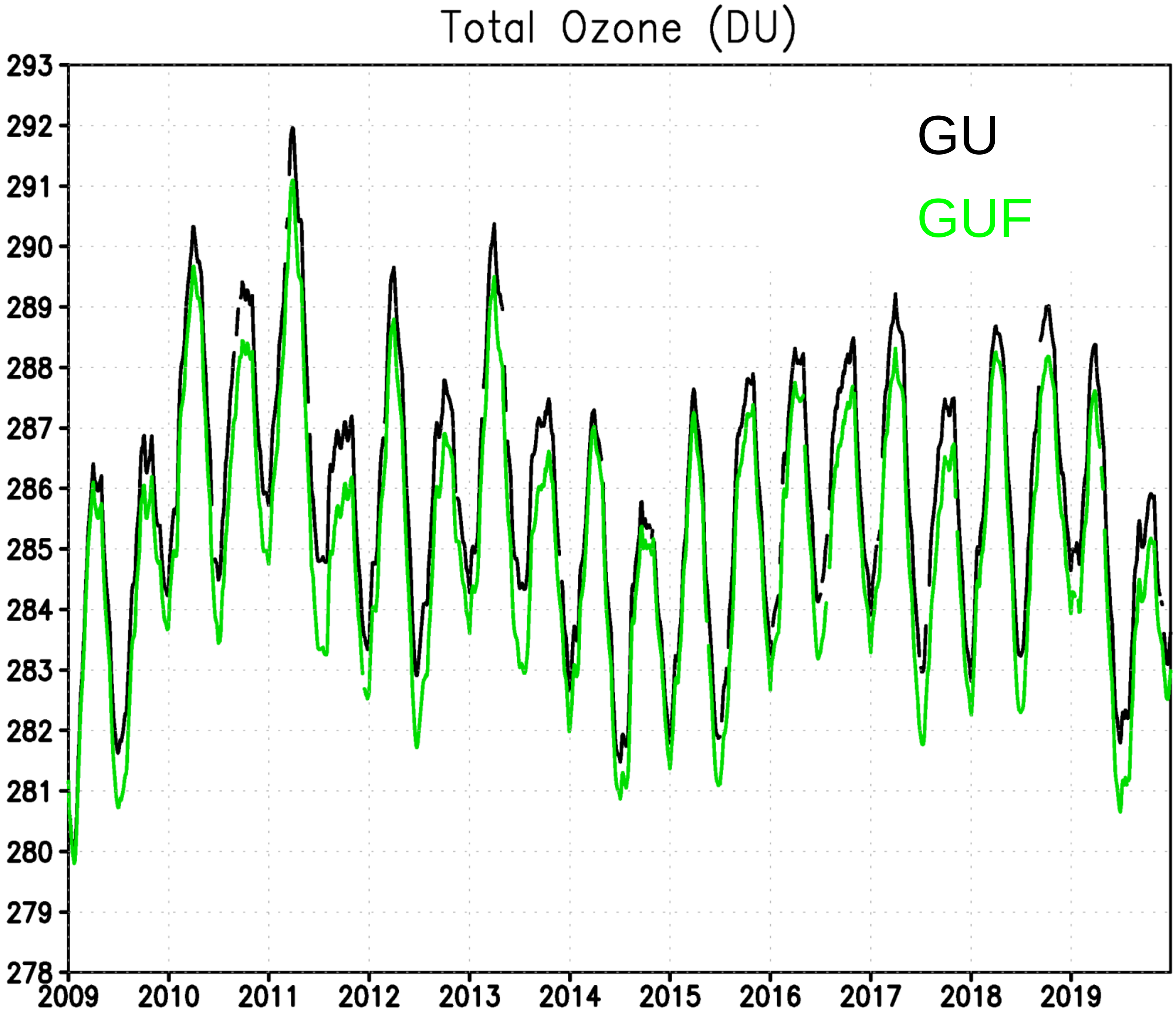
- GEM 4.6.0 – rc6
- Based on the GDPS config with Δt=30 min & psadj=on
- Global Uniform (360x180)
- Yin Yang (319x107) – minimum overlap – no blending
- Linearized chemistry (O_3 , CH_4) – non interactive
- Mass fixers : BC (ILMC monotonicity) – tracers only

<u>4 runs:</u>

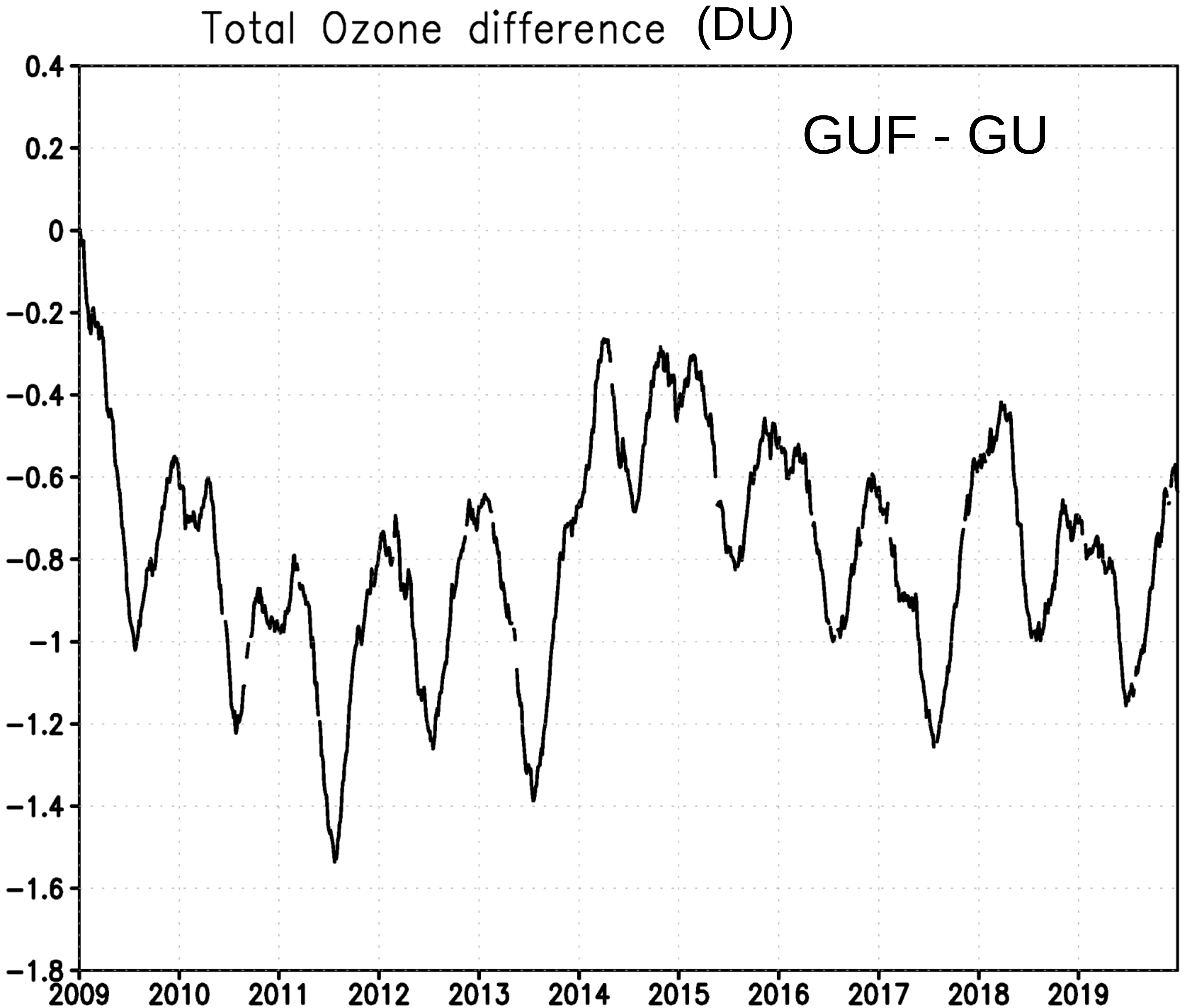
1) GEM Lat-Lon (GU) – 11y
2) GEM Lat-Lon + Mass fixers (GUF) – 11y
3) GEM Yin Yang (GY) – 3y
4) GEM Yin Yang + Mass fixers (GYF) -3y

Same dynamics


Total Ozone (DU)
GU
293
292
291
290
289
288
287
286
285
284
283
282
281
280
279
278
2009
2010
2011
2012
2013
2014
2015
2016
2017
2018
2019



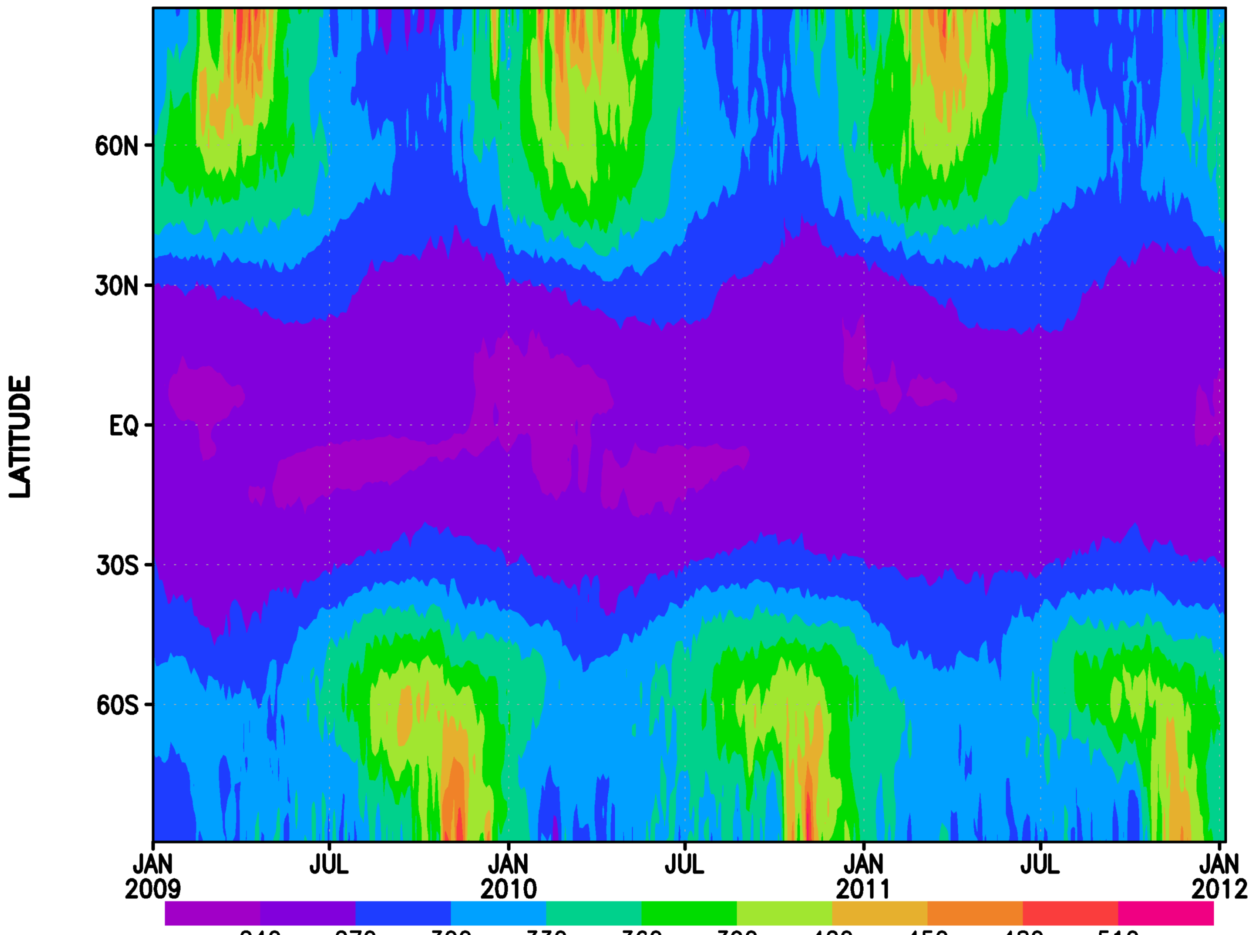
Total Ozone (DU)
GU
GUF
293
292
291
290
289
288
287
286
285
284
283
282
281
280
279
278
2009
2010
2011
2012
2013
2014
2015
2016
2017
2018
2019

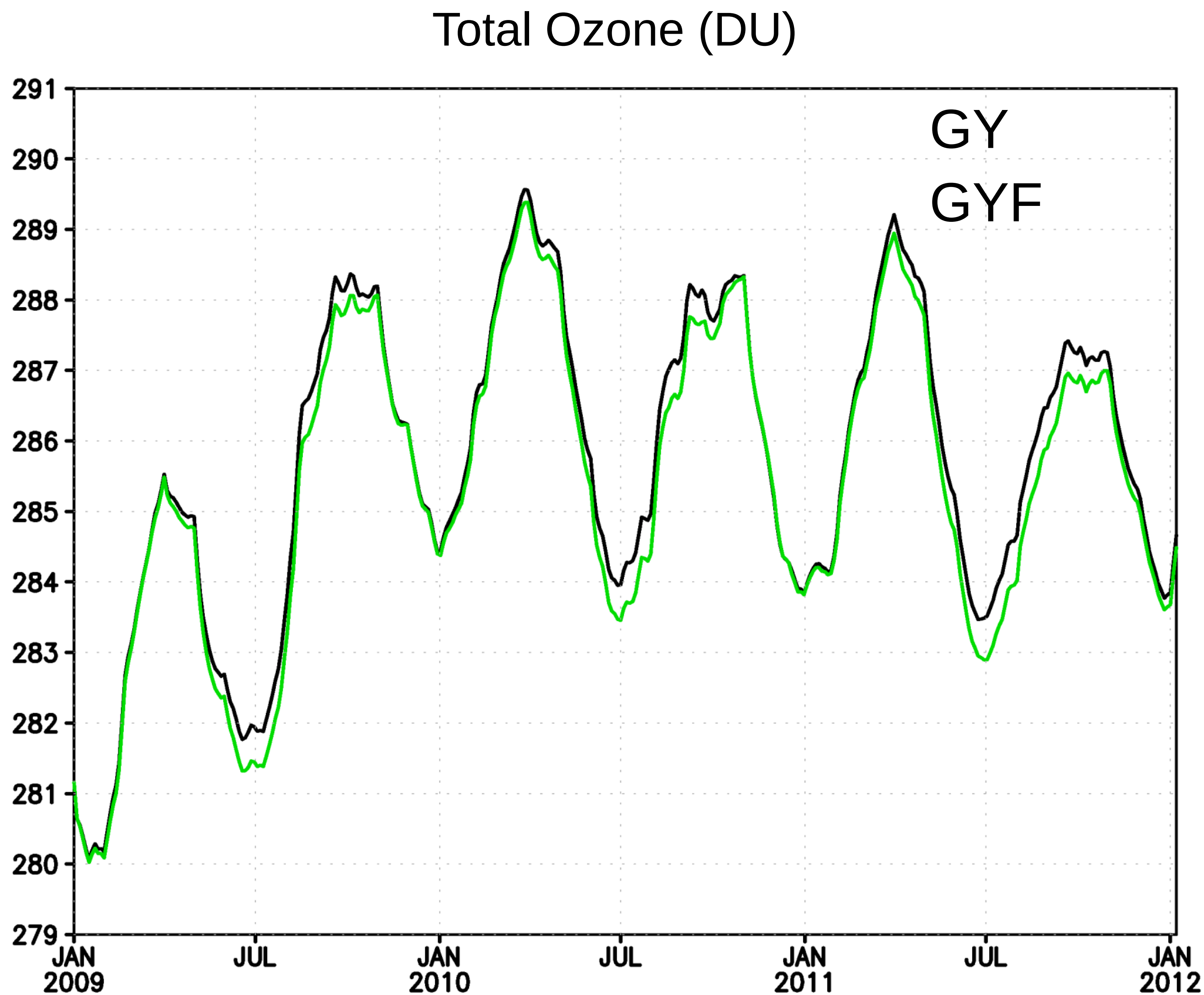


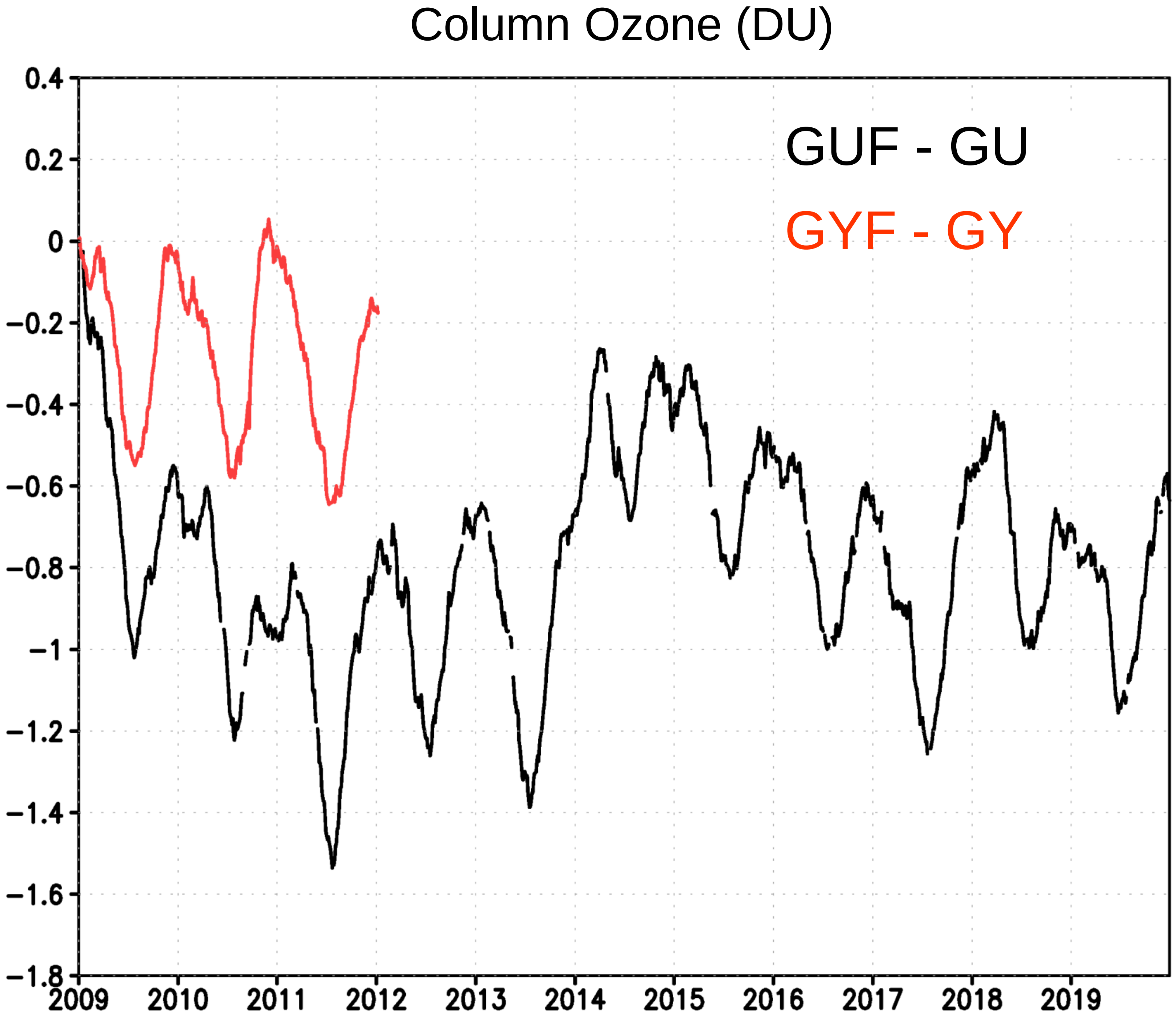
Total Ozone difference (DU)
GUF - GU
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
−1.2
−1.4
−1.6
−1.8
2009
2010
2011
2012
2013
2014
2015
2016
2017
2018
2019

Column Ozone (DU)

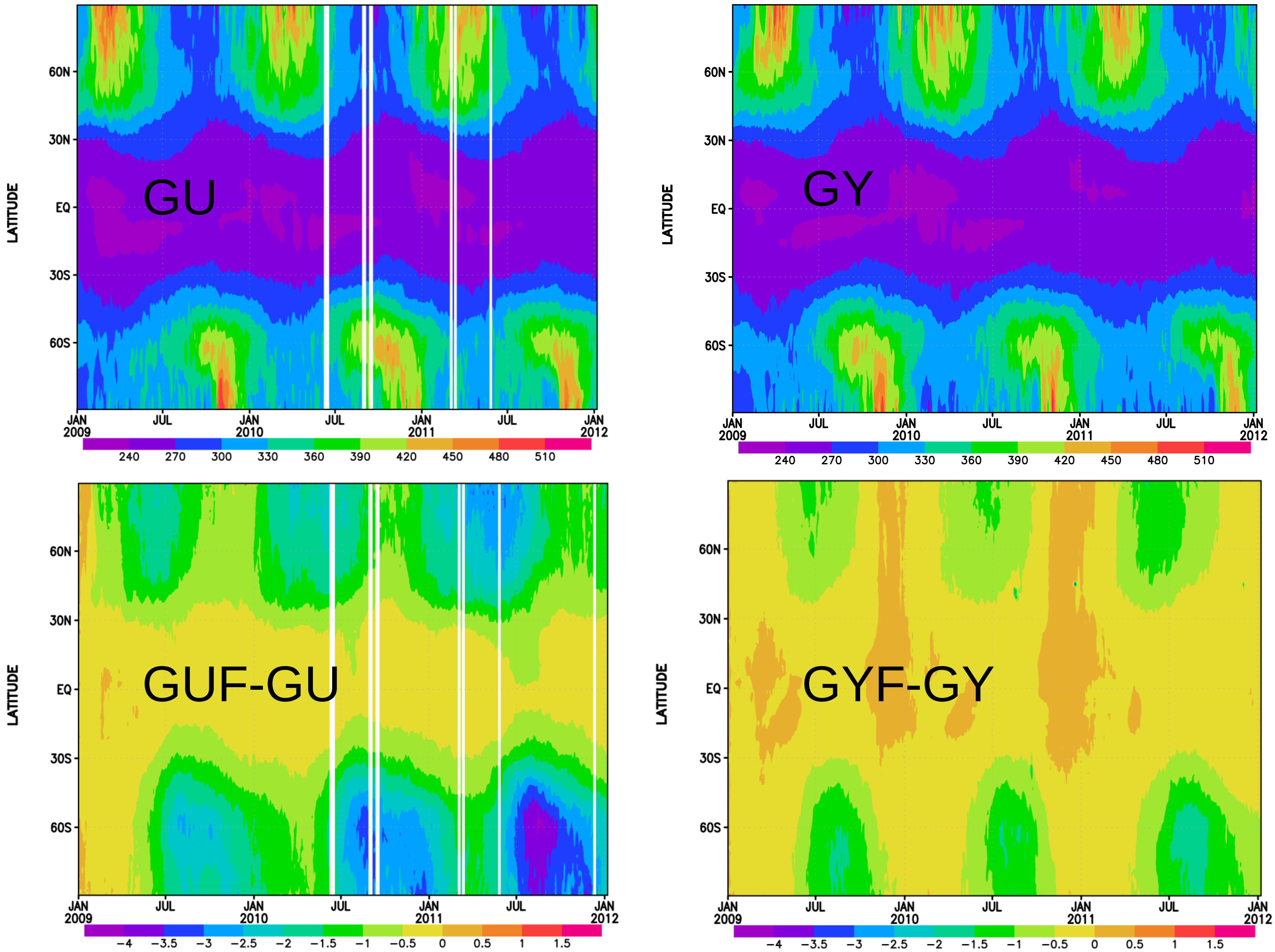
3 Year zonal mean time series



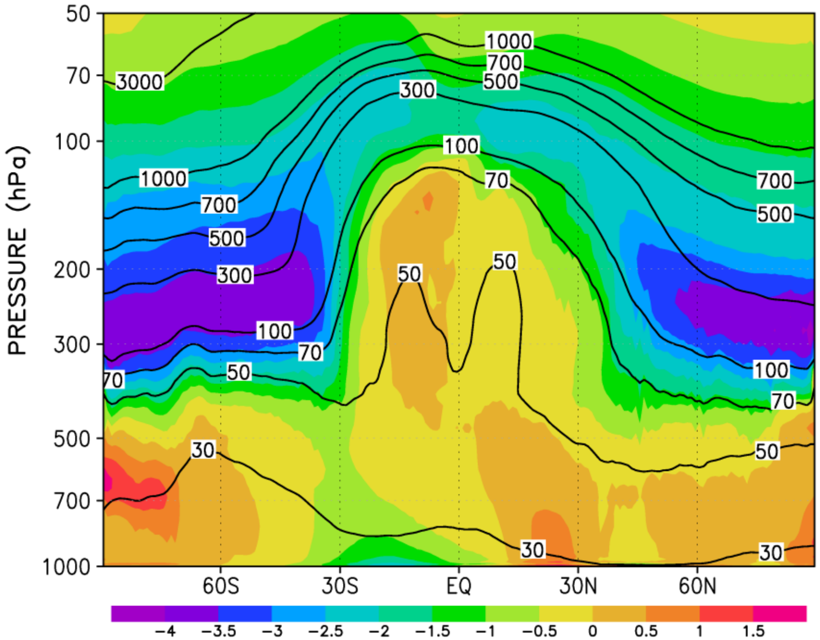

Total Ozone (DU)
GY
GYF
291
290
289
288
287
286
285
284
283
282
281
280
279
JAN 2009
JUL
JAN 2010
JUL
JAN 2011
JUL
JAN 2012



Column Ozone (DU)
GUF - GU
GYF - GY
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
−1.2
−1.4
−1.6
−1.8
2009
2010
2011
2012
2013
2014
2015
2016
2017
2018
2019

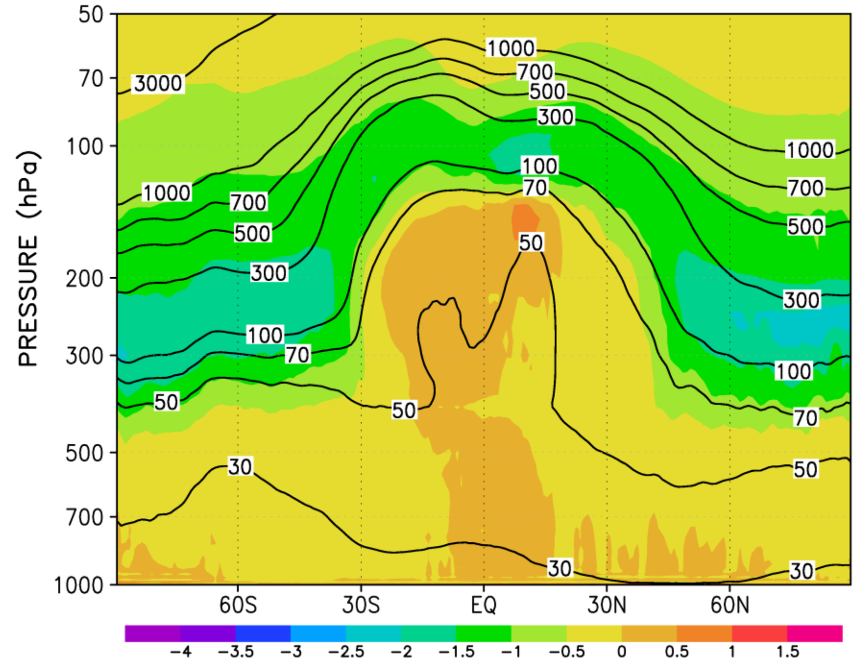

GU
GY
GUF-GU
GYF-GY
LATITUDE
60N
30N
EQ
30S
60S
JAN 2009
JUL
JAN 2010
JUL
JAN 2011
JUL
JAN 2012
240
270
300
330
360
390
420
450
480
510
−4
−3.5
−3
−2.5
−2
−1.5
−1
−0.5
0
0.5
1
1.5

O3 (ppbv) – July 2012



GUF-GU

O3 (ppbv) - July 2012



GYF-GY

Conclusions & next

- Mass fixers can be used to diagnose the deficiency of numerical schemes:

  S-L transport on the Yin-Yang grid system ensure better mass conservation for ozone. For methane, the impact is neutral.

  Next:

- Impact of Horizontal diffusion
- Evaluation of ILMC in GEM-MACH-v2 (GEM4 based)
- GEM-MACH-Global and mass fixers

REFERENCES

- Bermejo R., and J.Conde, 2002: A conservative quasi-monotone semi-Lagrangian scheme, Mon.Wea.Rev., 130, 423-430.
- Gravel S,. and A.Staniforth, 1994: A mass-conserving semi-Lagrangian scheme for the Shallow-Water equations, Mon.Wea.Rev., 122, 243-248
- Kaas, E., 2008: A simple and efficient locally mass conserving semi-Lagrangian transport scheme, Tellus, 60A, 305-320.
- Kent, J., P.A.Ullrich, and C.Jablonowski, 2013: Dynamical core model intercomparison project: Tracer transport test cases, Q.J.R.Meteorol.Soc., DOI: 10.1002/qj.2208
- Laprise, R., and A.Plante, 1995: A class of semi-Lagrangian integrated-Mass (SLIM) numerical transport algorithms, Mon.Wea.Rev.,123,553-565
- Lauritzen, P.H., and J.Thuburn, 2012: Evaluating advection/transport schemes using interrelated tracers, scatter plots and numerical mixing diagnostics, Q.J.R.Meteorol.Soc., 138, 665, 906–918
- Mahidjiba, A., A.Qaddouri, J.Côté, 2008: Application of the semi-Lagrangian inherently conserving and efficient (SLICE) transport method to divergent flows on a C grid, Mon.Wea.Rev., 136, 4850-4866
- Sorensen, B., E.Kaas, and U.S.Korsholm, 2013: A mass-conserving and multi-tracer efficient transport scheme in the online integrated Enviro-HIRLAM model, Geosci. Model Dev., 6, 1029-1042
- Zerroukat, M., and T.Allen, 2012: A three-dimensional monotone and conservative semi-Lagrangian (SLICE-3D) for transport problems, Q.J.R.Meteorol.Soc., 138, 1640–1651
- Zerroukat, M., 2006: SLICE-3D: A three-dimensional conservative semi-Lagrangian scheme for transport problems, SRNWP-NT, Zagreb (Presentation)