



Mixed phase clouds: recent progress in theoretical analysis and in-situ observations

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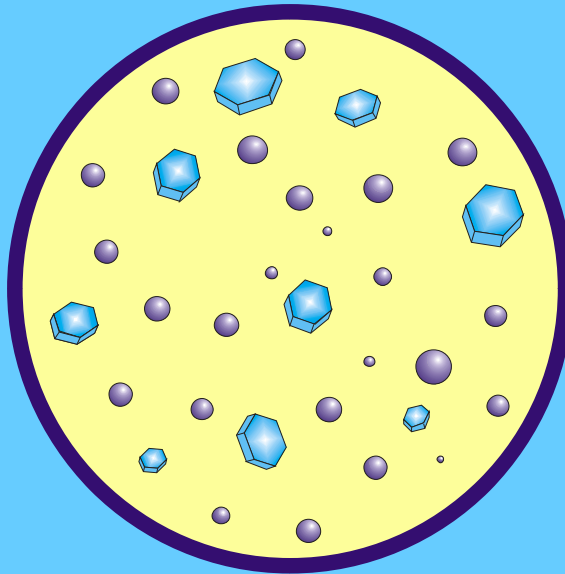
Outline:

- 1. In-situ observations of mixed phase clouds**
- 2. Theoretical consideration of mixed phase**

Cloud droplets may stay in a metastable liquid condition down to approximately -40°C .

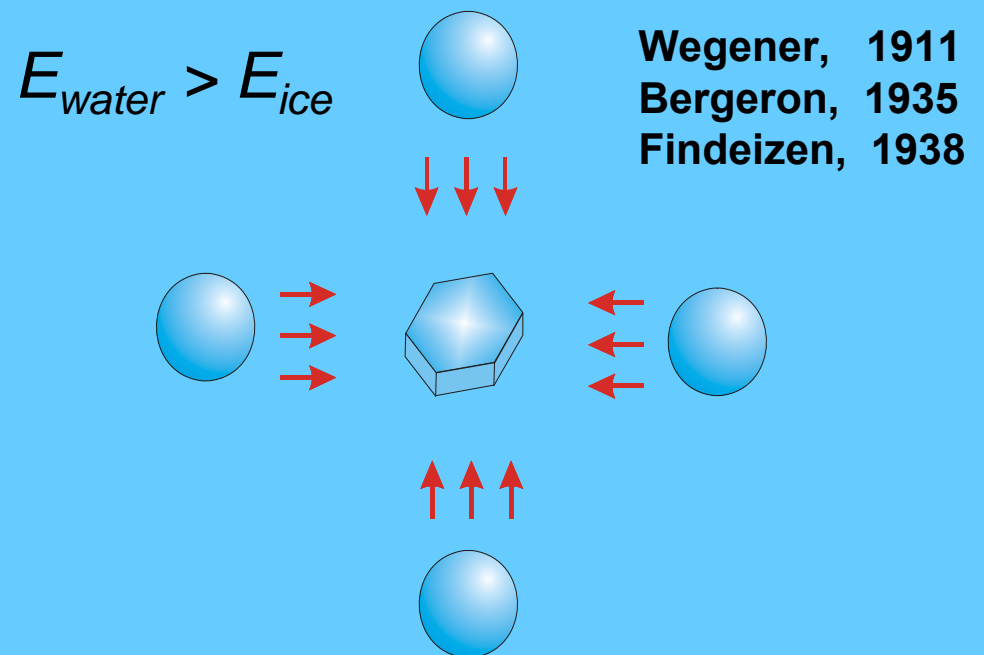


A population of cloud particles below 0°C may consist of a mixture of ice particles and liquid droplets.



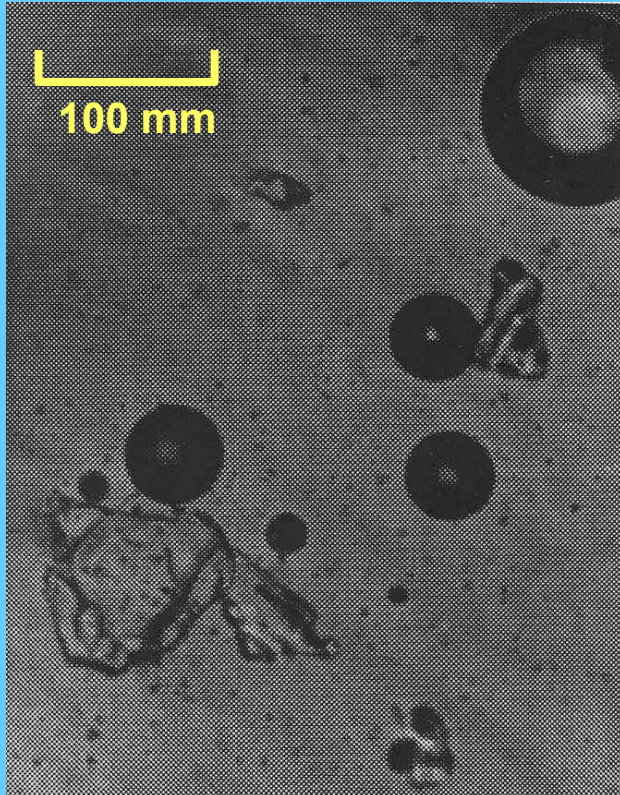
Mixed-phase clouds
 $-40 < T < 0\text{C}$

1. Mixed phase clouds is a three-phase system consisting of water vapour, liquid droplets and ice particles
2. Mixed phase clouds are fundamentally unstable
3. Important for precipitation formation, radiation budget
4. Aviation safety
5. Notoriously difficult for simulations in cloud and weather prediction and climate models



Early aircraft observations of mixed phase clouds:

Zak (1937), Peppler (1940), Weickmann (1945)



Ns, H=900m; T=-5C



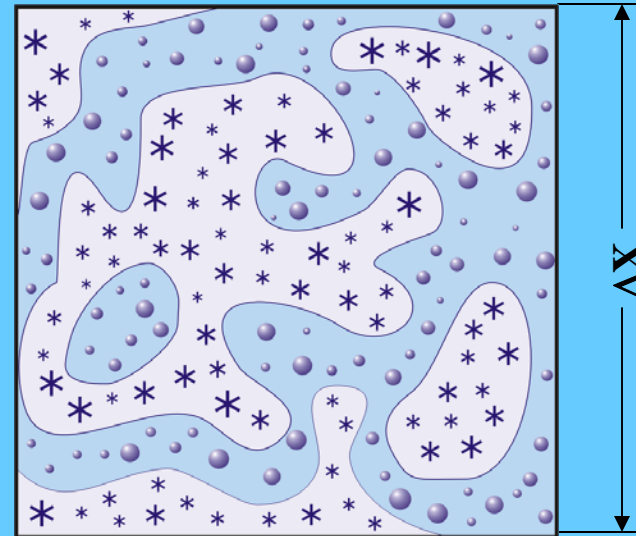
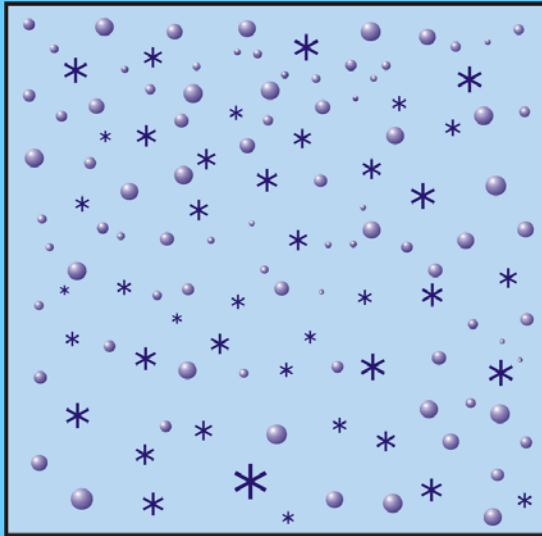
As, H=3060m; T=-19C



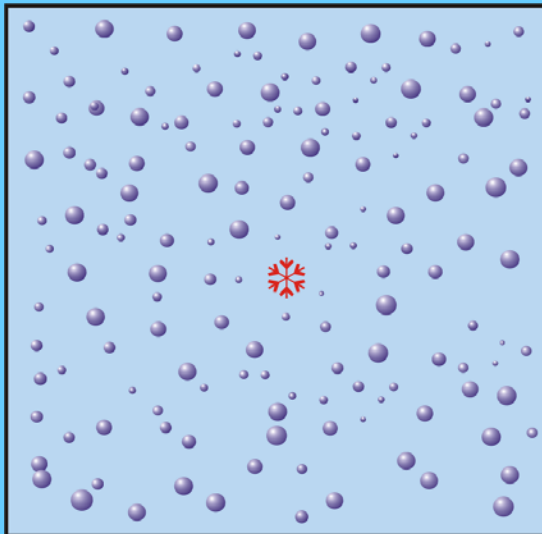
As, H=4300m; T=-27C

Borovokov, 1951

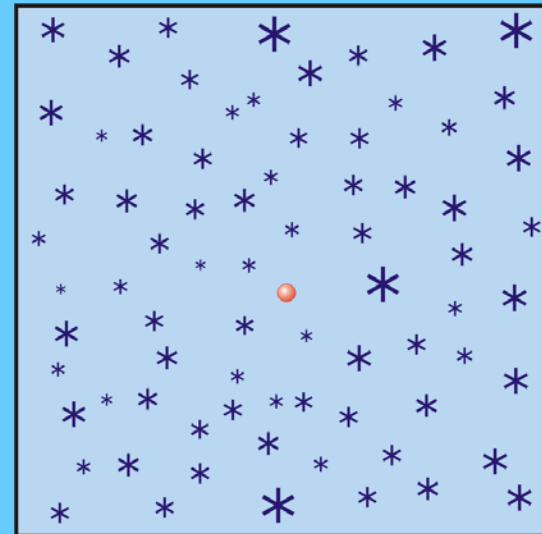
What is a mixed phase cloud?



$\Delta X = 1\text{cm?} \dots 10\text{km?}$



1 ice particle per 10^{100} droplets?



1 droplet per 10^{100} ice particles?

Definition of mixed phase clouds

$$\mu_n = \frac{\sum_j \alpha_{ice\ j} N_{ice\ j} D_{ice\ j}^n}{\sum_j \alpha_{ice\ j} N_{ice\ j} D_{ice\ j}^n + \sum_i \alpha_{liq\ j} N_{liq\ i} D_{liq\ i}^n} = \frac{\overline{\alpha_{ice} N_{ice} D_{ice}^n}}{\overline{\alpha_{ice} N_{ice} D_{ice}^n + \alpha_{liq} N_{liq} D_{liq}^n}}$$

Concentration (0th moment)

$$\mu_0 = \frac{N_{ice}}{N_{ice} + N_{liquid}}$$

Water content (3rd moment)

$$\mu_3 = \frac{W_{ice}}{W_{ice} + W_{liquid}}$$

Extinction coefficient (2nd moment)

$$\mu_2 = \frac{\beta_{ice}}{\beta_{ice} + \beta_{liquid}}$$

Radar reflectivity (6th moment)

$$\mu_6 = \frac{Z_{ice}}{Z_{ice} + Z_{liquid}}$$

range of changes

all liquid $0 < \mu < 1$ all ice

Definition of ice, liquid and mixed phase clouds in this study

TWC	$>0.01\text{g/m}^3$
Liquid cloud	$\text{IWC/TWC} \leq 0.1$
Mixed phase	$0.1 < \text{IWC/TWC} < 0.9$
Ice cloud	$\text{IWC/TWC} \geq 0.9$
Spatial resolution	100m

Type of clouds *St, Sc, Ns, As, Ac, Ci* associated with frontal systems

Total cloud length 61,770 km

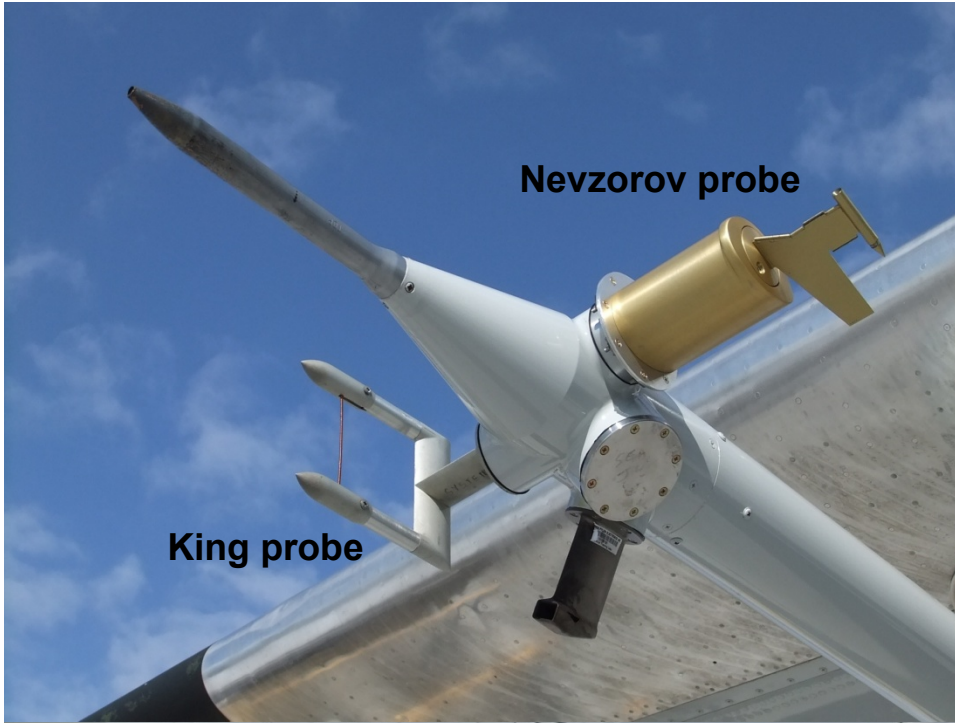
Temperature $-40 < T < 0\text{C}$

Height $0 < H < 7\text{km}$

Projects: BASE, CFDE1, CFDE3, FIRE.ACE, AIRS1, AIRS1.5, AIRS2

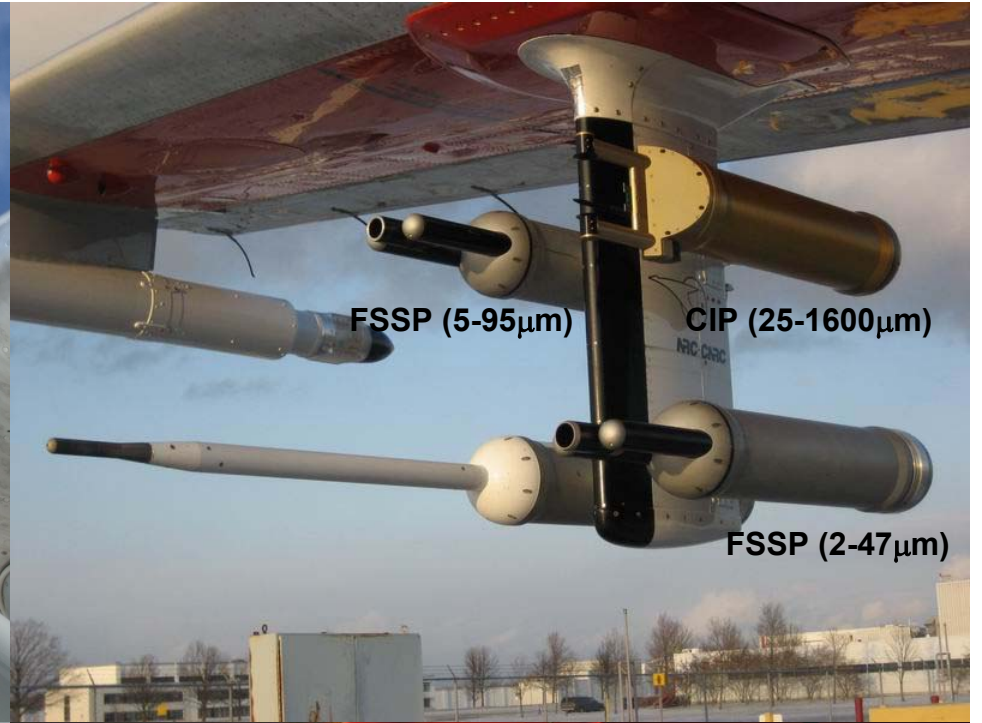
Convair 580
National Research Council of Canada





Nevzorov probe

King probe



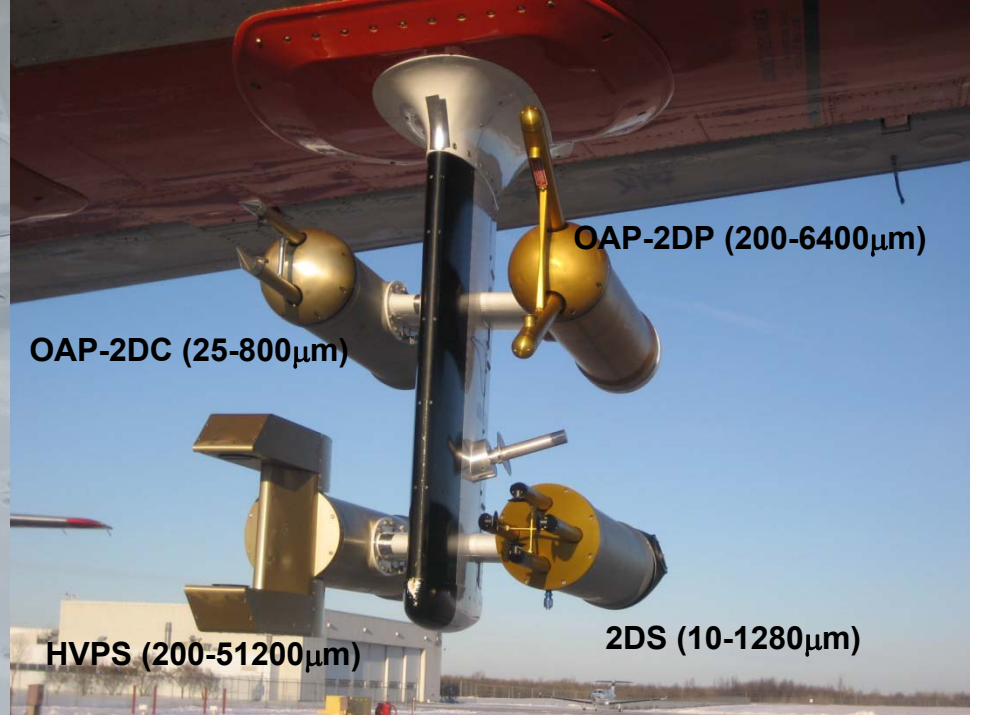
FSSP (5-95 μm)

CIP (25-1600 μm)

FSSP (2-47 μm)



Rosemount Icing Detector



OAP-2DC (25-800 μm)

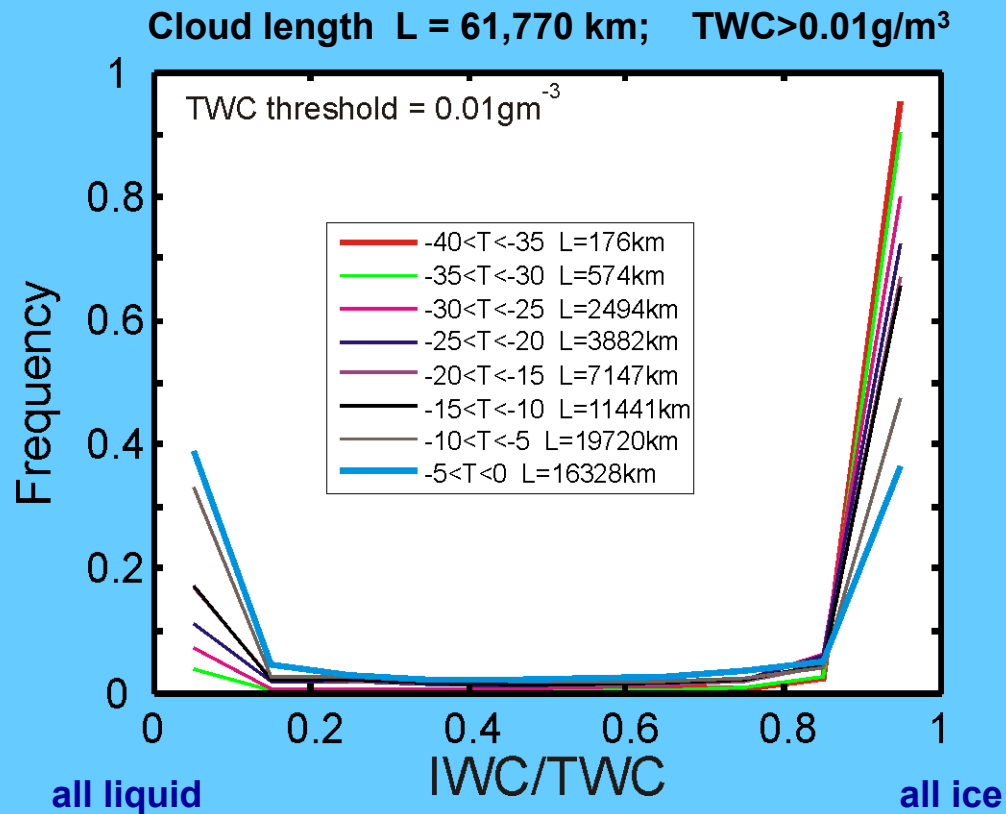
OAP-2DP (200-6400 μm)

HVPS (200-51200 μm)

2DS (10-1280 μm)

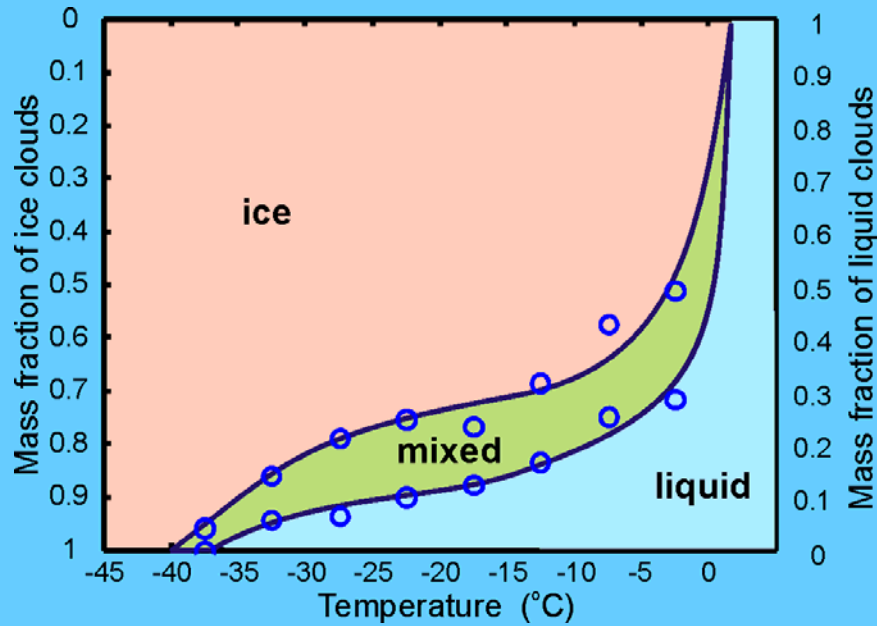


Frequency of occurrence of ice fraction in midlatitude stratiform clouds at $\Delta L=100\text{m}$



The probability density function of ice mass fraction in stratiform clouds has U-shape at $-35\text{C} < T < 0\text{C}$

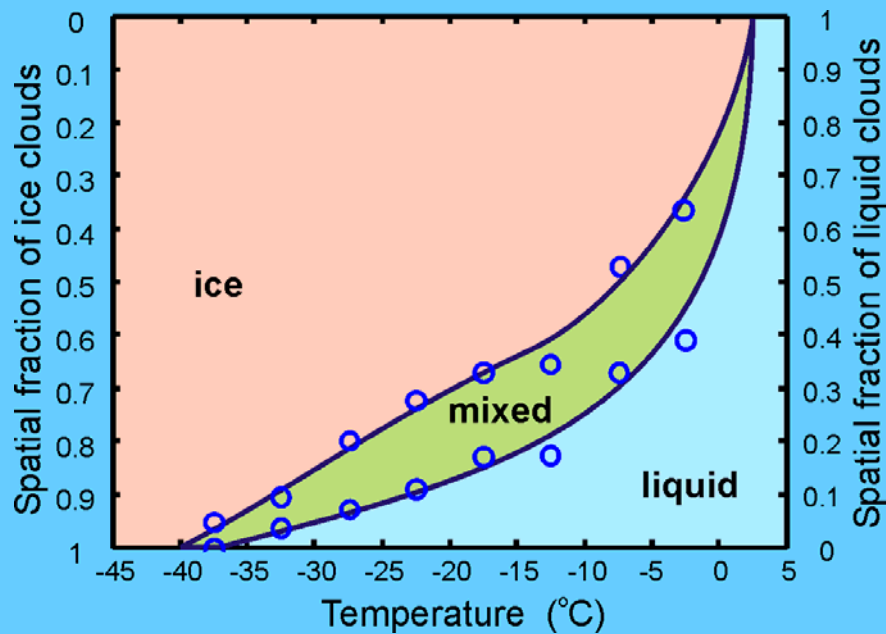
Frequency of occurrence of mixed phase clouds



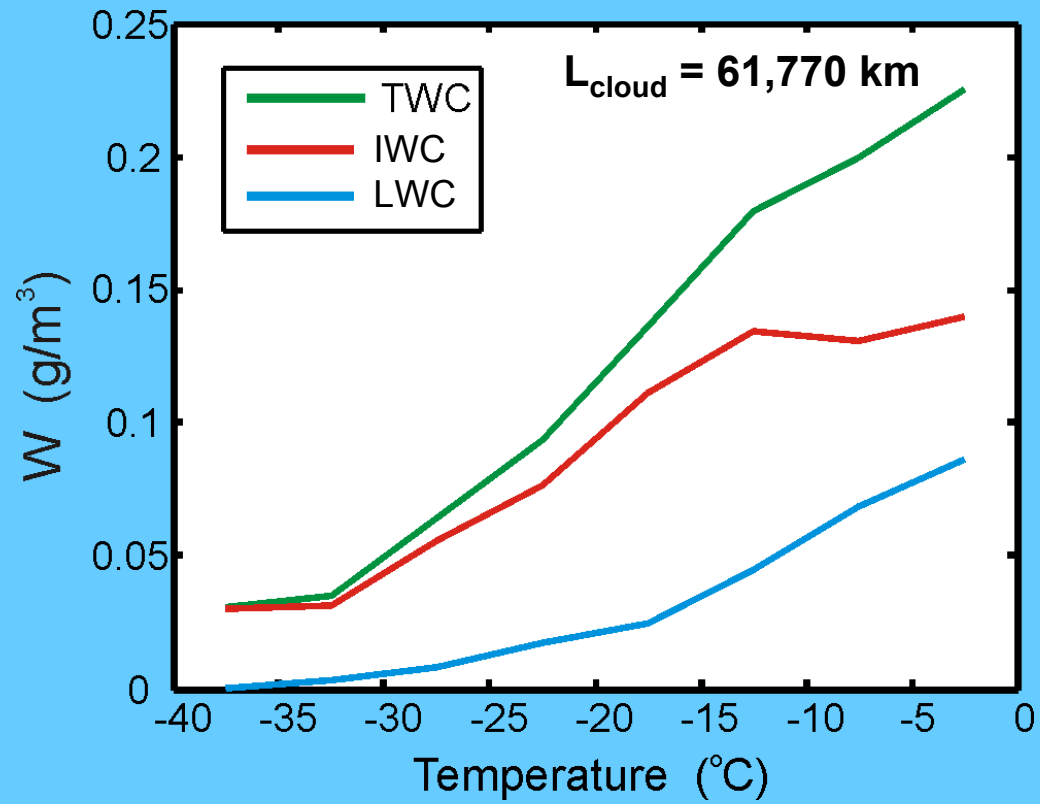
Cloud length $L = 61,770$ km

TWC threshold $= 0.01 \text{ kg/m}^3$

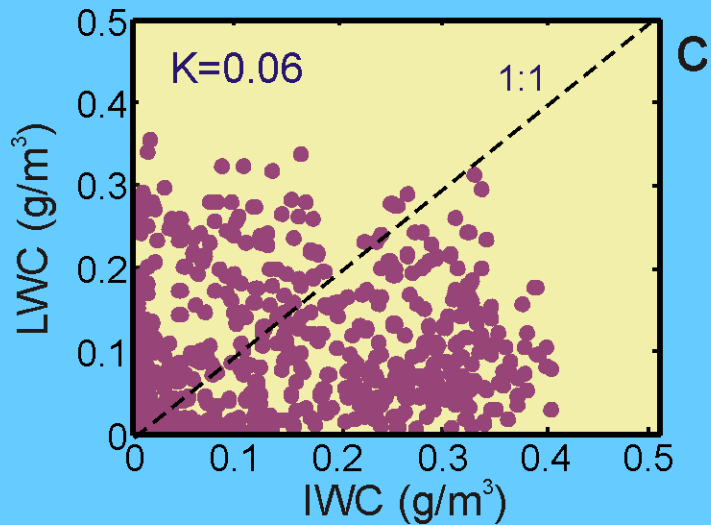
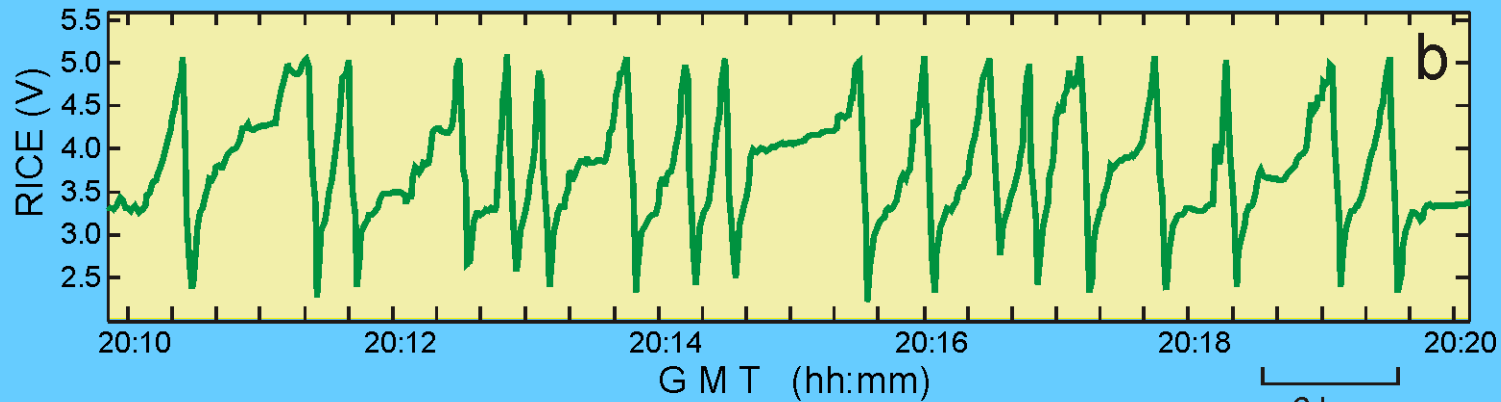
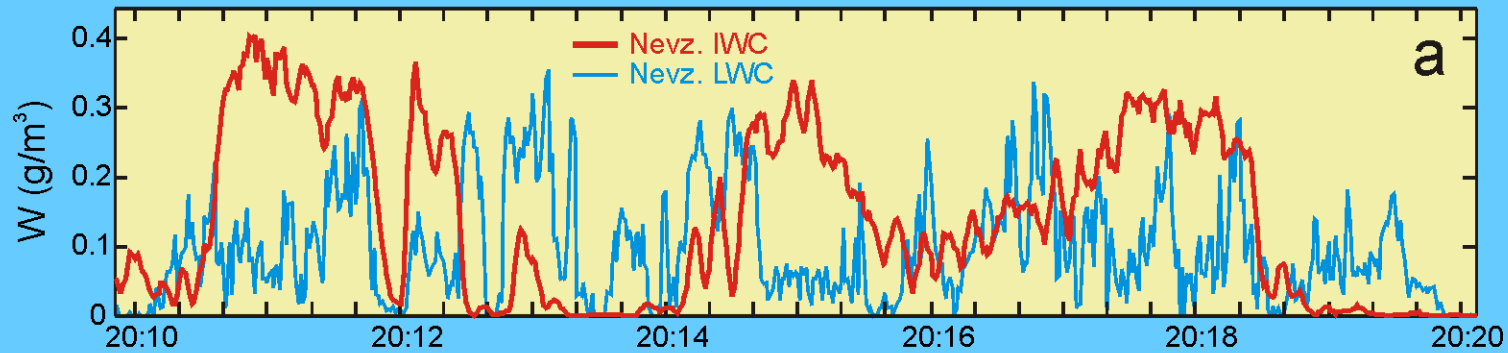
Spatial resolution $\Delta L = 100$ m



Average LWC, IWC, TWC in stratiform midlatitude clouds
(TWC > 0.01 g/m³)



16 December 1999; AIRS 20:09:45-20:20:12; T= -6°C; H=1200m

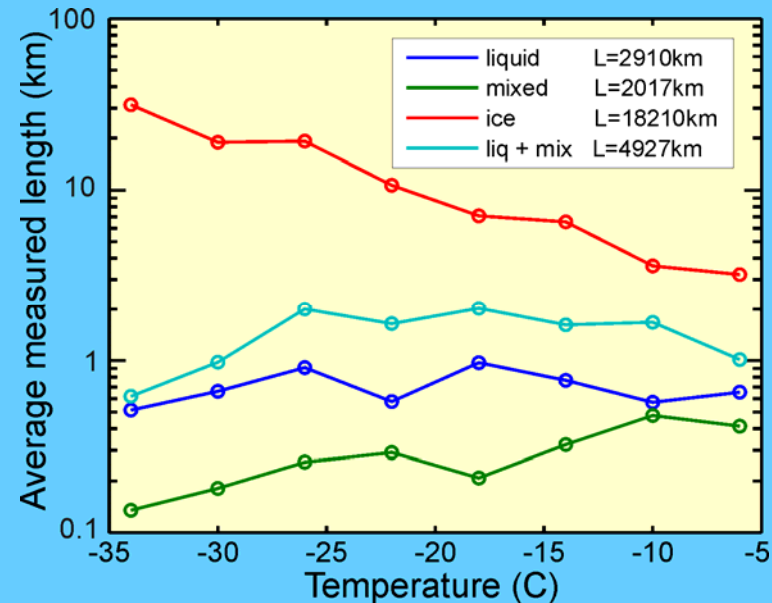
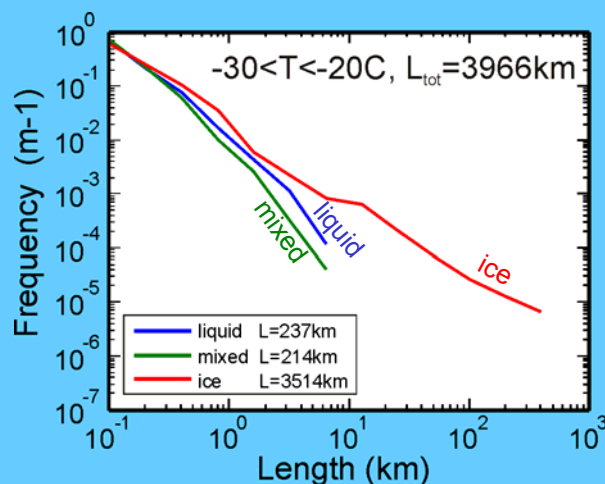
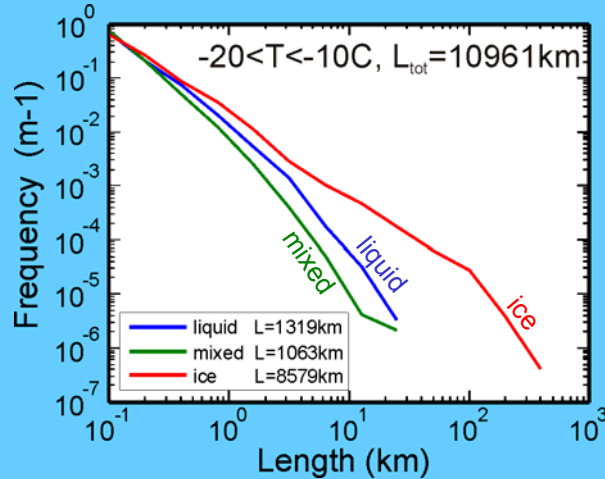
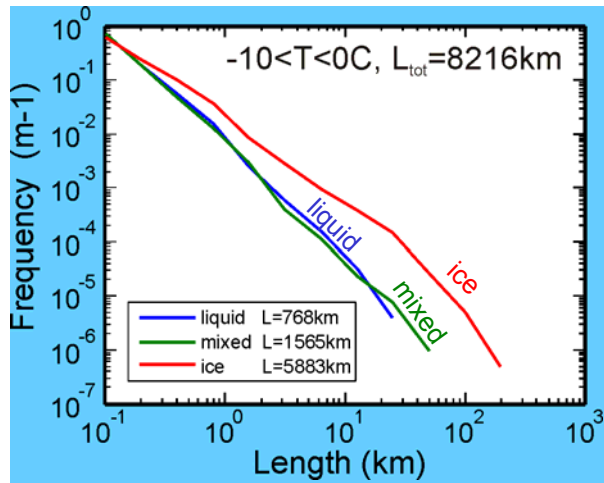


There is no correlation between liquid and ice phases in mixed phase clouds

Occurrence of lengths of liquid, mixed phase and ice cloud zones

TWC threshold=0.01kg/m³

Spatial resolution $\Delta L=100\text{m}$



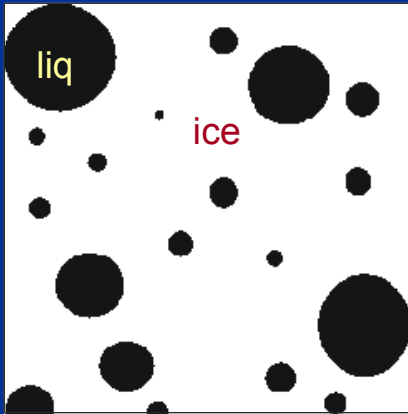
The average length of continuous liquid zones stays constant with temperature : $L_w \sim 500\text{m}$

The continuous length of mixed phase zones decreases with decrease of T : $100\text{m} < L_m < 500\text{m}$

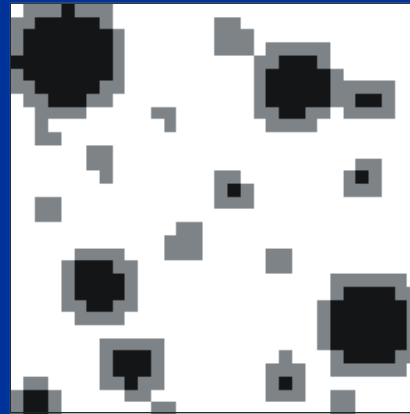
The continuous length of ice zones increases with the decrease of T : $4\text{km} < L_i < 40\text{km}$

Effect of averaging on the mixed phase measurements

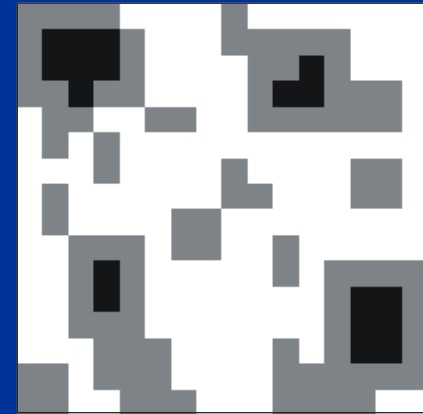
$\Delta L \sim 0$



$\Delta L = L/32$



$\Delta L = L/16$



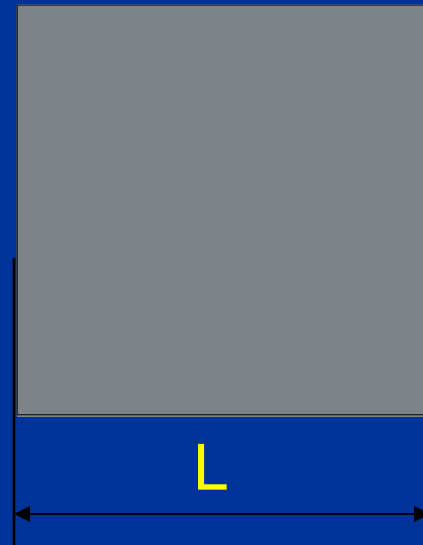
$\Delta L = L/8$



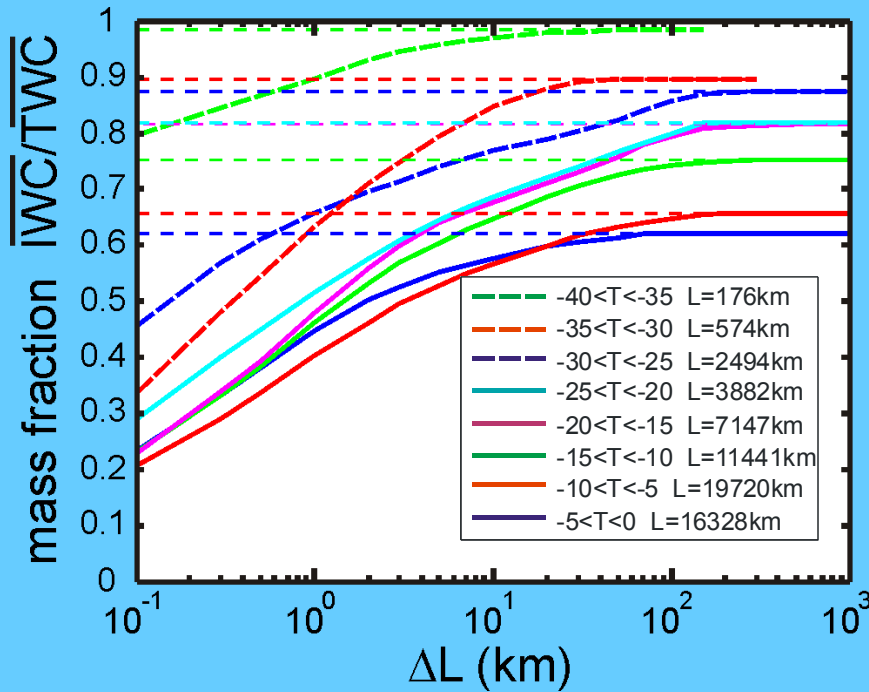
$\Delta L = L/4$ saturation scale



$\Delta L = L/2$

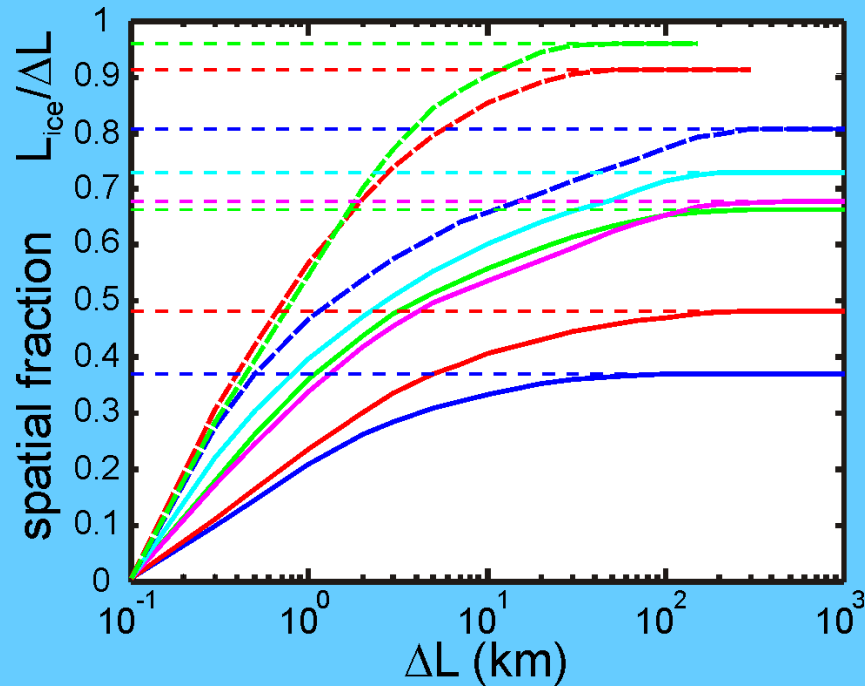


Spatial and mass fractions of ice in liquid containing clouds at different averaging scales ΔL



Cloud length $L = 61,770$ km
TWC threshold = 0.01 kg/m^3

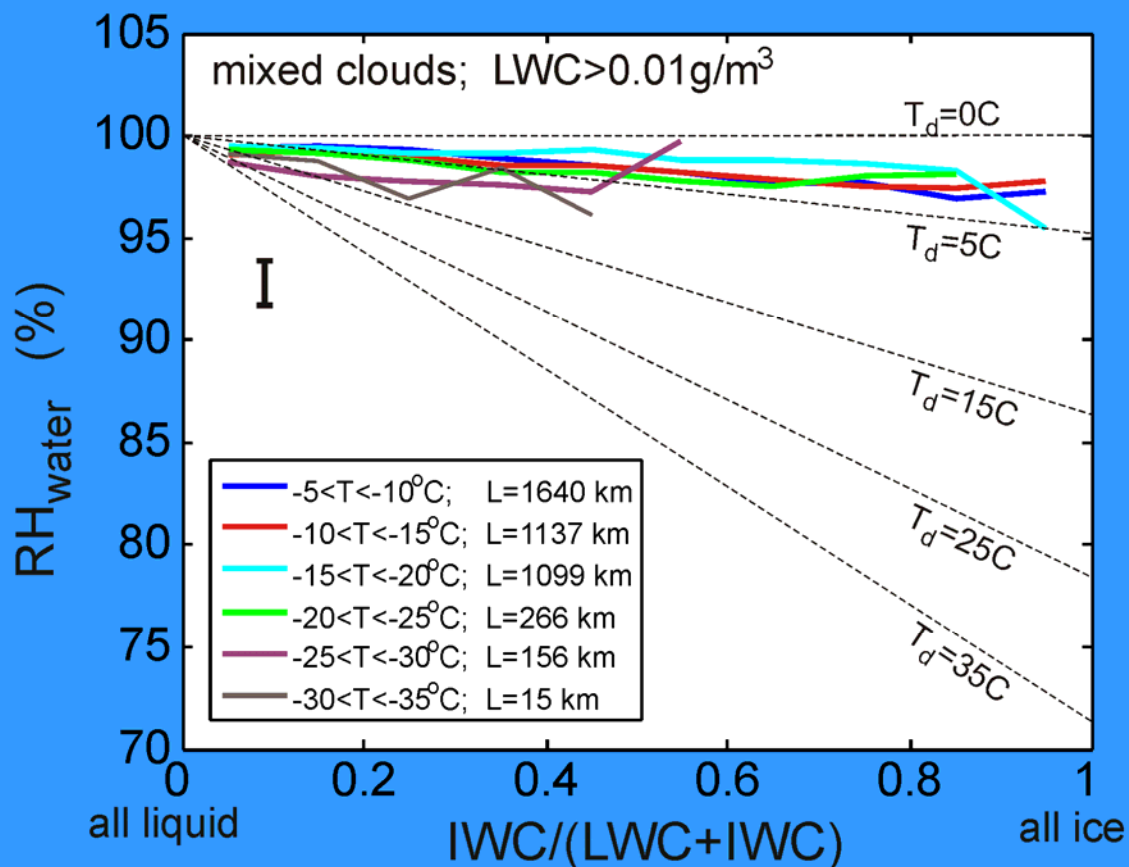
**Saturation spatial scale
 $\Delta L_s \sim 70 \div 200 \text{ km}$**



**Cold clouds ($-40 < T < 0 \text{ C}$)
at the averaging scale $L > \Delta L_s$
are always mixed phase**

Relative Humidity in Mixed Clouds

(averaging scale 100m)



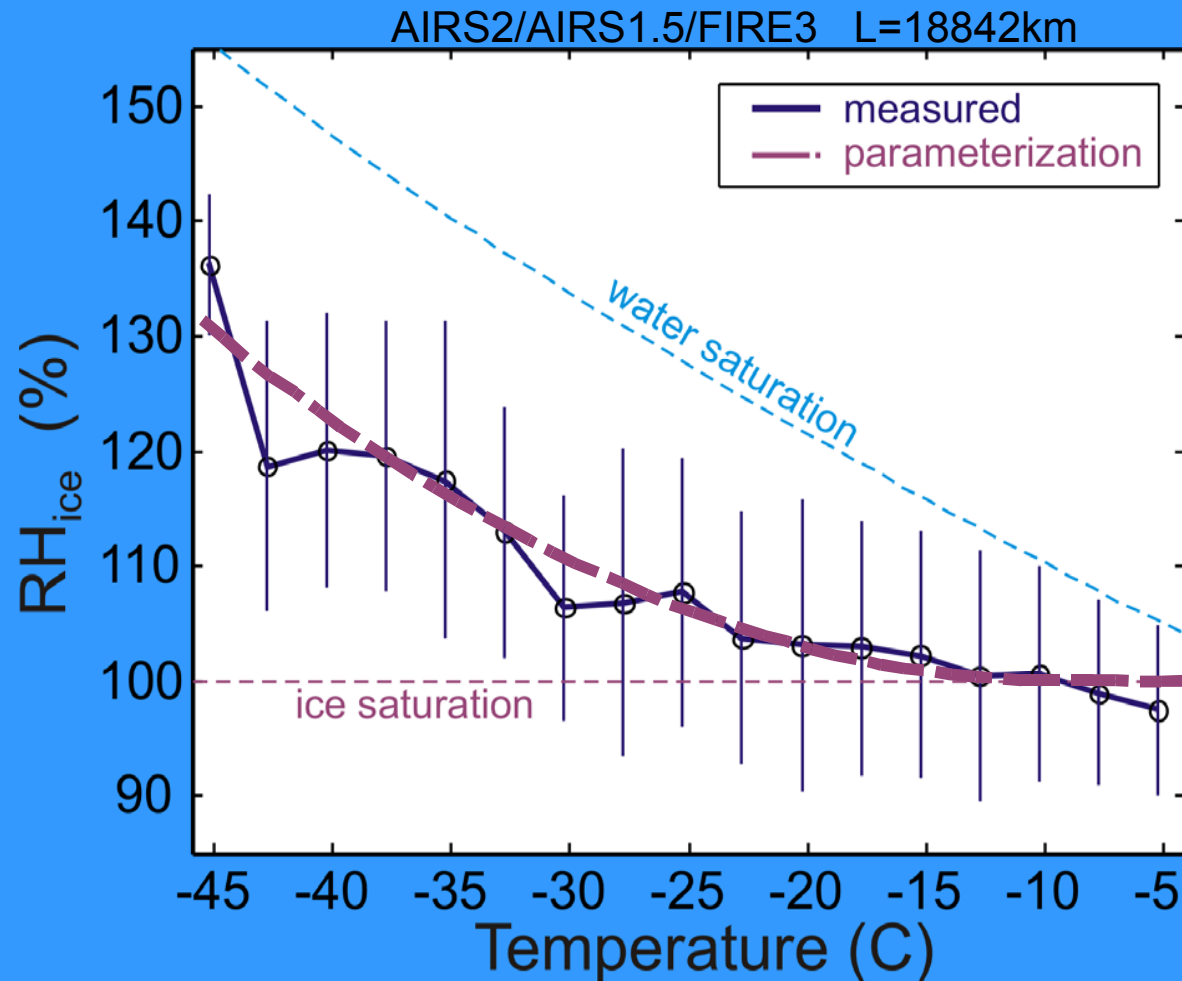
$$RH = \frac{RH_w m_w + RH_i m_i}{m_w + m_i}$$

parameterization of RH in mixed clouds
used in cloud and mesoscale simulations

Here m_w and m_i are the masses of
liquid and ice

Relative humidity in mixed phase clouds is close to saturation over water at all T and μ .

Relative Humidity in Ice Clouds ($\Delta x=100m$)



$$RH_{ice} = 0.0191 T^2 + 0.2093 T + 99$$

Theoretical considerations

1. Theoretical framework
2. Characteristic time scales of the phase transformation
3. Humidity in mixed phase clouds
4. Glaciation of mixed phase clouds
5. Conditions for maintenance of mixed phase

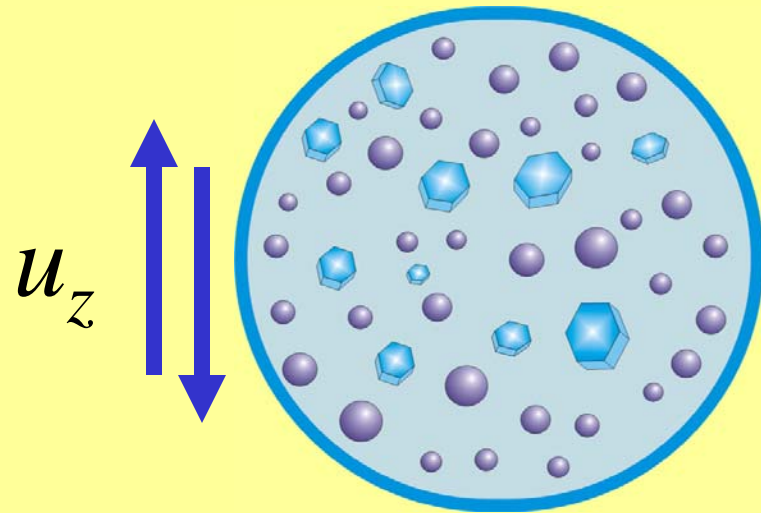
Basic assumptions:

1. adiabatic parcel
2. ice particles and droplets are uniformly mixed
3. ice concentration $N_{\text{ice}}(t) = \text{const}$
4. droplet concentration $N_{\text{droplets}}(t) = \text{const}$
5. temperature $T(x,y,z) = \text{const}$
6. water vapor $E(x,y,z) = \text{const}$

$$\tau_{\text{turb}} \ll \tau_{\text{diff}}; \quad K_{\text{turb}} \gg K_{\text{diff}} \Rightarrow$$

\Rightarrow regular condensation

Mixed-phase cloud parcel



Equation for supersaturation in mixed phase cloud

$$S_w = \frac{e - E_w}{E_w} \quad \text{supersaturation over liquid water (definition)}$$

$$\frac{dS_w}{dt} = \frac{1}{E_w} \frac{de}{dt} - \frac{e}{E_w^2} \frac{dE_w}{dt} \Rightarrow$$

$$\frac{1}{S_w + 1} \frac{dS_w}{dt} = a_0 u_z - a_2 N_i \sqrt{r_{i0}^2 + 2cA_i \int_0^t (\xi S_w(t') + \xi - 1) dt'} -$$

$$- \left(a_1 B_w N_w \sqrt{r_{w0}^2 + 2A_w \int_0^t S_w(t') dt'} + a_2 B_i N_i \sqrt{r_{i0}^2 + 2cA_i \int_0^t (\xi S_w(t') + \xi - 1) dt'} \right) S_w$$

Features:

1. Single variable equation
2. No analytical solution
3. Easy to integrate into numerical models
4. Gives accurate solution for $-1\text{km} < \Delta Z < 1\text{km}$

E_w	saturation vapor pressure over water
r_w, r_i	radii of droplets and ice particles
N_w, N_i	concentration of droplets and ice particles
u_z	vertical velocity
a, b, B	coefficients dependent on P, T

Equation for supersaturation in mixed phase cloud

$$\frac{1}{S_w + 1} \frac{dS_w}{dt} = a_0 u_z - a_2 B_i^* N_i \bar{r}_i - (a_1 B_w N_w \bar{r}_w + a_2 B_i N_i \bar{r}_i) S_w$$

$$\begin{aligned} \bar{r}_i &= \text{const} \\ \bar{r}_w &= \text{const} \end{aligned}$$

quasi-steady approximation

$$S_w = \frac{S_{qs\ w} - C_0 \exp(-t/\tau_p)}{1 + C_0 \exp(-t/\tau_p)}$$

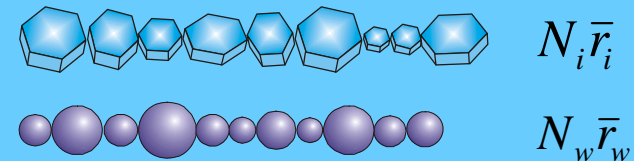
$$S_{qs\ w} = \frac{a_0 u_z - b_i^* N_i \bar{r}_i}{b_w N_w \bar{r}_w + b_i N_i \bar{r}_i}$$

quasi-steady supersaturation

$$\tau_p = \frac{1}{a_0 u_z + b_w N_w \bar{r}_w + (b_i + b_i^*) N_i \bar{r}_i}$$

time of phase relaxation

Equation for supersaturation
in mixed-phase clouds
(Korolev and Mazin, 2003, JAS)



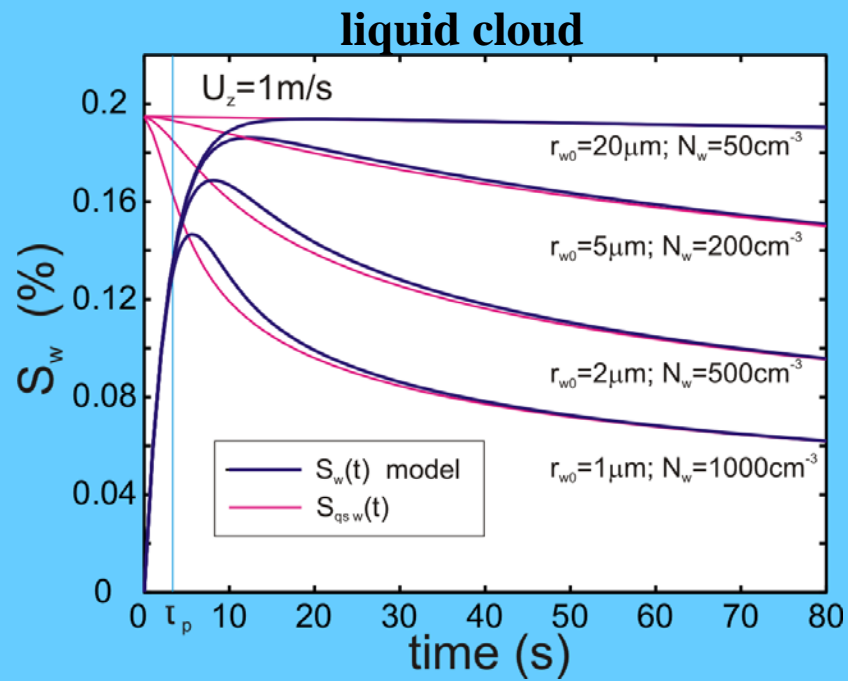
$N_i \bar{r}_i$ integral radius of ice particles

$N_w \bar{r}_w$ integral radius of droplets

u_z vertical velocity

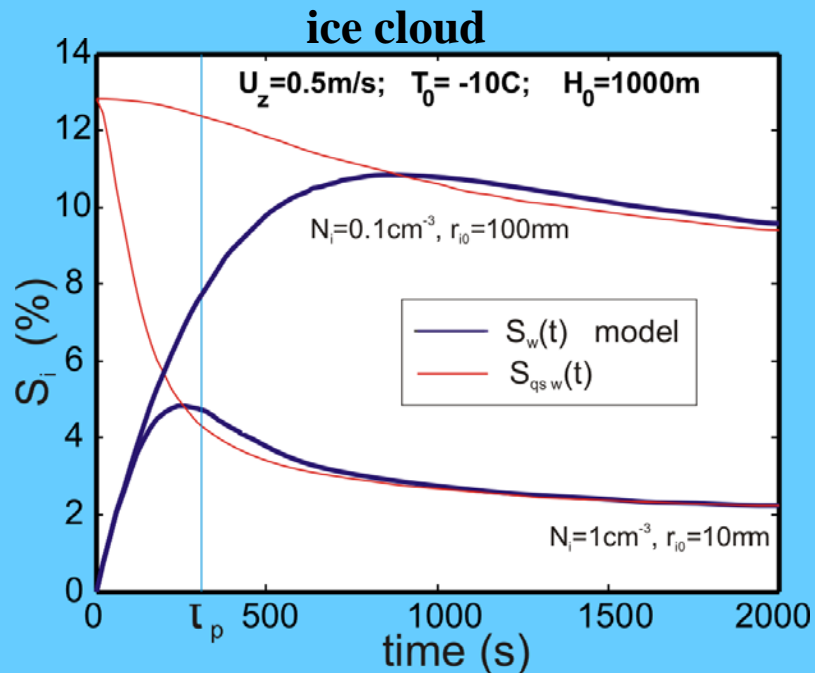
a, b, B coefficients dependent of P, T

Asymptotic behavior of S_{qs}



$$\lim_{t \rightarrow \infty} S_{qs}(t) = S(t)$$

$$S_{qs\ w} = \frac{a_0 u_z - b_i^* N_i \bar{r}_i}{b_w N_w \bar{r}_w + b_i N_i \bar{r}_i}$$

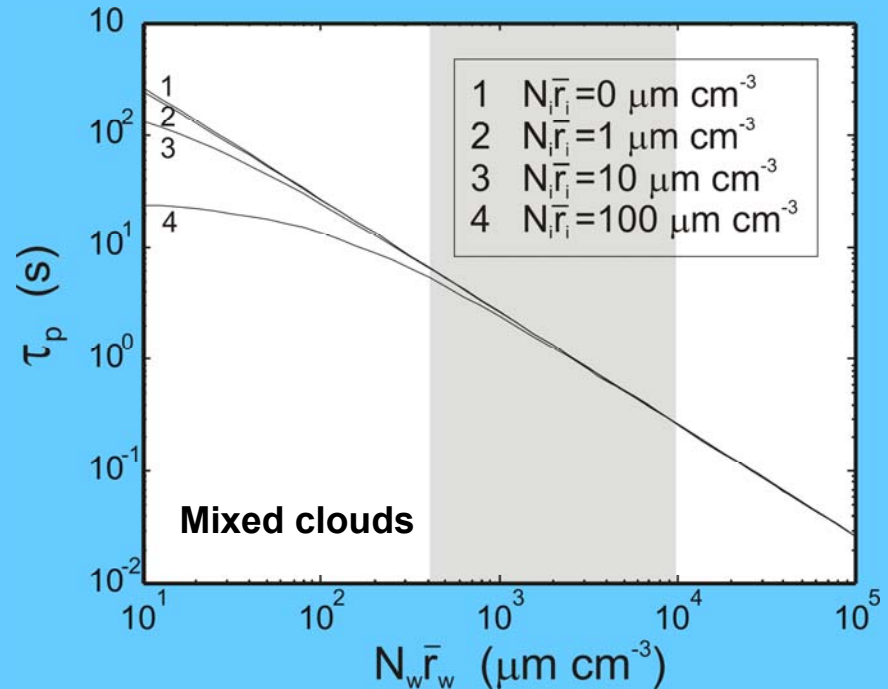
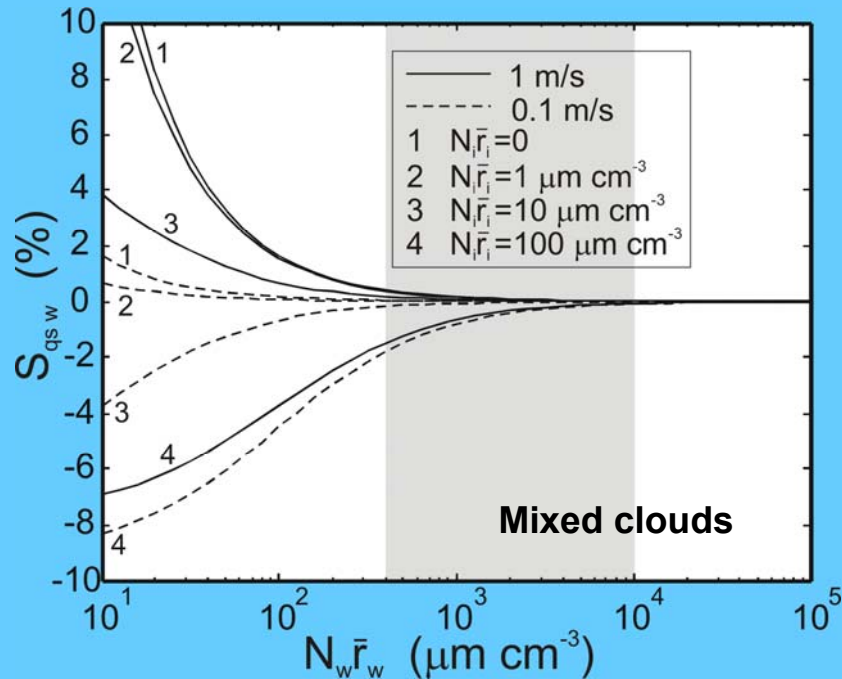


$$\tau_p = \frac{1}{a_0 u_z + b_w N_w \bar{r}_w + (b_i + b_i^*) N_i \bar{r}_i}$$

Supersaturation and time of phase relaxation in mixed phase clouds

$$S_{qs\ w} = \frac{a_0 u_z - b_i N_i \bar{r}_i}{b_w N_w \bar{r}_w + b_i N_i \bar{r}_i}$$

$$\tau_p = \frac{1}{a_0 u_z + b_w N_w \bar{r}_w + (b_i + b_i^*) N_i \bar{r}_i}$$



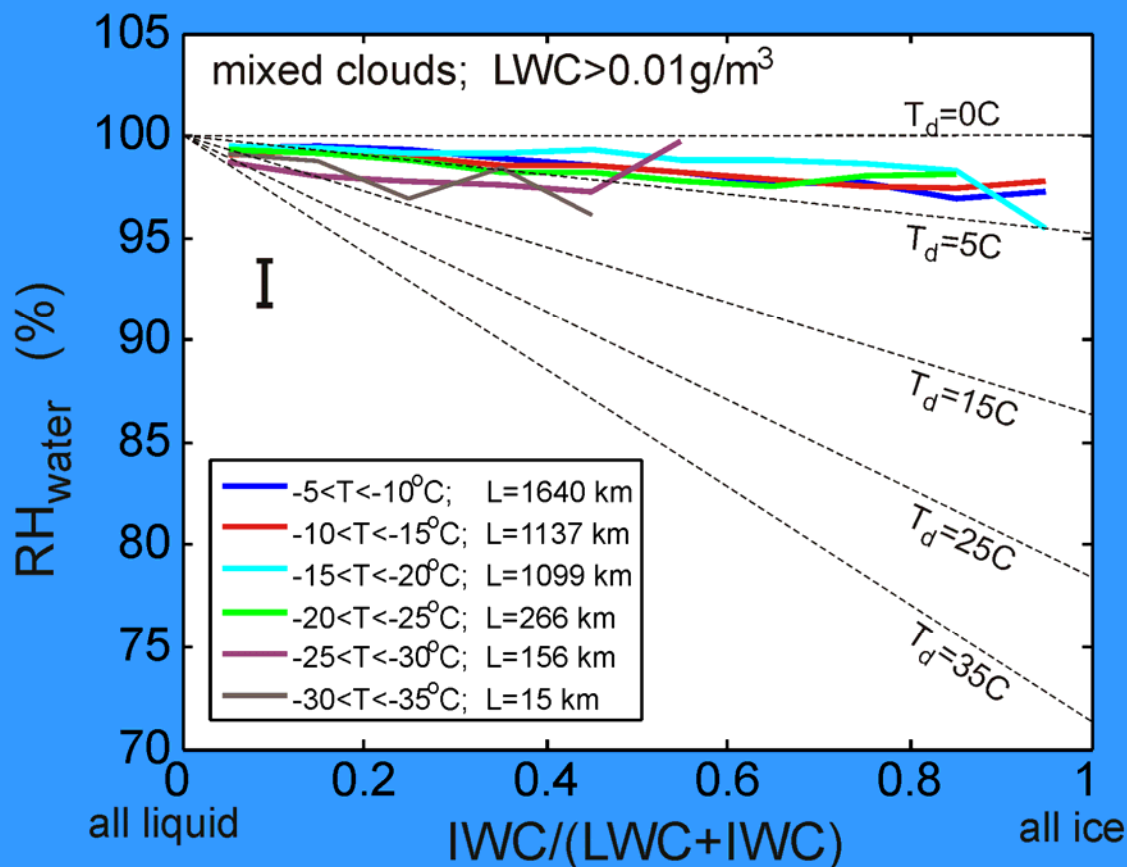
$RH_w \sim 100\%$

$\tau_p \sim 10^0 \text{ s}$

- RH in mixed phase clouds is close to saturation over water.
- Time of phase relaxation is of the order of seconds.

Relative Humidity in Mixed Clouds

(averaging scale 100m)



$$RH = \frac{RH_w m_w + RH_i m_i}{m_w + m_i}$$

parameterization of RH in mixed clouds
used in cloud and mesoscale simulations

Here m_w and m_i are the masses of
liquid and ice

Relative humidity in mixed phase clouds is close to saturation over water at all T and μ .

Rates of phase transformation in mixed-phase clouds

liquid droplets

$$\frac{dq_w}{dt} = B_w S_w N_w \bar{r}_w$$

ice particles

$$\frac{dq_i}{dt} = B_{i0} S_i N_i \bar{r}_i c$$

water vapor

$$\frac{dq_v}{dt} = -\frac{dq_w}{dt} - \frac{dq_i}{dt}$$

\Rightarrow

$$\dot{q}_w = \frac{(a_0 u_z - b_i^* N_i \bar{r}_i) B_w N_w \bar{r}_w}{b_w N_w \bar{r}_w + b_i N_i \bar{r}_i}$$

\Rightarrow

$$\dot{q}_i = \frac{\left(a_0 u_z - \frac{1-\xi}{\xi} b_w N_w \bar{r}_w \right) B_i N_i \bar{r}_i}{b_w N_w \bar{r}_w + b_i N_i \bar{r}_i}$$

\Rightarrow

$$\dot{q}_v = \frac{B_w B_i^* (a_1 - a_2) N_w \bar{r}_w N_i \bar{r}_i - a_0 u_z (B_w N_w \bar{r}_w + B_i N_i \bar{r}_i)}{b_w N_w \bar{r}_w + b_i N_i \bar{r}_i}$$

$$S_i = \xi S_w + \xi - 1$$

$$S_{qsw} = \frac{a_0 u_z - b_i^* N_i \bar{r}_i}{b_w N_w \bar{r}_w + b_i N_i \bar{r}_i}$$

q_w mixing ratio of liquid

q_i mixing ratio of ice

q_v mixing ratio of water vapor

Three equilibrium points of phase transformation in mixed-phase clouds

Liquid water equilibrium: $\dot{q}_w = 0$

$$\dot{q}_w = \frac{(a_0 u_z - b_i^* N_i \bar{r}_i) B_w N_w \bar{r}_w}{b_w N_w \bar{r}_w + b_i N_i \bar{r}_i}$$

\Rightarrow

$$u_z^* = \frac{E_w - E_i}{E_i} \eta N_i \bar{r}_i$$

$$10^{-2} < u_z^* < 10^0 \text{ m/s}$$

threshold velocity for
liquid water equilibrium

Ice equilibrium: $\dot{q}_i = 0$

$$\dot{q}_i = \frac{\left(a_0 u_z - \frac{1-\xi}{\xi} b_w N_w \bar{r}_w \right) B_i N_i \bar{r}_i}{b_w N_w \bar{r}_w + b_i N_i \bar{r}_i}$$

\Rightarrow

$$u_z^o = \frac{E_i - E_w}{E_w} \chi N_w \bar{r}_w$$

$$-10^3 < u_z^o < -10^0 \text{ m/s}$$

threshold velocity for
ice equilibrium

Water vapor equilibrium: $\dot{q}_v = 0$

$$\dot{q}_v = \frac{B_w B_i^* (a_1 - a_2) N_w \bar{r}_w N_i \bar{r}_i - a_0 u_z (B_w N_w \bar{r}_w + B_i N_i \bar{r}_i)}{b_w N_w \bar{r}_w + b_i N_i \bar{r}_i} \Rightarrow$$

$$u_z^+ = \frac{(\xi - 1)(B_w b_i - b_w B_i) N_w \bar{r}_w N_i \bar{r}_i}{a_0 \xi (B_w N_w \bar{r}_w + B_i N_i \bar{r}_i)}$$

$$10^{-4} < u_z^+ < 10^{-2} \text{ m/s} \sim 0 \text{ m/s}$$

threshold velocity for
water vapor equilibrium

Wegener, 1911: *Thermodynamik der Atmosphäre*

“The vapour tension will adjust itself to a value in between the saturation values over ice and over water. The effect of this must then be, that condensation continuously will take place on the ice, whereas at the same time liquid water evaporates, and this process must go on until the liquid phase is entirely consumed”

$$E_i < e < E_w$$
$$\dot{q}_w < 0$$
$$\dot{q}_i > 0$$

The Glossary of Meteorology (2000)

“Bergeron-Findeisen process”:

*“... The basis of this theory is in fact that the equilibrium water vapour pressure with respect to ice is less than that with respect to liquid at the same subfreezing temperature. Thus within an admixture of these (ice and liquid) particles, and provided that the total water content were sufficiently high, **the ice crystals would gain mass by vapour deposition at the expense of the liquid drops that would lose their mass by evaporation**”.*



Four scenarios of mixed phase evolution

$$u_z^o < u_z^+ < u_z^*$$

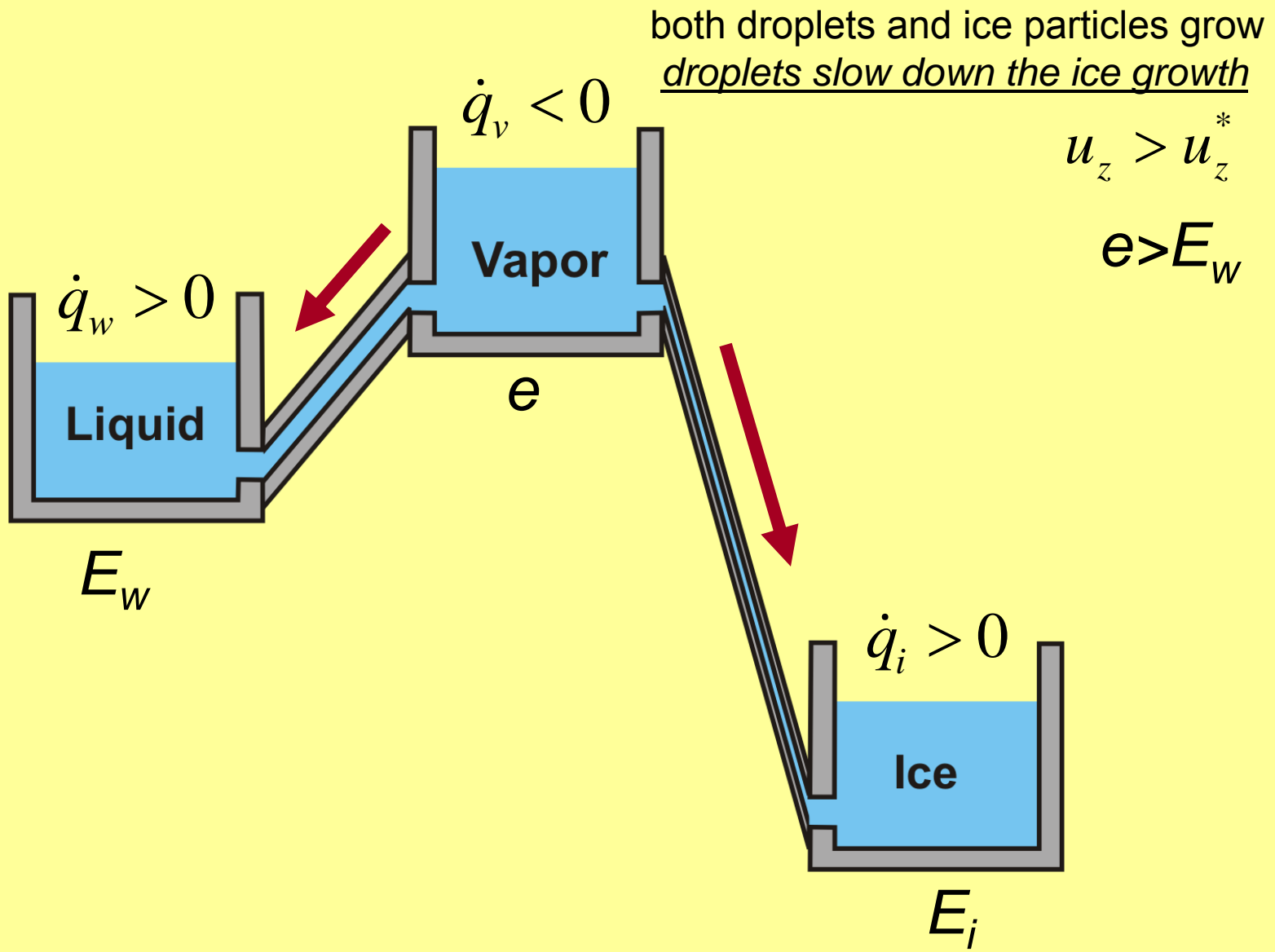
ice vapor liquid

always true
for any $N_i r_i, N_w r_w$

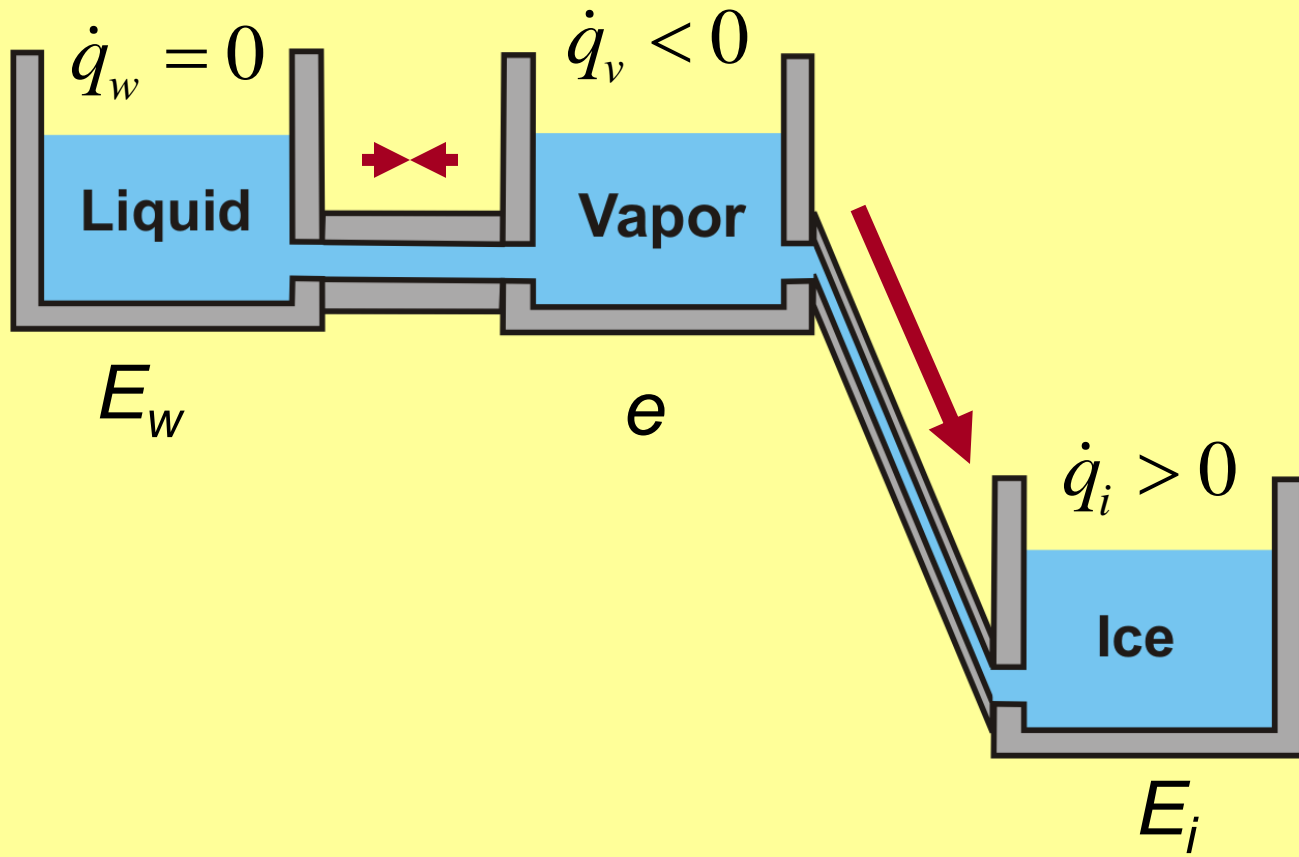
\Rightarrow four possible inequalities for u_z

u_z	e	liquid 	vapour	ice 	
$u_z < u_z^o$	$e < E_i$	evaporate	increase	evaporate	<u>not WBF</u>
$u_z^o < u_z < u_z^+$	$E_i < e < E_v$	evaporate	increase	grow	WBF
$u_z^+ < u_z < u_z^*$	$E_v < e < E_w$	evaporate	decrease	grow	WBF
$u_z > u_z^*$	$e > E_w$	grow	decrease	grow	<u>not WBF</u>

water vapor pressure



water vapor pressure ↑



droplets in equilibrium
ice particles grow

$$u_z = u_z^*$$

$$e = E_w$$

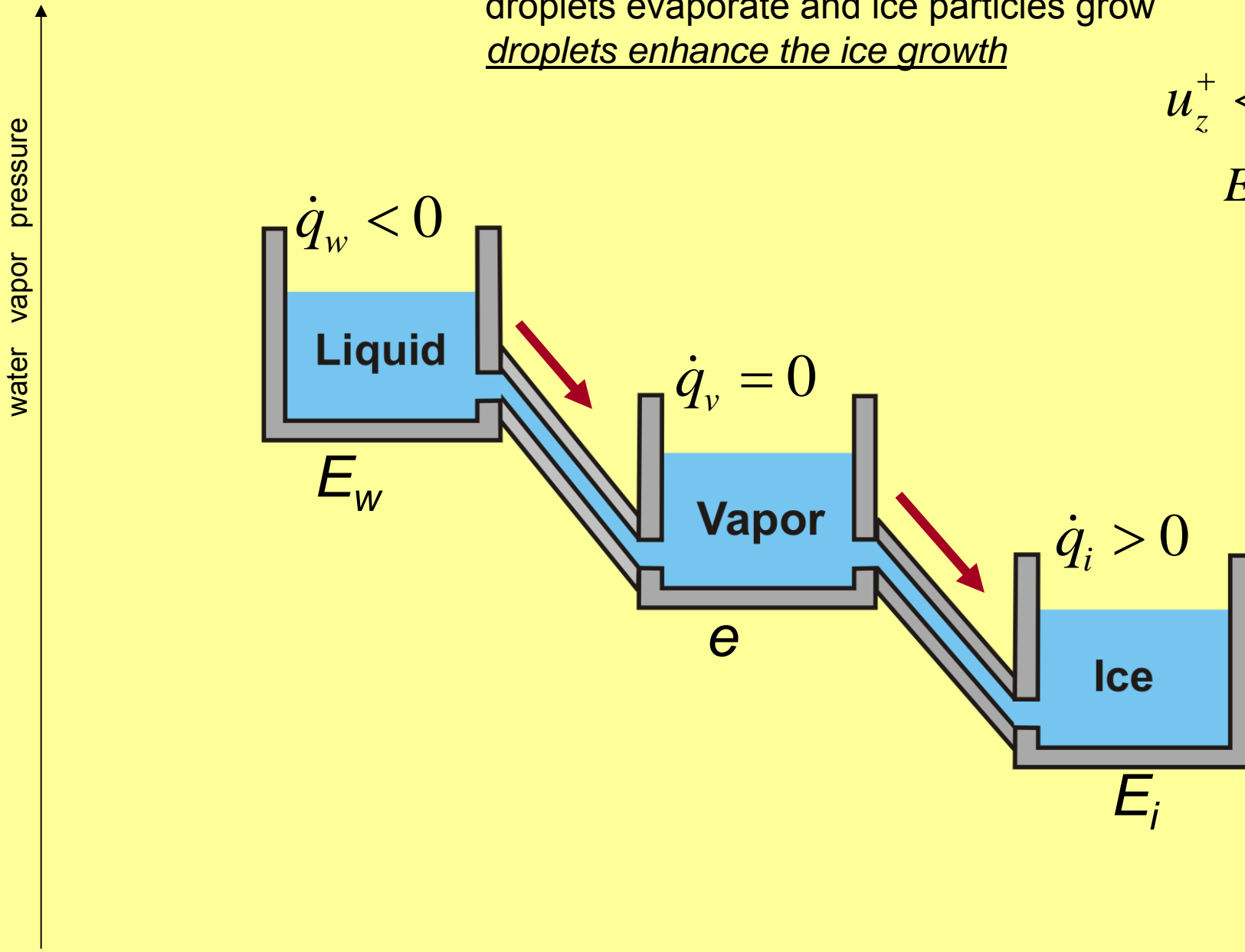
condition for Wegener-Bergeron-Findeisen mechanism

droplets evaporate and ice particles grow

droplets enhance the ice growth

$$u_z^+ < u_z < u_z^*$$

$$E_i < e < E_w$$

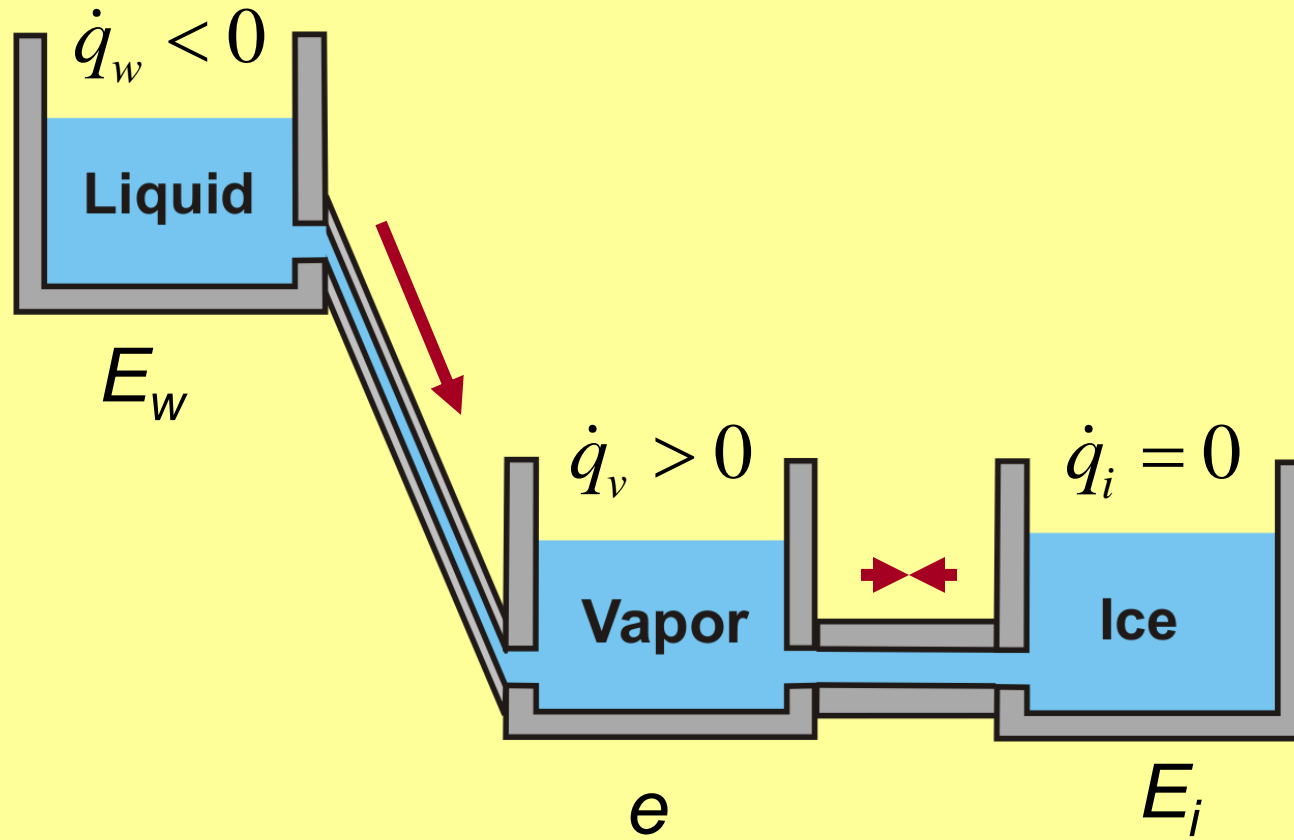


droplets evaporate
ice particles in equilibrium

$$u_z = u_z^o$$

$$e = E_i$$

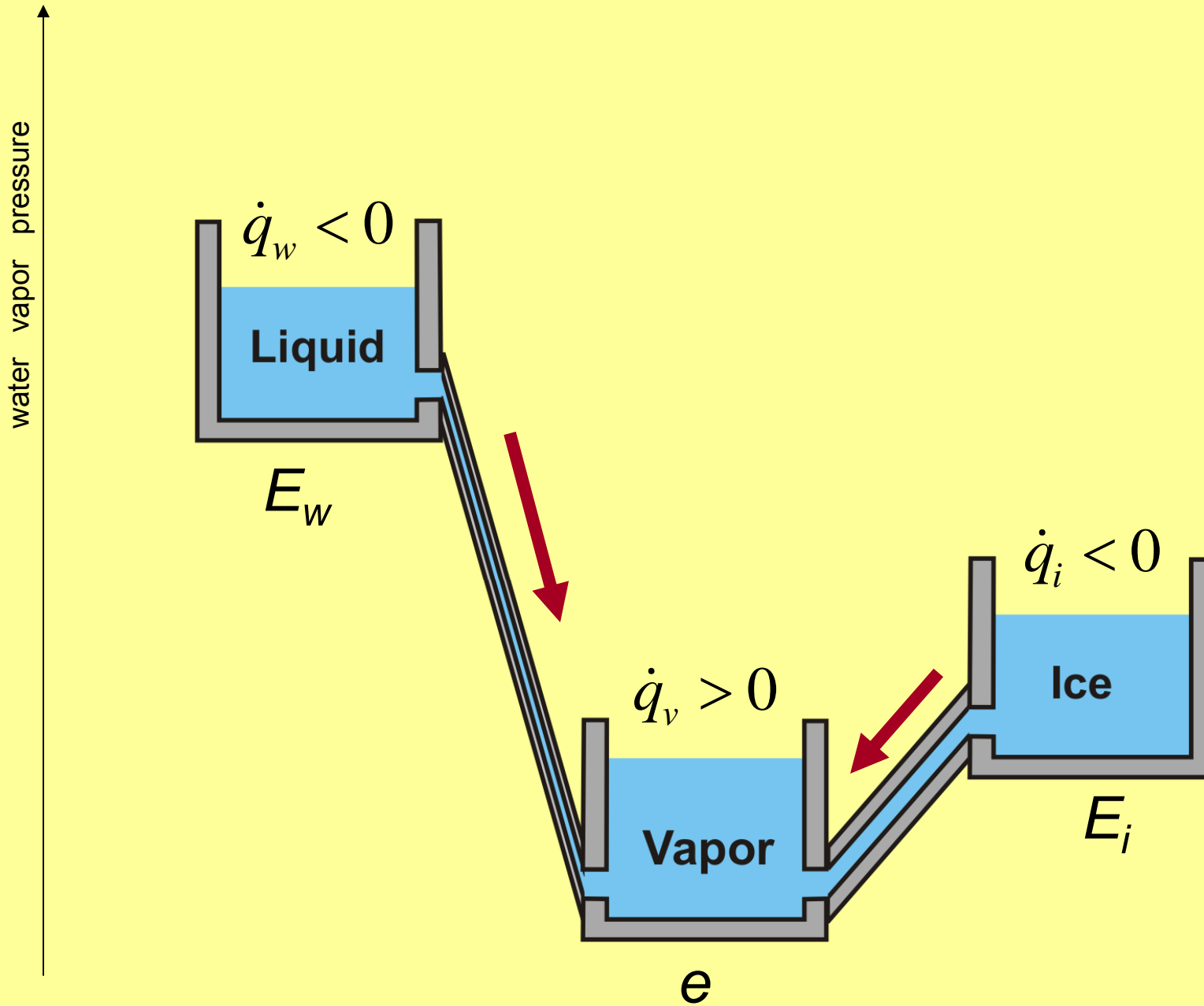
water vapor pressure



both ice particle and droplets evaporate

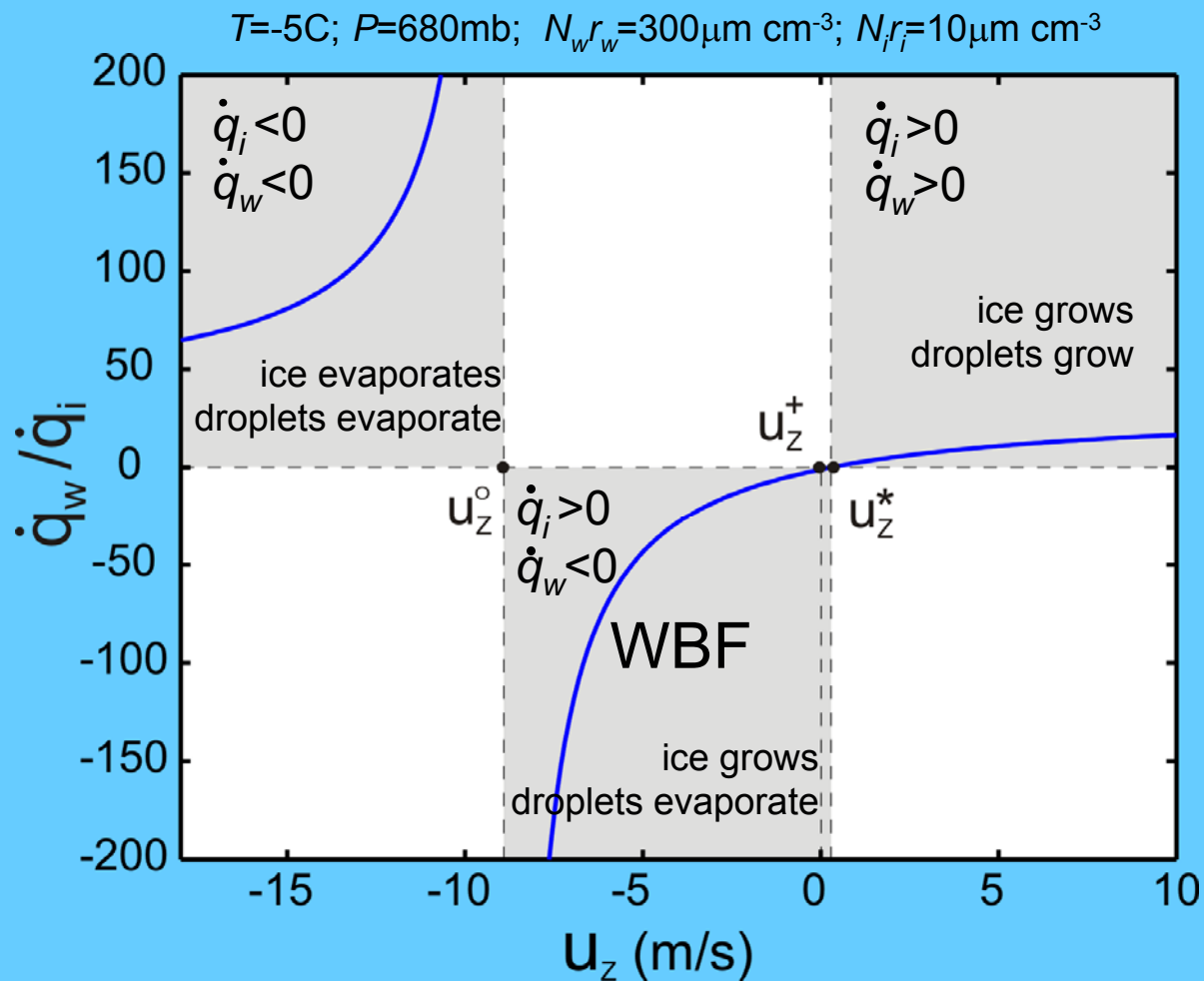
$$u_z < u_z^o$$

$$e < E_i$$

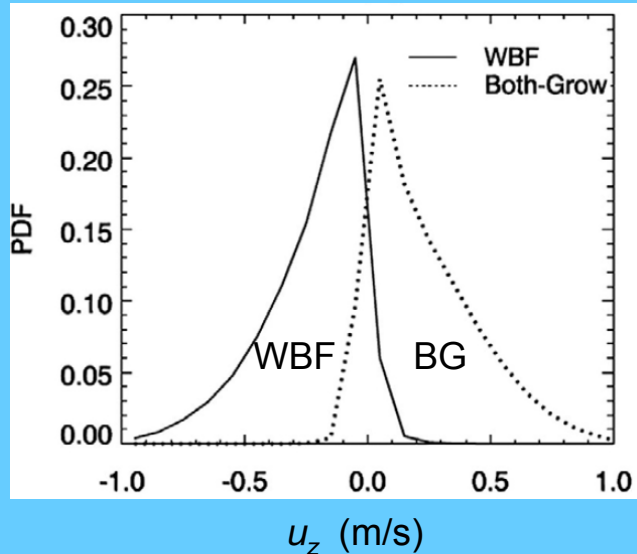


Limited range of conditions for the WBF process

$$\frac{\dot{q}_w}{\dot{q}_i} = \frac{(a_0 u_z - b_i^* N_i \bar{r}_i) B_w N_w \bar{r}_w}{\left(a_0 u_z - \frac{1-\xi}{\xi} b_w N_w \bar{r}_w \right) B_i N_i \bar{r}_i}$$

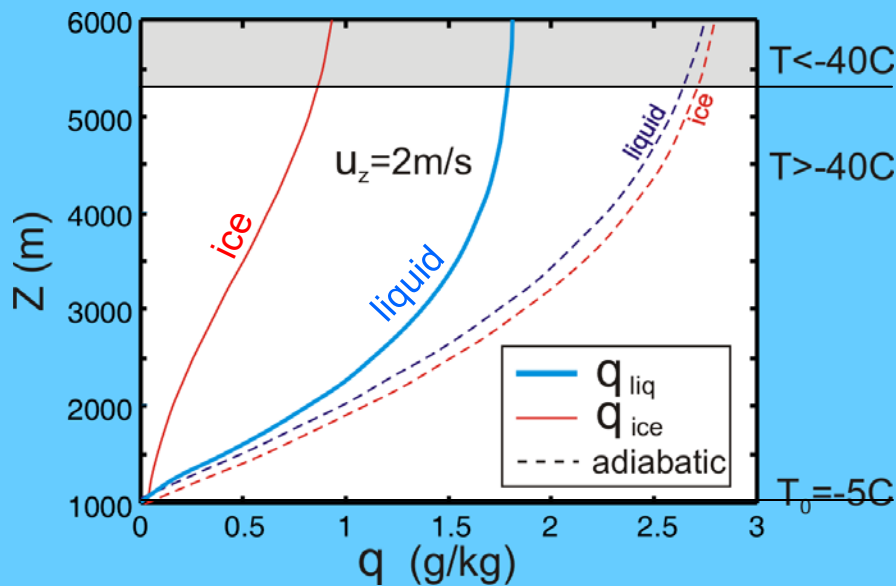


WBF process versus “Both-Grow” process in mixed phase



In shallow stratiform mixed phase clouds the WBF process is enabled in approximately 50% of time

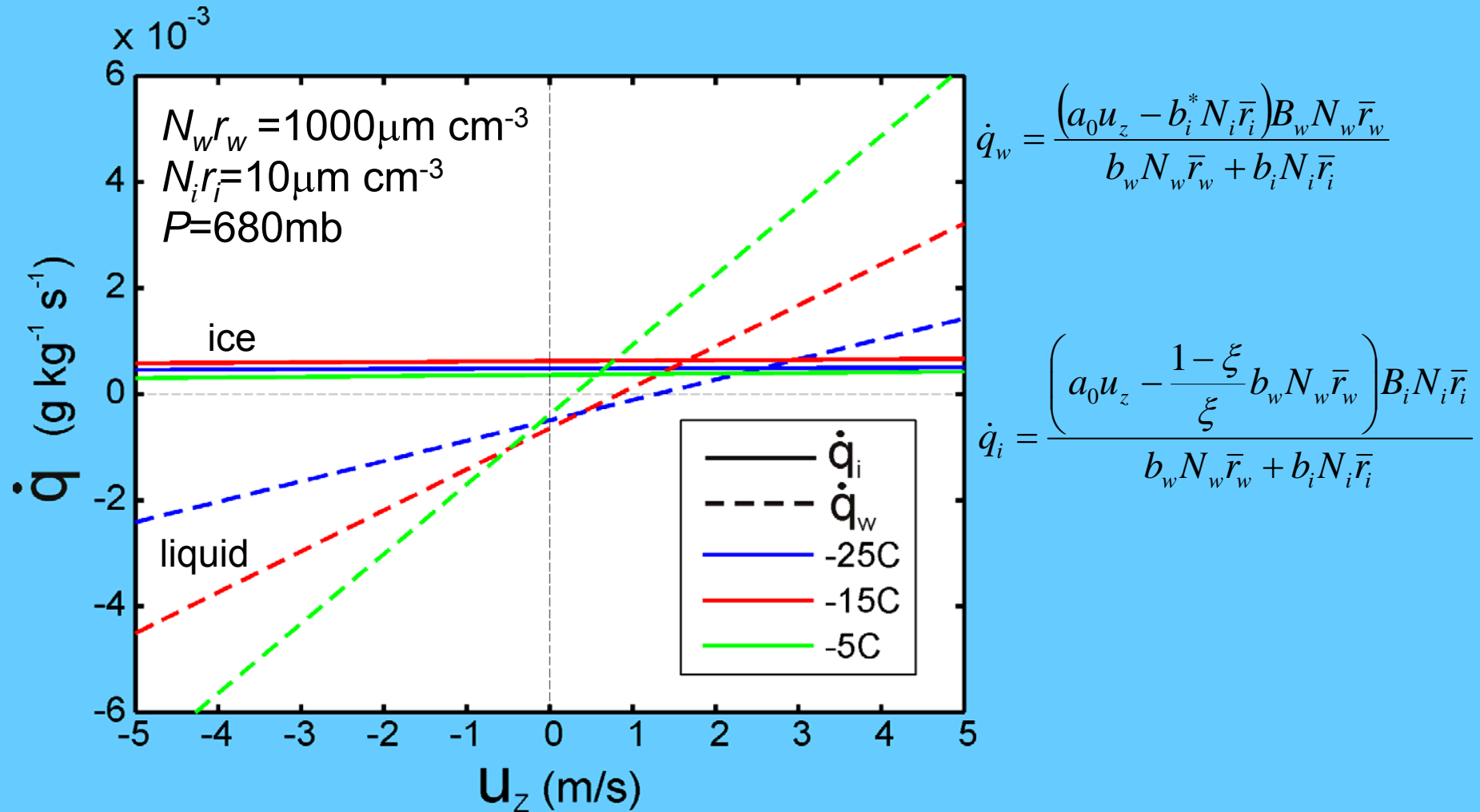
Fan et al., 2011, JGR



In convective cores in cumulus clouds the WBF process is disabled even during a moderate ascent

Korolev, 2007, JAS

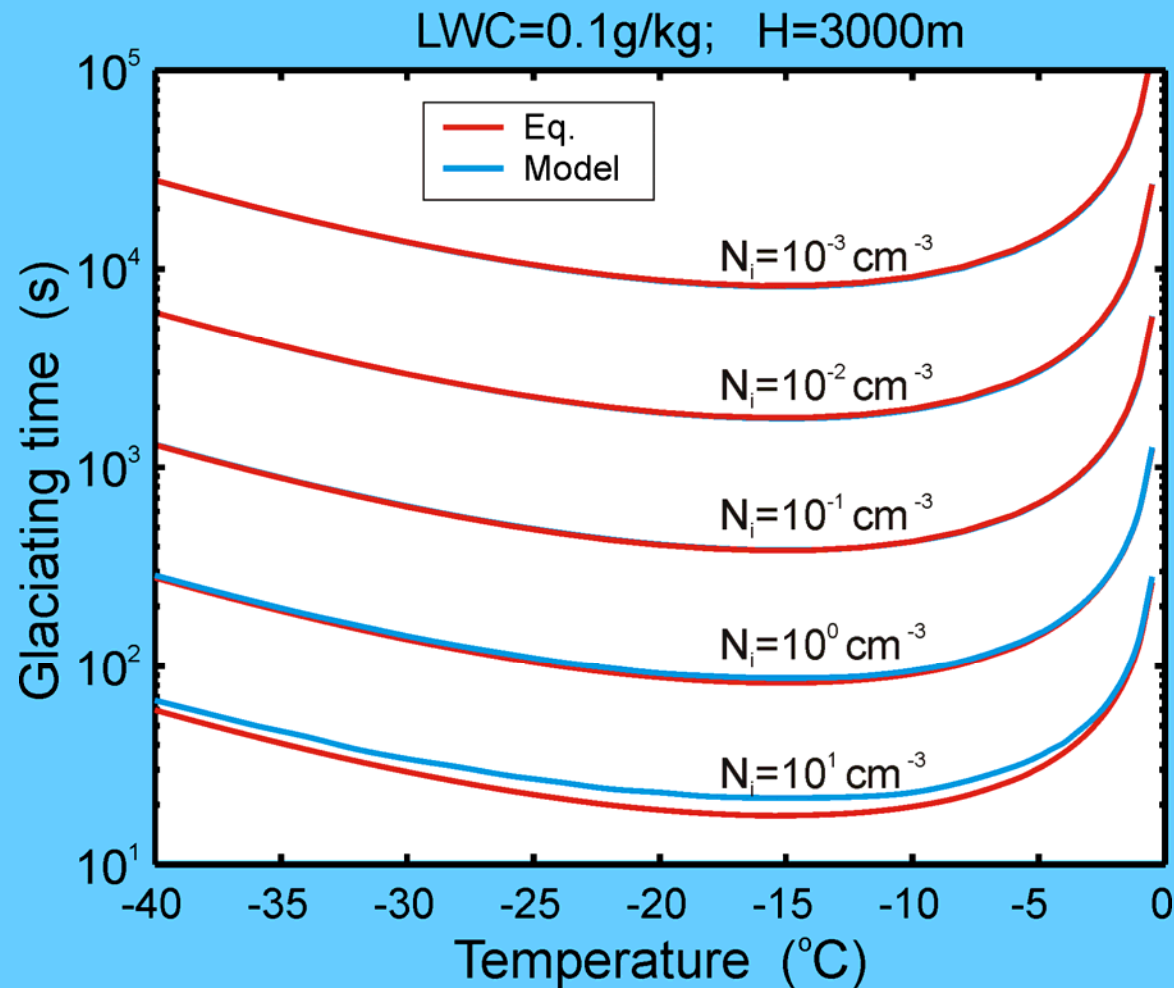
Growth rate of ice and liquid water content in mixed phase cloud



In mixed phase clouds the growth rate of liquid droplets is more sensitive to the vertical velocity in comparisons to that of ice particles

Glaciation time for $U_z=0$

$$\tau_{gl} = \frac{1}{4\pi c A_i S_i} \left(\frac{9\pi\rho_i}{2} \right)^{1/3} \left(\left(\frac{W_w(t_0) + W_i(t_0)}{N_i} \right)^{2/3} - \left(\frac{W_i(t_0)}{N_i} \right)^{2/3} \right)$$

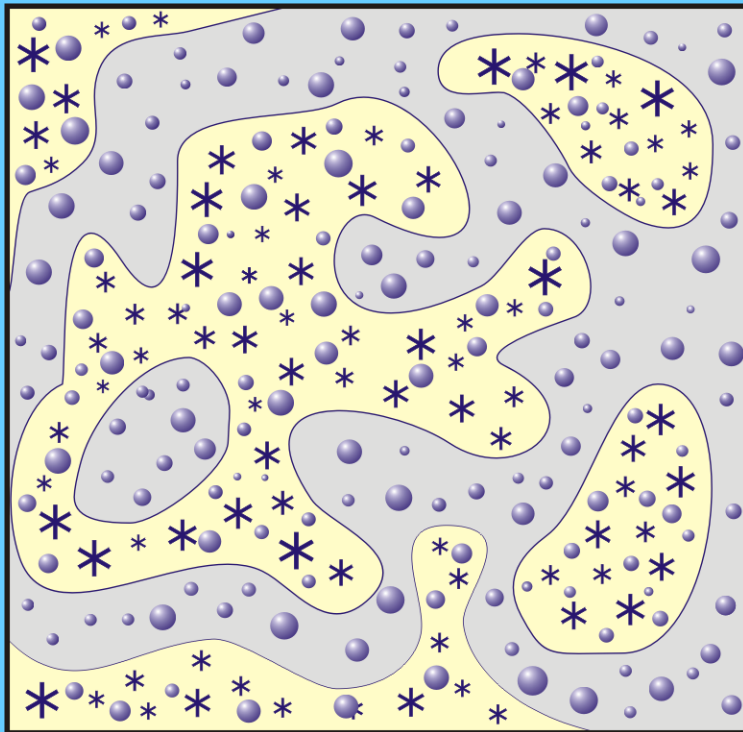


$$\tau_{gl} = k \left(\frac{W_{LWC}}{N_{ice}} \right)^{2/3}$$

Two competing processes: glaciating and mixing

$$\tau_{gl} > \tau_t$$

The condition for the existence of isolated single-phase liquid and ice zones with the characteristic scale L in clouds with isotropic turbulence



$$\tau_t = \left(\frac{L^2}{\varepsilon} \right)^{1/3} \quad \tau_{gl} = k \left(\frac{W_{LWC}}{N_{ice}} \right)^{2/3}$$

$$\tau_{gl} \sim 10^2 - 10^3 \text{ s};$$

$$\varepsilon \sim 10^{-3} - 10^{-4} \text{ m}^2/\text{s}^3$$

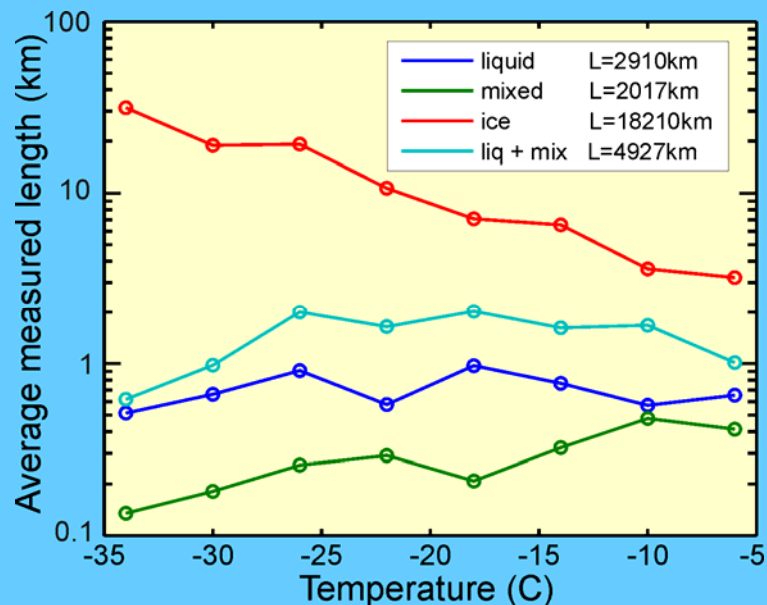
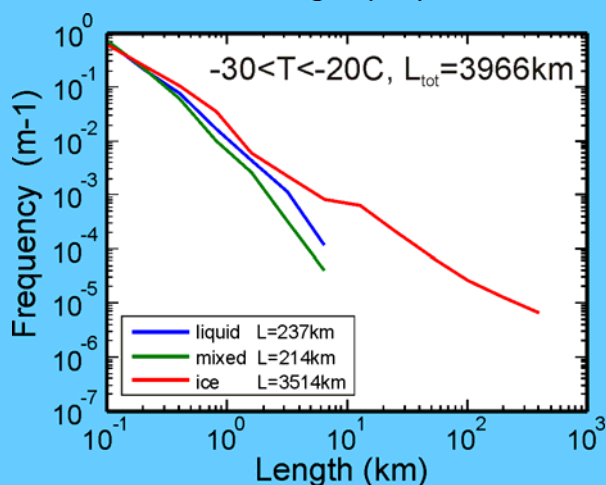
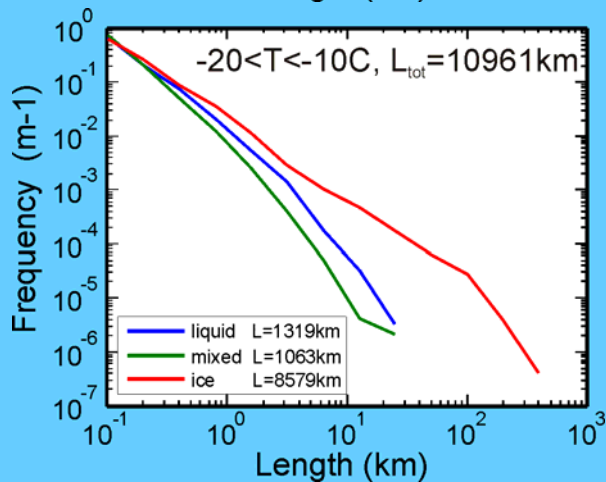
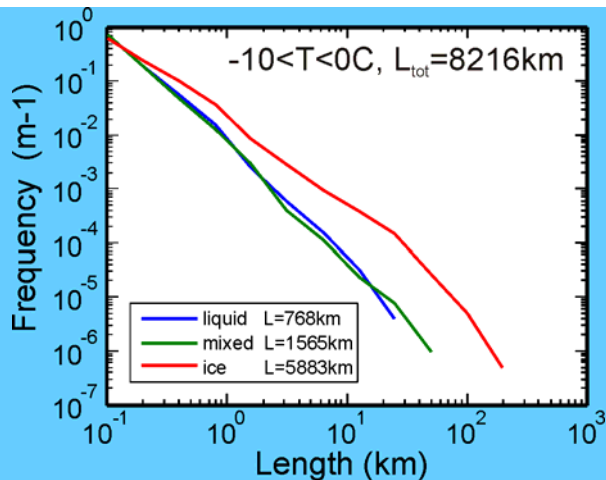
The characteristic spatial scale of the single phase zones in mixed phase clouds

$$L_{ph} \sim k \varepsilon^{1/2} \frac{W_{LWC}}{N_{ice}} \sim 10^1 - 10^3 \text{ m}$$

Sustained single phase zones may exist at the spatial scales $L < L_{ph}$

Occurrence of lengths of liquid, mixed phase and ice cloud zones

TWC threshold=0.01kg/m³



The average length of continuous liquid zones stays constant with temperature $L_w \sim 500\text{m}$

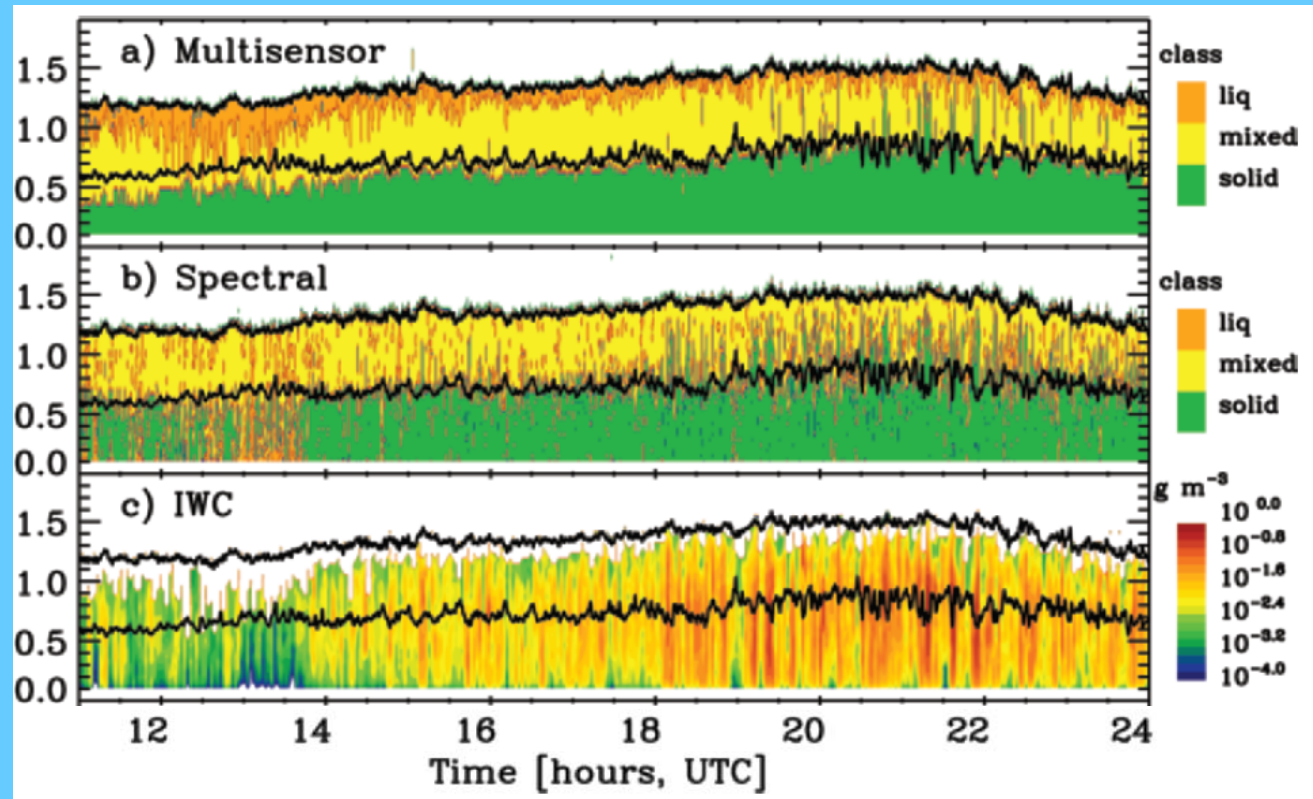
The continuous length of mixed phase zones decreases with decrease of T $100\text{m} < L_m < 500\text{m}$

The continuous length of ice zones increases with the decrease of T $4\text{km} < L_i < 40\text{km}$

The documentation of long-living mixed phase stratiform layers at temperatures as low as -30C

Rauber, R.M. and A. Tokay, 1991: *J. Atmos. Sci.*,
Pinto, J.O., 1998: *J. Atmos. Sci.*

09 Oct 2004, Barrow, Alaska, NSA ARM site,



Shupe et al. 2008, BAMS.

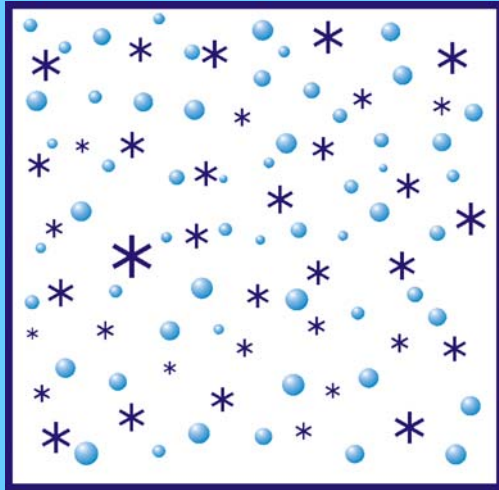


mixed

ice

In stratiform clouds $U_z \sim 0$, therefore they are expected to glaciate within about one hour.

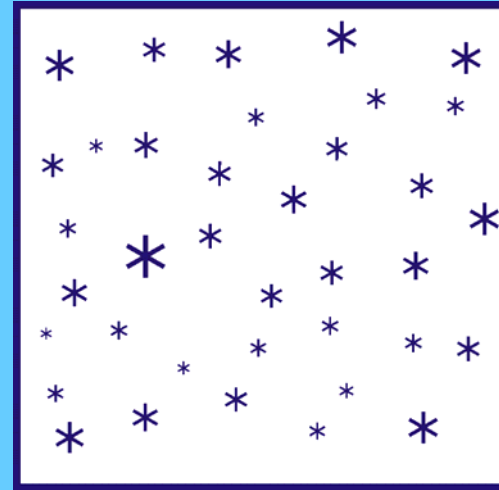
Observation of long-living clouds conflicts with the theoretical estimation of glaciation time



WBF process



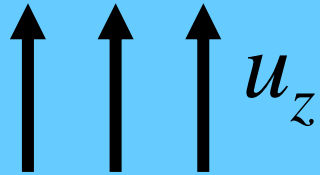
?



$$u_z^* = \frac{b_i^* N_i \bar{r}_i}{a_{0w}} \quad \text{threshold vertical velocity, when } e = E_w$$



mixed



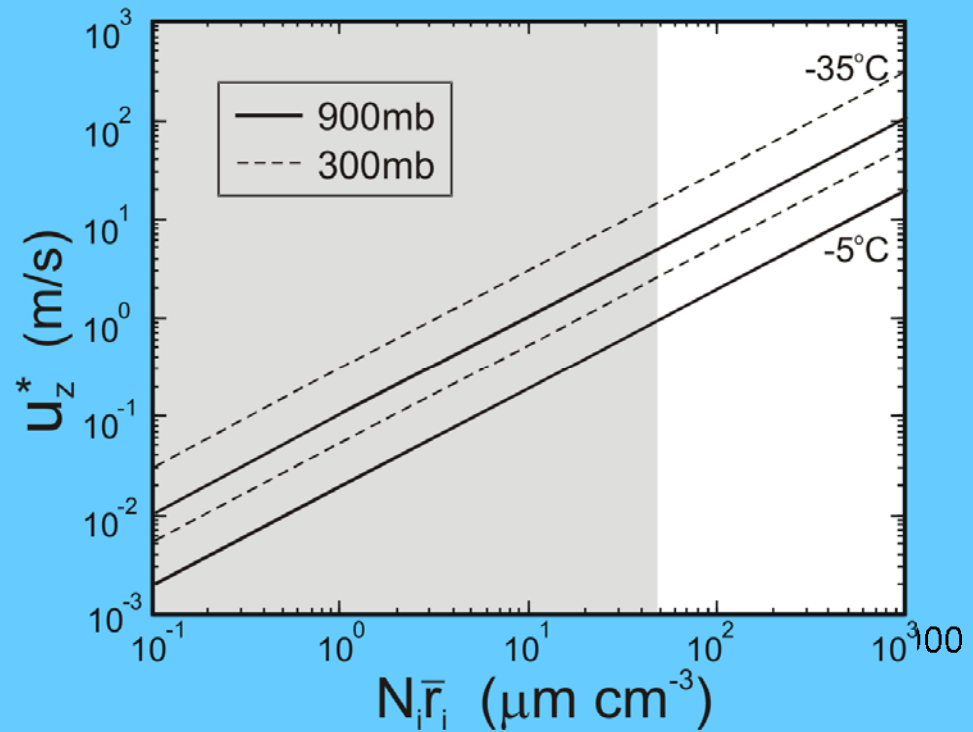
u_z



ice

$$u_z > u_z^*$$

condition for activation of liquid water in ice cloud

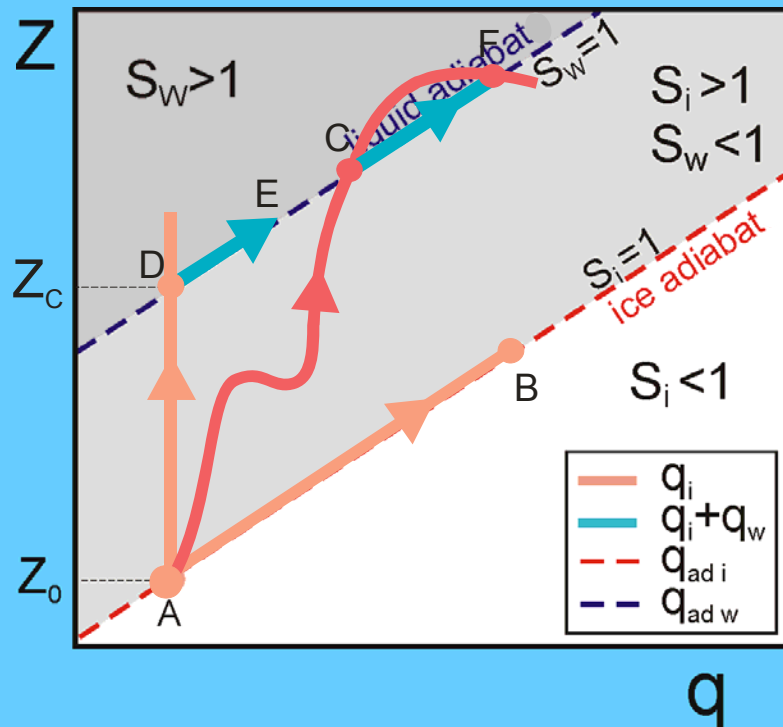
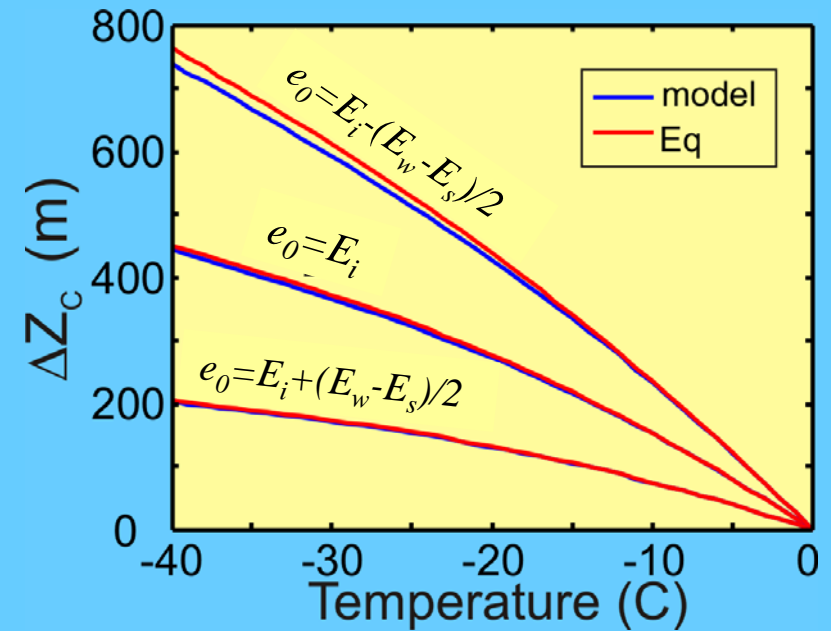


$$\tau_{ph} \approx \frac{b(T)}{N_i \bar{r}_{i0}} \quad \text{time of phase relaxation}$$

$$\tau_z \approx \frac{\Delta Z}{u_z} \quad \text{characteristic time of vertical motion}$$

$$\tau_z \gg \tau_{ph} \rightarrow \text{line AB}$$

$$\tau_z \ll \tau_{ph} \rightarrow \text{line AD}$$



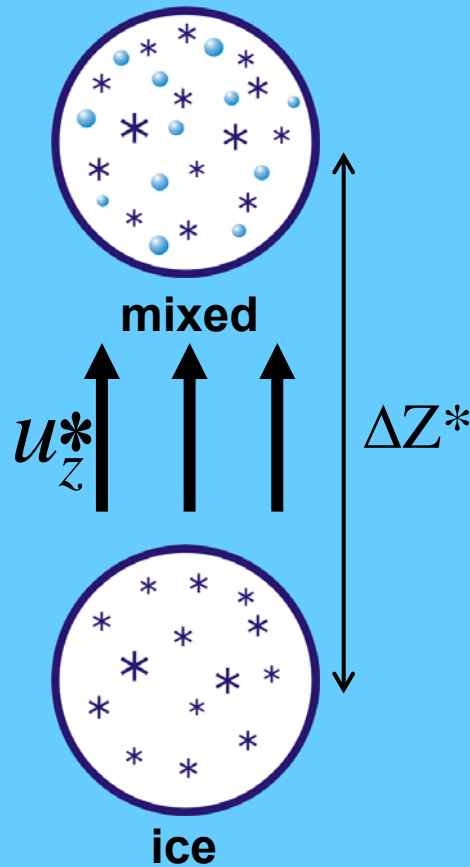
$$\frac{1}{S_w + 1} \frac{dS_w}{dz} = a_w \rightarrow \Delta Z_c = a_w^{-1} \ln \left(\frac{E_w}{e_0} \right)$$

$$\Delta Z > \Delta Z^* \quad \text{condition for activation of liquid water in ice cloud}$$

$$\Delta Z_{\min}^* = \Delta Z_c \quad \text{for } u_z \rightarrow \infty$$

$$\Delta Z < \Delta Z_c \quad \text{no activation of liquid is possible}$$

Dynamic forcing of mixed phase in preexisting ice cloud: formulation of the necessary and sufficient conditions for the activation of liquid in ice clouds



1st Necessary Condition: The vertical velocity of an ice cloud parcel must exceed a threshold velocity to activate liquid water.

$$u_z > u_z^*$$

$$u_z^* = \frac{b_i^* N_i \bar{r}_i}{a_{0w}}$$

2nd Necessary Condition: The activation of liquid water within an ice cloud parcel, below water saturation, requires a vertical ascent (ΔZ) above some threshold altitude (ΔZ^*) to bring the vapor pressure of the parcel to water saturation:

$$\Delta Z > \Delta Z^*$$

$$\Delta Z_c^* = a_w^{-1} \ln \left(\frac{E_w}{e_0} \right)$$

1st and 2nd conditions give a set of necessary and sufficient conditions of activation of liquid in ice cloud

Lower sun pillar, subsun

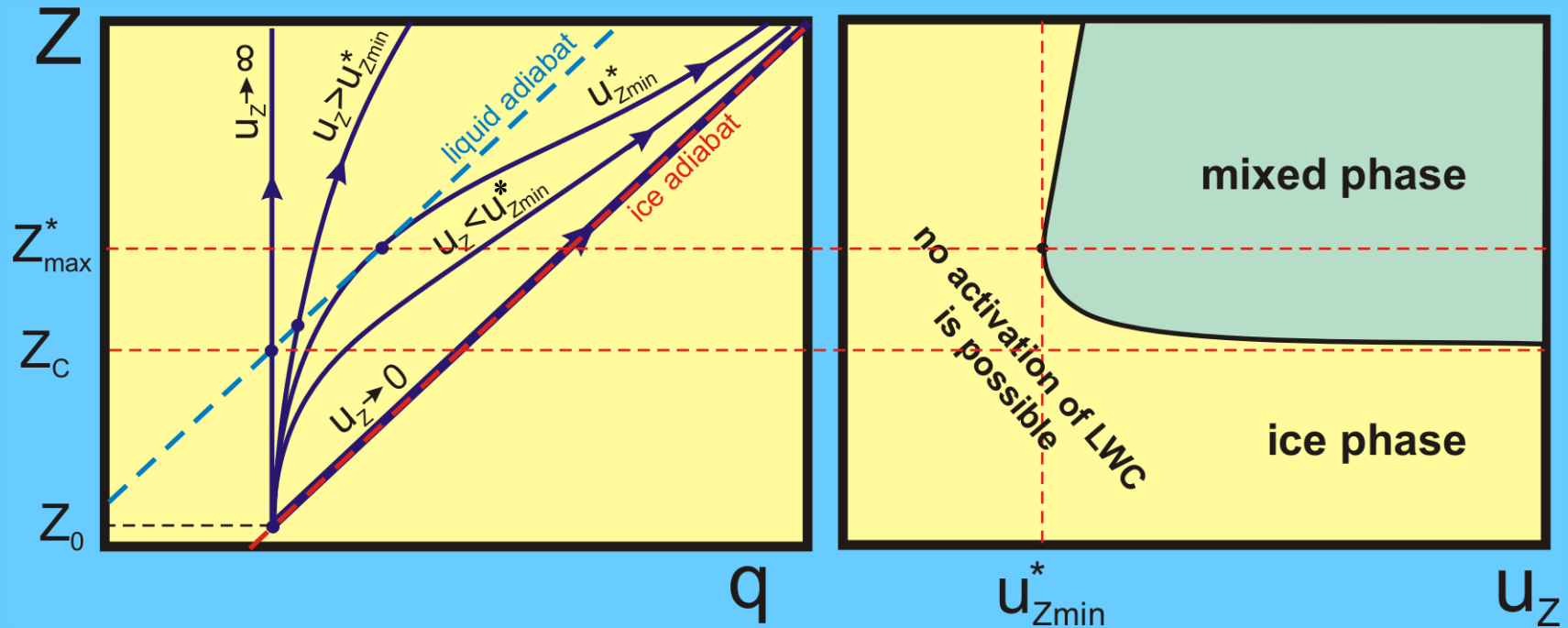


Lower sun pillar, subsun



UNIFORM ASCENT

(necessary and sufficient conditions)



Necessary and sufficient condition for the activation of liquid water in ice clouds for *uniform ascent*.

$$u_z > u_z^*$$

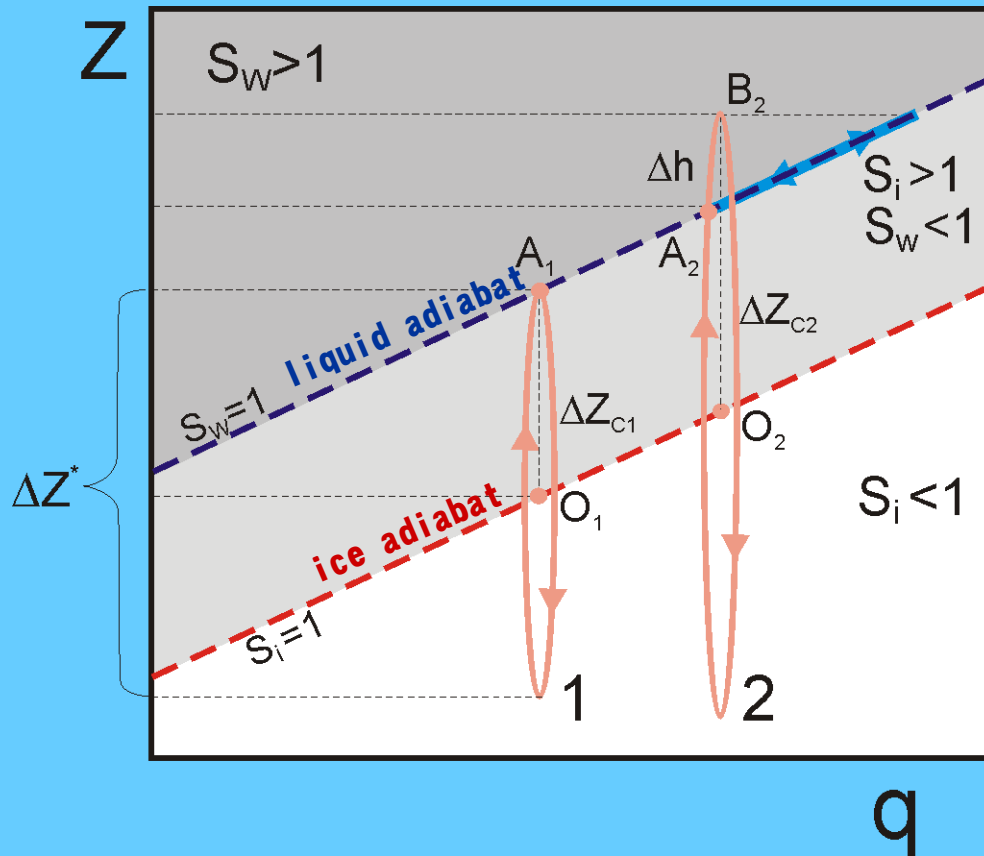
$$\Delta Z > \Delta Z^*$$

$$u_{zmin}^*(N_i, r_{i0}, T_0)$$

$$\Delta Z_{max}^*(N_i, r_{i0}, T_0)$$

$$\Delta Z_c(T_0)$$

Maintenance of mixed phase clouds during vertical fluctuations of U_z



$$u_0^* = \frac{k_0 b_m (E_w - E_i) N_i \bar{r}_m \Delta Z}{E_i \sqrt{\Delta Z^2 - 4a_w^2 \ln^2 \left(\frac{E_w}{E_i} \right)}}$$

$$u_0 > u_0^*$$

$$\Delta Z > 2\Delta Z_c$$

Necessary and sufficient conditions for the indefinitely long maintenance of mixed phase during harmonic oscillations.

HARMONIC OSCILLATIONS

modeling of the activation of liquid water in ice cloud during vertical harmonic oscillations.

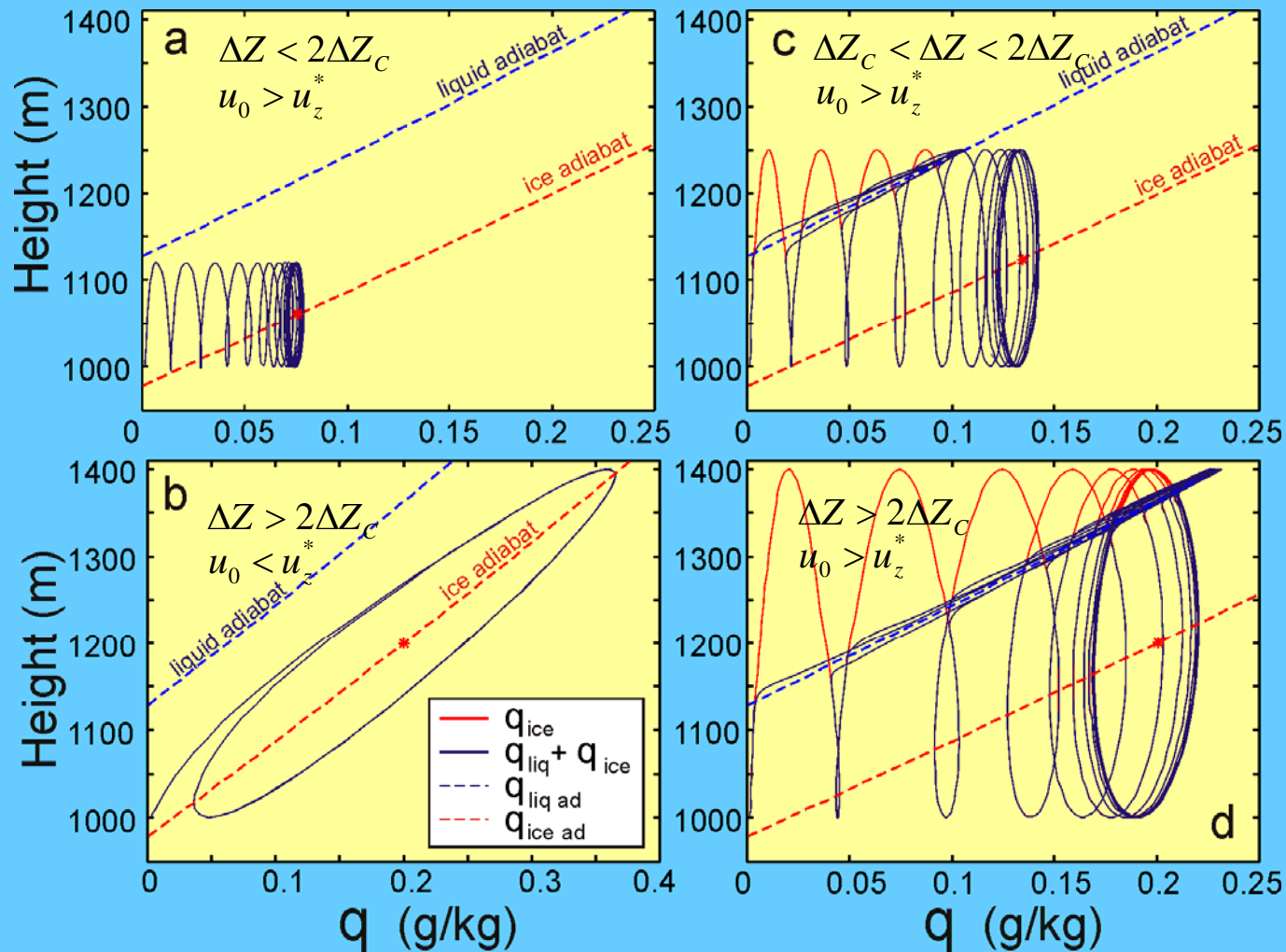
$N_{ice}=50l^{-1}$, $r_{i0}=20\mu m$, $S_{i0}=1.01$, $T_0=-10C$, $\Delta Z_C=153m$.

(a) $\Delta Z=125m$, $u_0=0.5m/s$, $u_0^*=0.08m/s$; $\Delta Z_C=153m$

(c) $\Delta Z=250m$, $u_0=1m/s$, $u_0^*=0.10m/s$; $\Delta Z_C=153m$

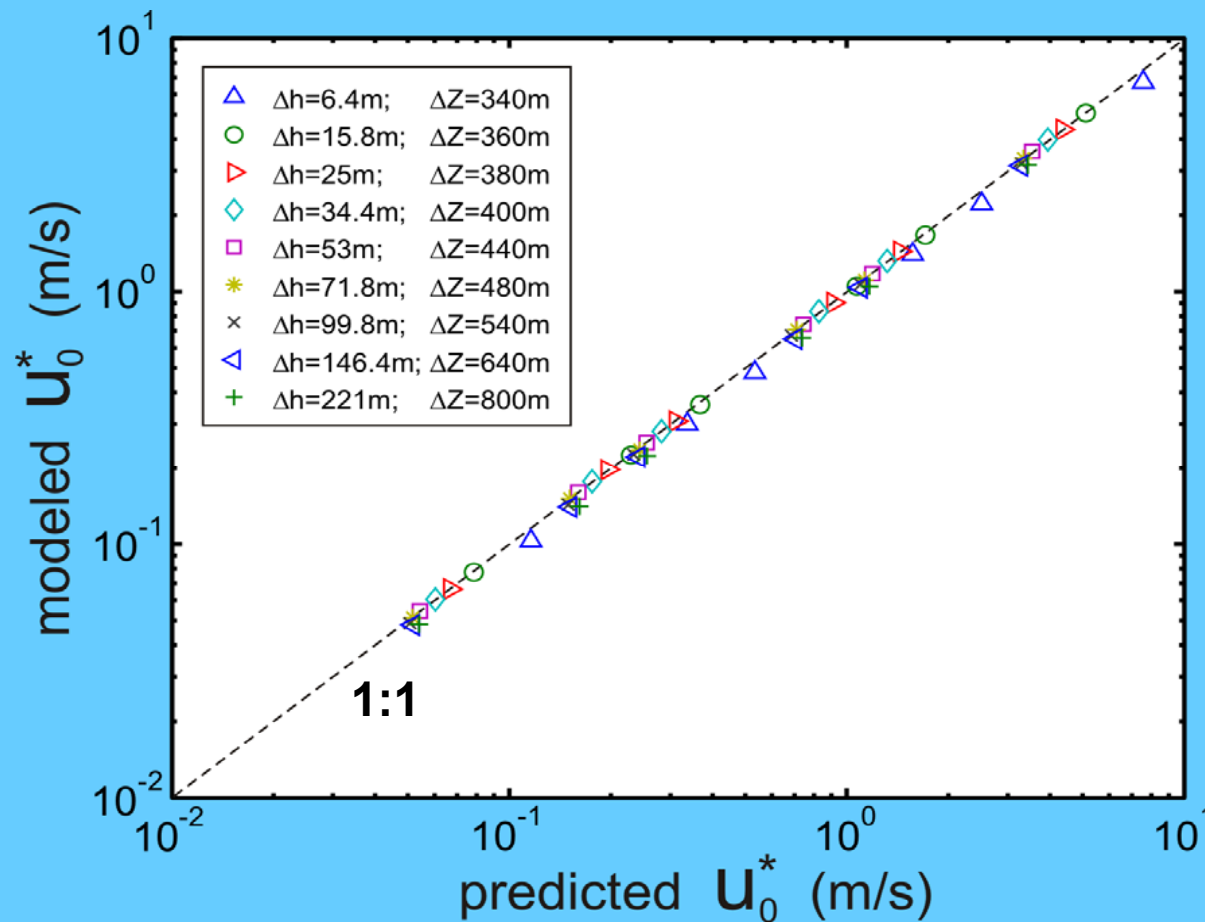
(b) $\Delta Z=400m$, $u_0=0.05m/s$, $u_0^*=0.12m/s$; $\Delta Z_C=153m$

(d) $\Delta Z=400m$, $u_0=1m/s$; $u_0^*=0.12m/s$ $\Delta Z_C=153m$



HARMONIC OSCILLATIONS

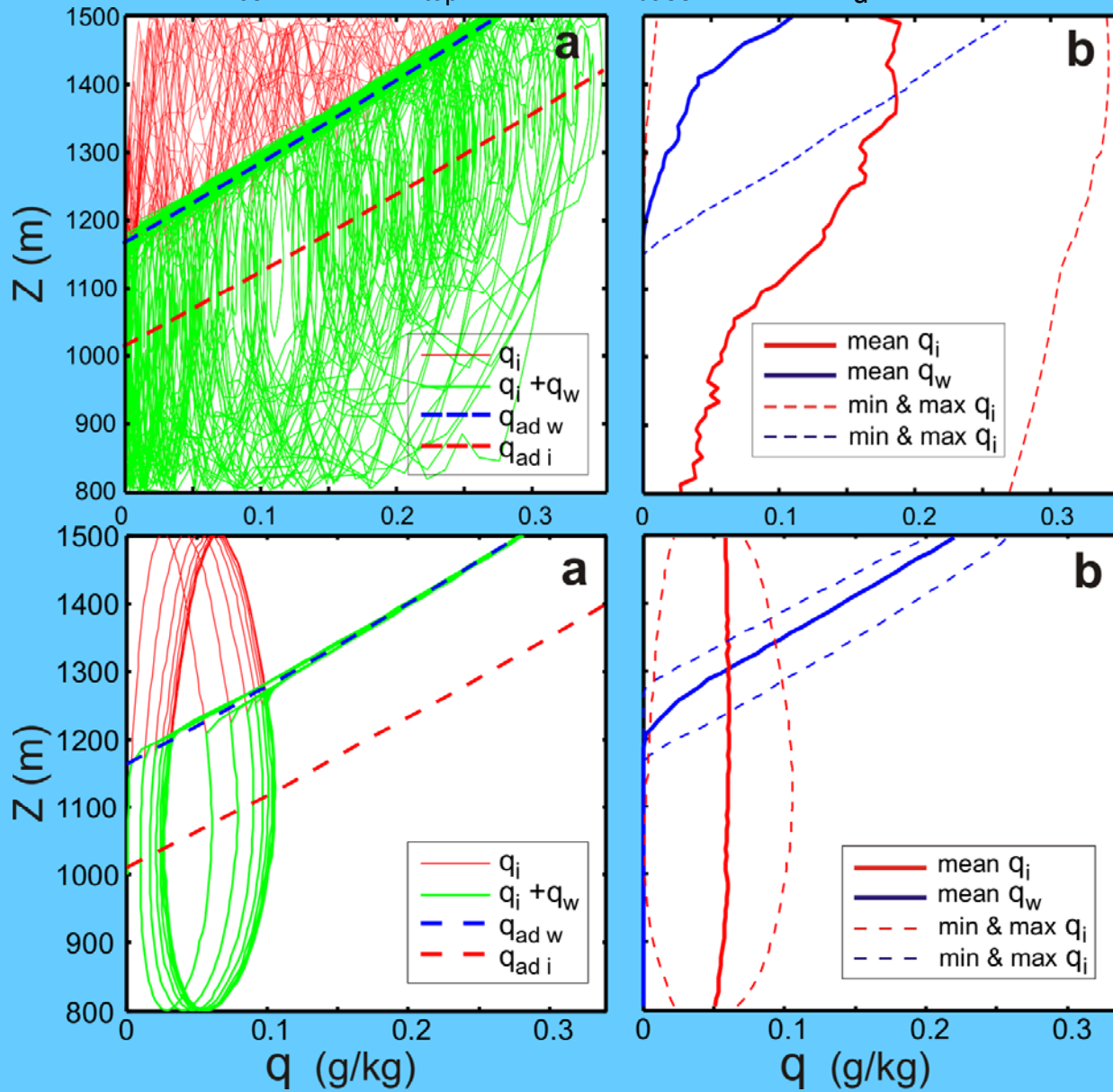
Comparison of the theoretical and modeled threshold velocity U_0^*
 $T_0 = -10\text{C}$, $340\text{m} < \Delta Z < 800\text{m}$; $0.05\text{m/s} < u_0 < 6\text{m/s}$;
 $20\mu\text{m} < r_{i0} < 200\mu\text{m}$; $50\text{l}^{-1} < N_i < 5000\text{l}^{-1}$



$$u_0^* = \frac{k_0 b_m (E_w - E_i) N_i \bar{r}_m \Delta Z}{E_i \sqrt{\Delta Z^2 - 4a_w^2 \ln^2 \left(\frac{E_w}{E_i} \right)}}$$

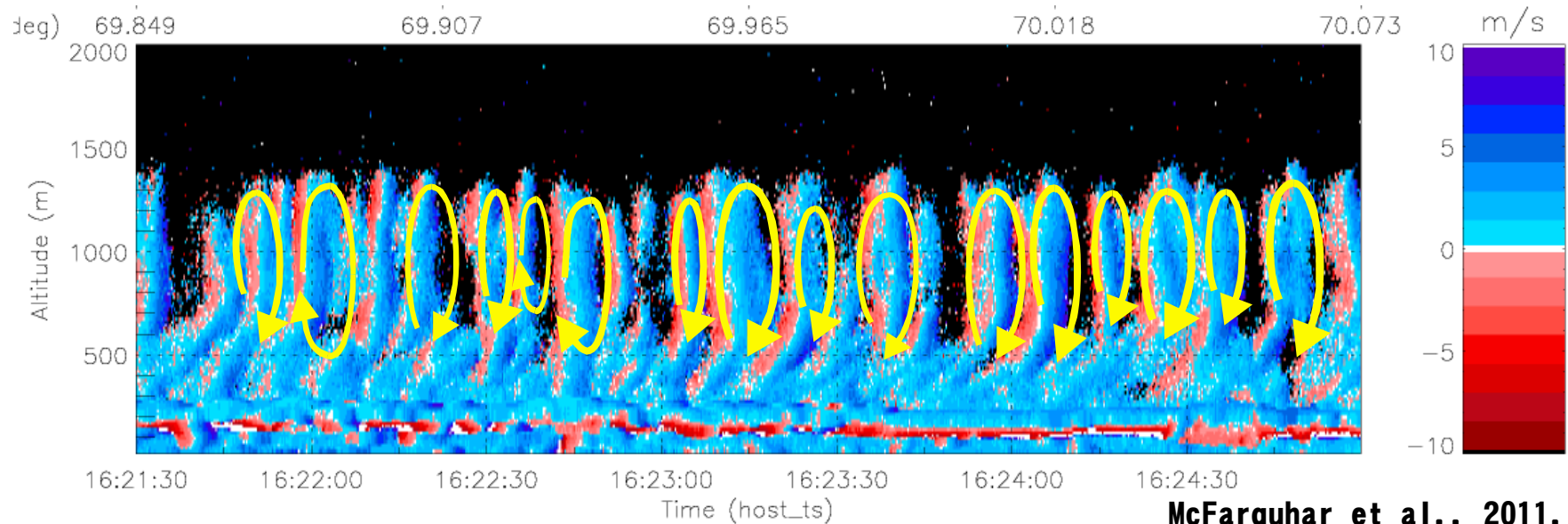
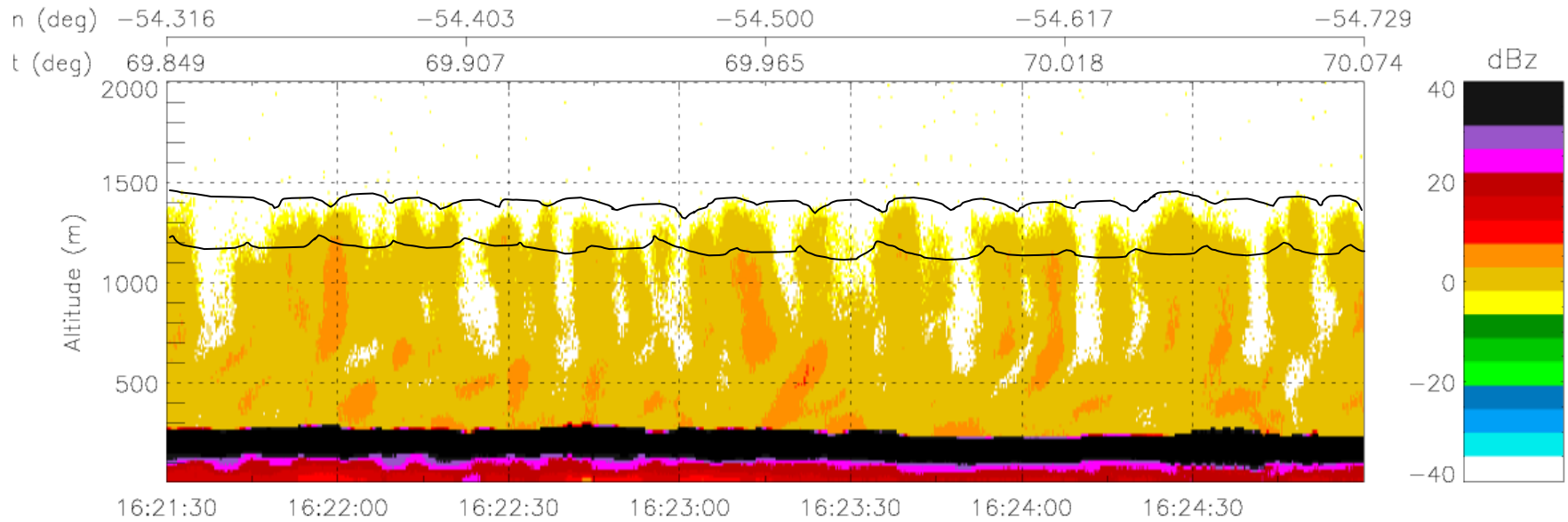
Random vertical fluctuations

$N_{ice} = 50l^{-1}$; $T_{top} = -14^{\circ}C$; $T_{base} = -8^{\circ}C$; $\sigma_u = 0.75m/s$



ISDAC, Alaska, NRC NAX radar stratiform mixed phase layer

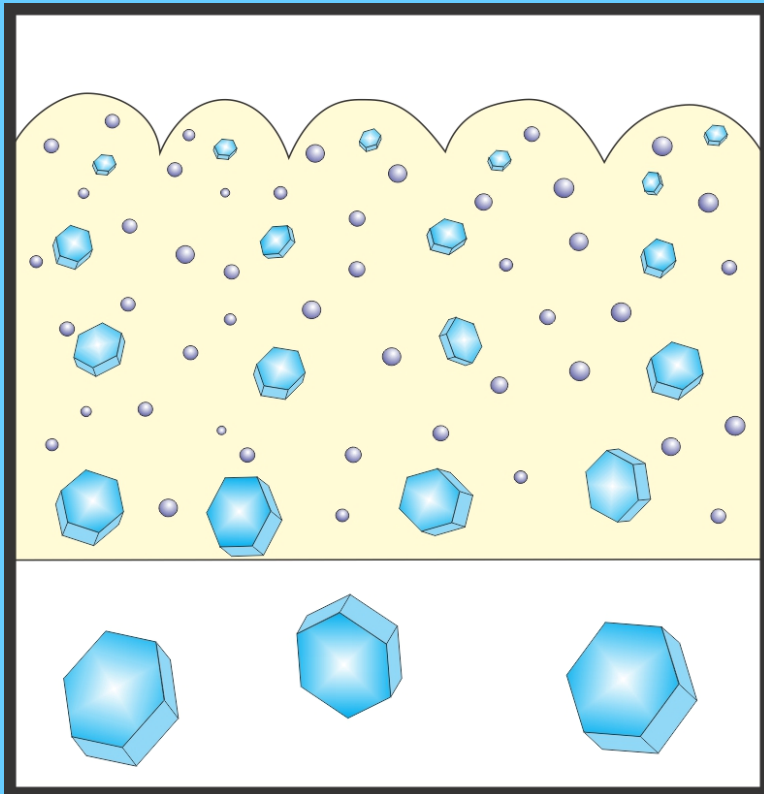
Tue Apr 8 2008
NAWX: Band: X Variable: Z



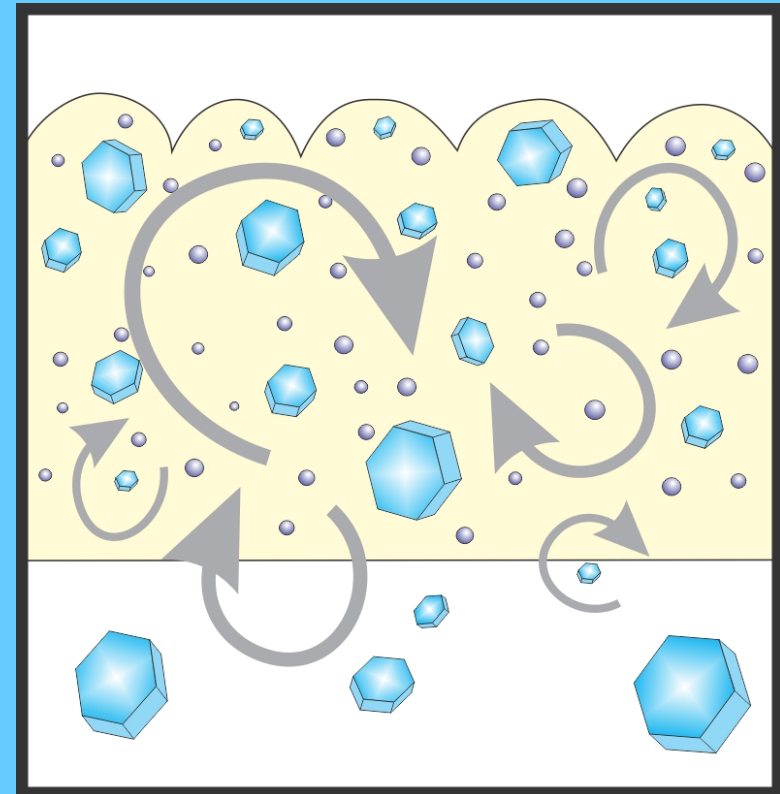
McFarquhar et al., 2011, BAMS

Effect of dynamics on the formation of stratiform mixed phase clouds

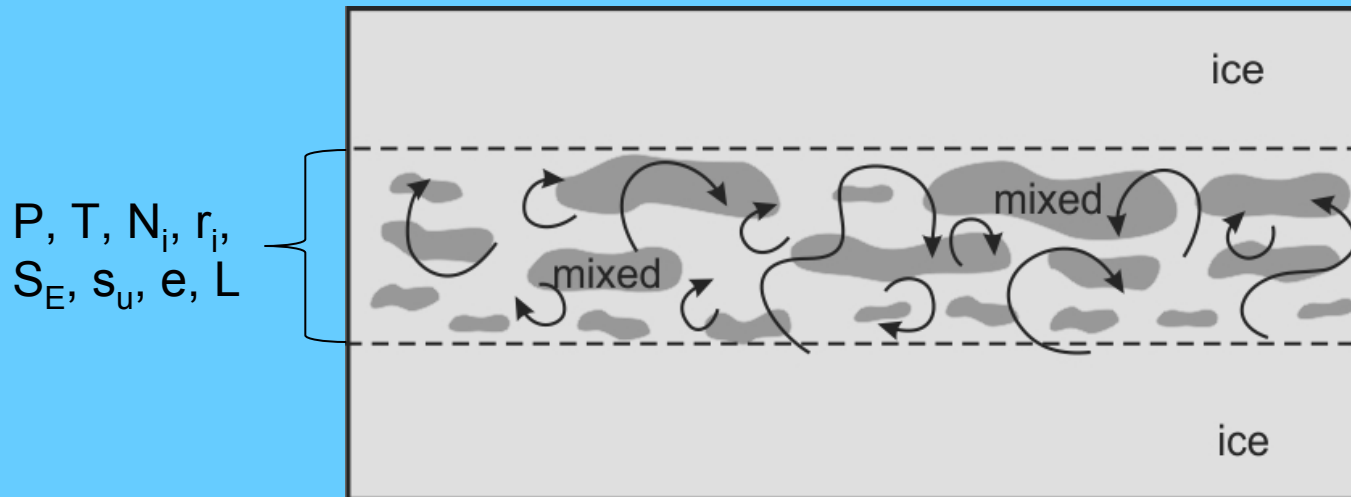
Static concept



Dynamic concept



Stochastic approach to dynamic forcing of mixed phase



$$\frac{1}{S_i + 1} \frac{dS_i}{dt} = a_i u_z - b_i B_0 M_1 S_i$$

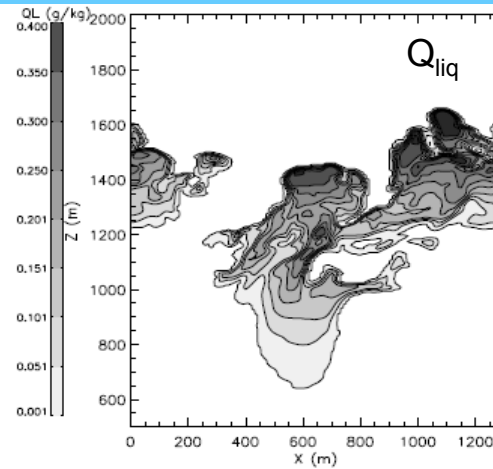
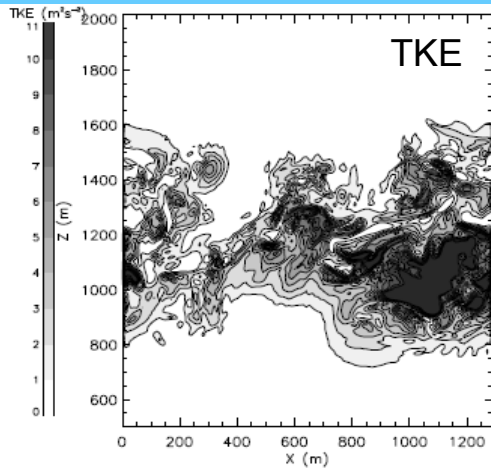
Differential equation for supersaturation in a Lagrangian parcel

$$\frac{dS_i}{dt} = a_i \sigma_u(t) - b_i B_0 M_1 S_i + \left(\frac{\varepsilon}{L^2} \right)^{1/3} (S_E - S_i)$$

Stochastic differential equation for supersaturation averaged over an ensemble of parcels

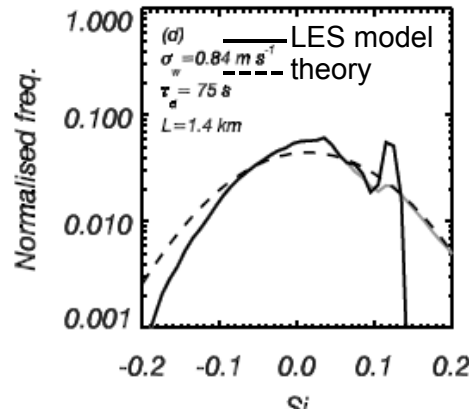
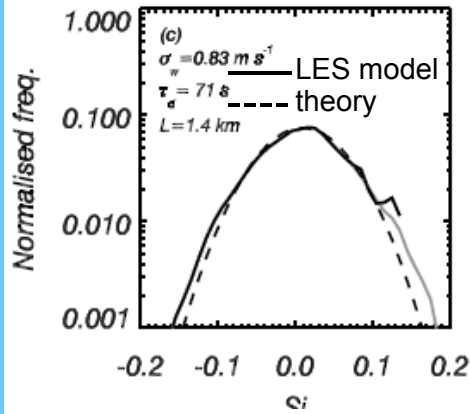
$$S_i(t) = \exp(-(B + C)t) \left[S_0 + S_E \frac{C}{B + C} (1 - \exp(-(B + C)t)) + \int_0^t \exp(-(B + C)(t - r)) \xi dr \right]$$

Hill et al. 2013 QJRMS (in press)



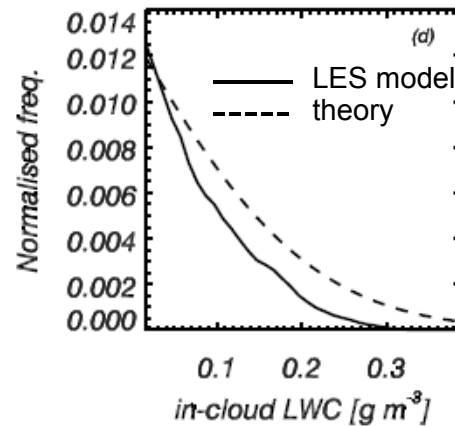
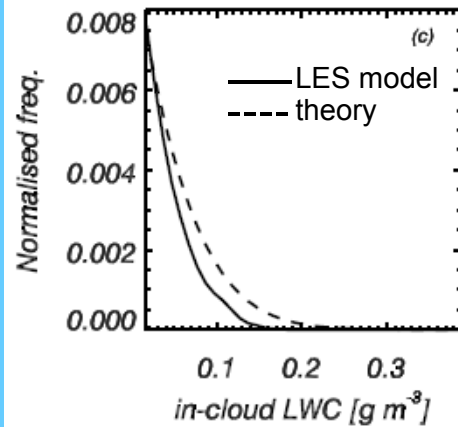
$100 L^{-1}$, mid

$1 L^{-1}$, mid



$100 L^{-1}$, mid

$1 L^{-1}$, mid



$$f(S_i) = \frac{1}{\sigma_S \sqrt{2\pi}} \exp\left(-\frac{(S_i - \langle S_i \rangle)^2}{2\sigma_S^2}\right)$$

$$\sigma_S^2 = \frac{S_E a_i^2 \sigma_u^2 \tau_d}{2 \left(b_i B_0 M_1 + \left(\frac{\epsilon}{L^2} \right)^{1/3} \right)}$$

$$\langle S_i \rangle = S_E \frac{\left(\frac{\epsilon}{L^2} \right)^{1/3}}{b_i B_0 M_1 + \left(\frac{\epsilon}{L^2} \right)^{1/3}}$$

$$\langle q \rangle = \int_{S_{iu}}^{\infty} (S_i - S_{iu}) f(S_i) \rho_a q_{si} dS_i$$

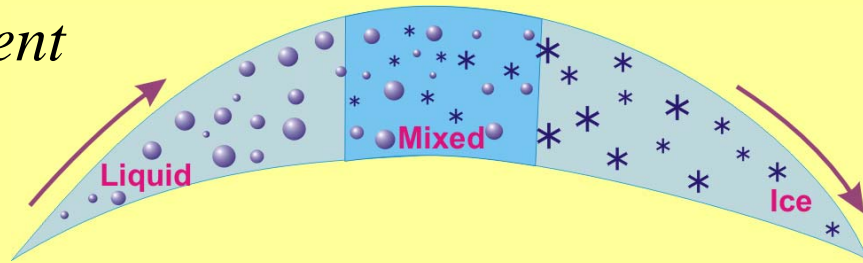
Field et al. 2013 QJRMS (in press)

Theoretical and experimental problems in mixed phase physics:

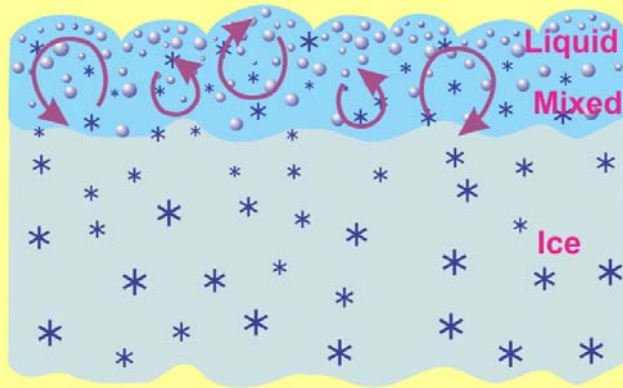
1. **Absence of physically based definition of mixed phase**
2. **Assumption of regular condensation in the description of mixed phase does not have theoretical or experimental justification**
 $(K_t \rightarrow \infty; \tau_{\text{mix}} \rightarrow 0)$
3. **Poor understanding of diffusional ice growth under varying environmental conditions (RH, T, P)**
4. **Characteristic spatial scales of mixed and single phase clouds at $L < 100\text{m}$**

Instead of conclusion

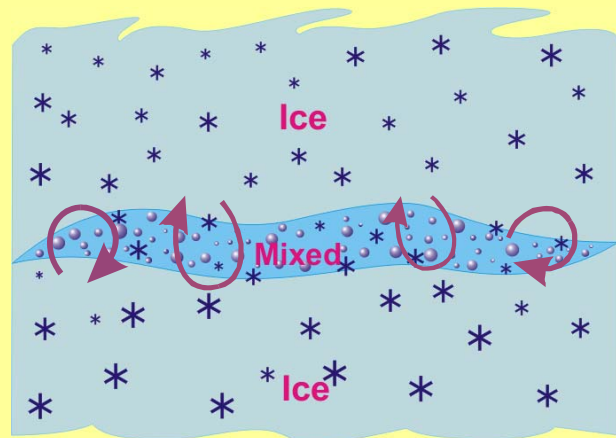
Ac lent



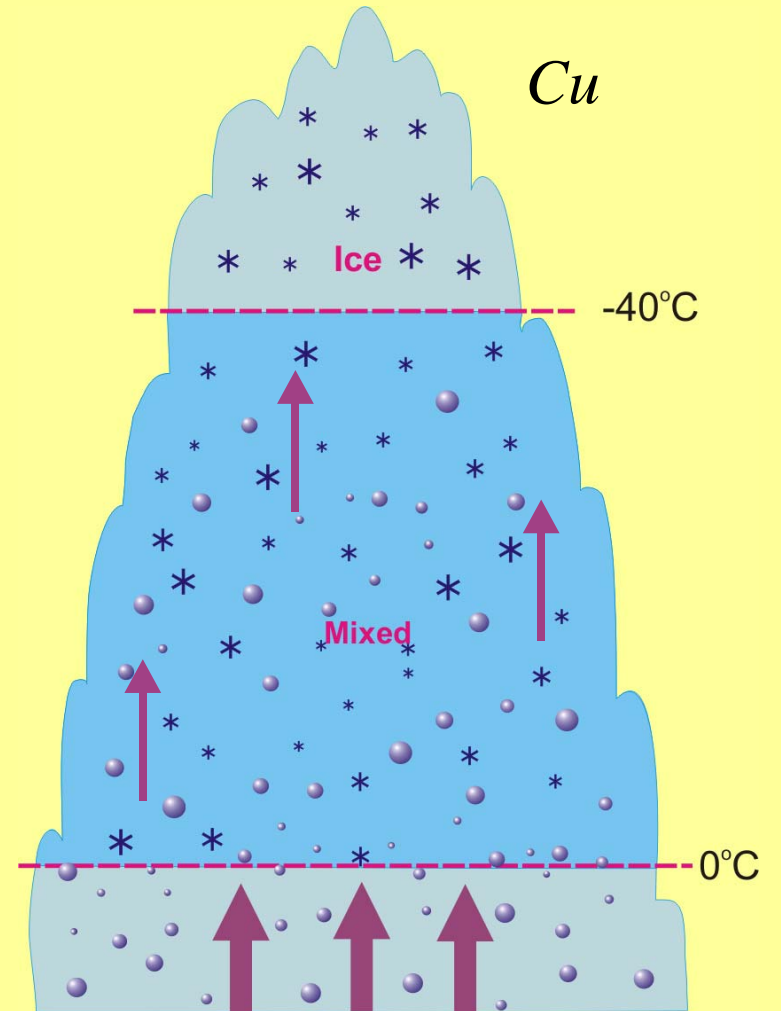
Ac, Sc



Cs-Ns



Cu



Thank you



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