Parameterization of Cloud Microphysics

– Update on Current Research –

Jason Milbrandt

Atmospheric Numerical Prediction Research Section (RPN-A)
Environment Canada
OUTLINE of PRESENTATION

• Background

• Current research on bulk microphysics schemes (BMS)
  1. Prognostic snow density
  2. Sedimentation-induced errors (in BMS)
  3. Comparison of 2-moment schemes
Modelling Systems and Applications at RPN/CMC

- regional climate models
- global climate models
- seasonal forecast
- global weather forecast
- regional weather forecast
- limited-area models
- urban-scale models
- building scale models

**time scale** (forecast range)
- 10 years
- 1 year
- 10 days
- 48 h
- 24 h
- 1 h

**spatial scale** (horizontal resolution)
- 10 m
- 1 km
- 10 km
- 100 km
Physical Processes and Systems (PPS) Group
https://wiki.cmc.ec.gc.ca/wiki/PPS

ERPS
Extended Range Prediction System (GEM-CLIM)

GDPS
Global Deterministic Prediction System (GEM-Global)

RDPS
Regional Deterministic Prediction System (GEM-REG)

HRDPS
High Resolution Deterministic Prediction System (GEM-LAM 2.5)
<table>
<thead>
<tr>
<th>System</th>
<th>Description</th>
<th>Resolution</th>
<th>STCOND</th>
</tr>
</thead>
<tbody>
<tr>
<td>ERPS</td>
<td>Extended Range Prediction System</td>
<td>Δx = 2 deg (220 km)</td>
<td>* Sundqvist</td>
</tr>
<tr>
<td>GDPS</td>
<td>Global Deterministic Prediction System</td>
<td>Δx = 33 km</td>
<td>Sundqvist</td>
</tr>
<tr>
<td>RDPS</td>
<td>Regional Deterministic Prediction System</td>
<td>Δx = 15 km</td>
<td>Sundqvist</td>
</tr>
<tr>
<td>HRDPS</td>
<td>High Resolution Deterministic Prediction System</td>
<td>Δx = 2.5 km</td>
<td>Milbrandt-Yau (1-moment)</td>
</tr>
</tbody>
</table>

*STCOND = grid-scale condensation/precipitation scheme
## Physical Processes and Systems (PPS) Group
https://wiki.cmc.ec.gc.ca/wiki/PPS

### ERPS
Extended Range Prediction System
- $\Delta x = 2$ deg (220 km)
- STCOND: Sundqvist

### GDPS
Global Deterministic Prediction System
- $\Delta x = 33$ km
- STCOND: Sundqvist
  - $\rightarrow \Delta x = 25$ km (possibly)
  - $\rightarrow$ simplified 2-moment (M-Y)

### RDPS
Regional Deterministic Prediction System
- $\Delta x = 15$ km
- STCOND: Sundqvist
  - $\rightarrow \Delta x = 10$ km
  - $\rightarrow$ simplified 2-moment (M-Y)

### HRDPS
High Resolution Deterministic Prediction System
- $\Delta x = 2.5$ km
- STCOND: Milbrandt-Yau (1-moment)
  - $\rightarrow$ 2-moment (M-Y)

**NEAR FUTURE**

Role of BMS is increasing in EC modelling systems
HRDPS: Current Configuration

- GEM v3.2.2
- 24-h runs (1 run daily)
- $\forall \Delta x = 2.5 \text{ km}$
- single-moment M-Y BMS
HRDPS: Next upgrade (January 2011)

- GEM v4.1.4
- 24-h runs (1 run daily)
- $\forall \Delta x = 2.5 \text{ km}$
- **double-moment** M-Y BMS

**Extension to grids**

- Persons per km²
- ≥ 50
- 10 to < 50
- 1 to < 10
- 0.4 to < 1
- Sparsely populated
- 100 km from the southern border
1. Prognostic Snow Density
2. Sedimentation-Induced Errors
3. Comparison of 2-Moment Schemes
1. Prognostic Snow Density

2. Sedimentation-Induced Errors

3. Comparison of 2-Moment Schemes
MOTIVATION:

How much snow will fall?
Accumulated Precipitation
(Liquid-Equivalent)

Accumulated Precipitation
(Unmelted - i.e. Snowfall Amount)
Observed SOLID-LIQUID ratios:

- average value approximately **10:1**
- can range from **3:1** to **100:1**
- varies geographically

Source: Ware et al. (2006), *Wea and Forecasting*
APPROACHES TO PREDICTION:

• 10:1 rule

• Climatology

• Neural network diagnostic (statistics of environmental conditions)
  e.g. Roebber et al. (2003)

• Decision tree algorithm (based on physical principles and environment)
  e.g. Dubé (2006)

• Prognostic from the microphysics of a NWP model
Cloud Microphysics Scheme:

6 hydrometeor categories

Size distribution of each category $x$:

$$N_x(D) = N_{0x}D^{\alpha_x}e^{-\lambda_x D}$$

Prognostic quantities:
- mass mixing ratio ($q_x$)
- total number concentration ($N_x$)

* Milbrandt and Yau, 2005a,b (J. Atmos. Sci.)
Cloud Microphysics Scheme:

Representation of “snow”: (i.e. solid, white precipitation at ground)

Size distribution of each category $x$:

$$N_x(D) = N_{0x} D^{\alpha_x} e^{-\lambda_x D}$$

Prognostic quantities:

- mass mixing ratio ($q_x$)
- total number concentration ($N_x$)

“Snow” is represented by 3 categories:

ICE (pristine crystals)

SNOW (large crystals / aggregates)

GRAUPEL (heavily rimed crystals)
“Snow” is represented by 3 categories:

**ICE** (pristine crystals), \( \rho_i = 500 \text{ kg m}^{-3} \)

**GRAUPEL** (rimed crystals), \( \rho_g = 400 \text{ kg m}^{-3} \)

**SNOW** (large crystals / aggregates), \( \rho_s = f(D_s) \)

*For **SNOW**:

Use of \( m_s(D) = c D_s^d \)

\( \Rightarrow \rho_s(D) = e D_s^f \)

(for the bulk density of an equivalent-mass sphere)

**Approach:**

For each category \( x \) (\( x = i, g, s \)):

Compute solid (unmelted) volume fluxes, \( F_{v_x} \)

\[
\frac{F_{v_x}}{F_{m_x}} = \frac{\int_0^\infty V(D) \cdot \text{vol}(D) \cdot N(D) \, dD}{\int_0^\infty V(D) \cdot m(D) \cdot N(D) \, dD} = \frac{\int_0^\infty V(D) \cdot \frac{m(D)}{\rho(D)} \cdot N(D) \, dD}{\int_0^\infty V(D) \cdot m(D) \cdot N(D) \, dD} = \frac{1}{\rho_x} \frac{\int_0^\infty V(D) \cdot m(D) \cdot N(D) \, dD}{\int_0^\infty V(D) \cdot m(D) \cdot N(D) \, dD} = \frac{1}{\rho_x}
\]

\[
F_{v_x} = F_{m_x} \frac{m_x}{\rho_x}
\]

**BUT** – only true for constant \( \rho_x \)

(OK for \textit{ICE} and \textit{GRAUPEL})

For \textit{SNOW}, \([\rho = \rho(D)]\) - must compute \( F_v \) directly (from integral)

\[
\rightarrow F_{v_s} = \int_0^\infty V(D) \cdot \text{vol}(D) \cdot N(D) \, dD
\]
**Estimation of liquid fraction (during melting):**

**Actual model representation:**

\[ \rho_s = f(D_s) \quad \rho_L = 1000 \text{ kg m}^{-3} \]

**Conceptual view of melting snow:**

\[ \frac{q_r}{q_r + q_s} \rightarrow \text{liquid fraction of melting snow} \]
Adjustments:

if $T < 0^\circ C$:

\[
f_{\text{liq}} = \frac{q_r}{q_r + (q_i + q_g + q_s)}
\]

\[
F_v = (1 - f_{\text{liq}}) \cdot F_v + f_{\text{liq}} \cdot F_{v_{\text{liq}}}
\]

E.g. Assume $D_s = 5 \text{ mm} \rightarrow \rho_s(D_s) = 26 \text{ kg m}^{-3}$:

$\rho_{s_{\text{melting}}} = 0.95(26 \text{ kg m}^{-3}) + 0.05(1000 \text{ kg m}^{-3}) = 75 \text{ kg m}^{-3}$

$\rho_{s_{\text{melting}}} = 0.50(26 \text{ kg m}^{-3}) + 0.50(1000 \text{ kg m}^{-3}) = 513 \text{ kg m}^{-3}$
Thus, instantaneous precipitation rates are given by:

\[
F_{v_{\text{liq}}} = F_{m_i} \rho_L + F_{m_g} \rho_L + F_{m_s} \rho_L
\]

→ total solid (liquid-equivalent) precipitation rate

\[
F_{v_{\text{liq}}} = \frac{F_{m_i}}{\rho_i} + \frac{F_{m_g}}{\rho_g} + \int_0^{\infty} V_s(D) \cdot \text{vol}_s(D) \cdot N_s(D) dD
\]

\[
F_v = (1 - f_{\text{liq}}) \cdot F_{v_{\text{liq}}} + f_{\text{liq}} \cdot F_{v_{\text{liq}}}
\]

(if \( T < 0^\circ\mathrm{C} \))

\[
\rightarrow \quad \text{SOLID-to- LIQUID}_{\text{inst}} = \frac{F_v}{F_{v_{\text{liq}}}}
\]
Accumulated Precipitation
(liquid-equivalent)

Accumulated Precipitation
(unmelted)
- snowfall amount -

x 10?
Accumulated Precipitation (liquid-equivalent)

Accumulated Precipitation (unmelted)
- snowfall amount -
Accumulated Precipitation (liquid-equivalent)
Number of Grid Points

Solid-to-Liquid Ratio

10:1

Source: Roebber et al. (2003), Weather and Forecasting

1650 Snowfall Events
28 Stations
1973–1994

Percentage of Total

Source: Roebber et al. (2003), Weather and Forecasting
Case: 12 March 2009 (00 z)

10:1

Solid-to-Liquid Ratio

New Microphysics
15 cases
(all grid points)

Whistler Station
Jan-March 2009

Number of Cases
Number of Points

Solid-to-Liquid Ratio

QPF

mm

160
140
120
100
80
60
40
20
0

Whistler Station
Solid-to-Liquid Ratio

**Diagnostic**
(Dubé algorithm)

**Explicit**
(Milbrandt-Yau)

Forecaster* at Cypress Bowl reported

“FLUFFY SNOWFLAKES”
(early afternoon)

*Michael Gélinas
2010 Olympics forecaster

2100 UTC (3:00 pm)
23 Feb 2010
Solid-to-Liquid Ratio

Diagnostic
(Dubé algorithm)

Explicit
(Milbrandt-Yau)

Forecaster* at Cypress Bowl reported
“FAST-FALLING (LIKE RAIN) SNOW PELLETS”
(early evening)

*Michael Gélinas
2010 Olympics forecaster

0400 UTC (8:00 pm)
23 Feb 2010
Explicit Diagnostic

FLUFFY SNOWFLAKES

SNOW PELLETS
Explicit Diagnostic

FLUFFY SNOWFLAKES

SNOW PELLETS
CONCLUSION – Part 1

• The cloud microphysics scheme predicts the individual quantities and size distributions of pristine crystals, aggregates, graupel

• This information can be exploited to compute the instantaneous solid (unmelted) precipitation rate → it need not be simply inferred (or diagnosed)

• Real-time simulations (during 2010 Olympics) indicate that this method produces a realistic results
1. Prognostic Snow Density

2. Sedimentation-Induced Errors

3. Comparison of 2-Moment Schemes
MODEL PREDICTION OF A PROGNOSTIC MOMENT

e.g. the mass mixing ratio, $q_x$, of category $x$ (where $x = c, r, i, ...$)

$$
\frac{\partial q_x}{\partial t} = -\frac{1}{\rho} \nabla \cdot \left( \rho q_x \vec{U} \right) + TURB(q_x) + \left. \frac{dq_x}{dt} \right|_S + \frac{1}{\rho} \frac{\partial}{\partial z} \left( \rho q_x \vec{V}_{xq} \right)
$$

- **ADVECTION / COMPRESSION**
- **TURBULENT MIXING**
- **SOURCES / SINKS**
- **SEDIMENTATION**

**MODEL DYNAMICS**

**MICROPHYSICS SCHEME**
MOTIVATION

1. To propose a method to quantify the sedimentation-induced errors in bulk microphysics schemes

2. To examine alternatives to the “standard” two-moment approach
BULK MICROPHYSICS SCHEMES

COMPUTATION OF SEDIMENTATION

\[ \frac{\partial q_x}{\partial t} \bigg|_{SEDI} = \frac{\partial (\rho q_x \bar{V}_{xq})}{\partial z} \]

\[ * \bar{V}_{xq} = \text{mass-weighted fall velocity} \]

\[ \frac{\partial N_x}{\partial t} \bigg|_{SEDI} = \frac{\partial (N_x \bar{V}_{xN})}{\partial z} \]

\[ \bar{V}_{xN} = \text{number-weighted fall velocity} \]

\[ \frac{\partial Z_x}{\partial t} \bigg|_{SEDI} = \frac{\partial (Z_x \bar{V}_{xZ})}{\partial z} \]

\[ \bar{V}_{xZ} = \text{reflectivity-weighted fall velocity} \]
BULK MICROPHYSICS SCHEMES

Sedimentation: 1-MOMENT scheme

\[ M_3(\rho q_r) \]

**Initial Conditions:**

\[ \rho q = 0.5 \text{ g m}^{-3} \]
\[ N_0 = 8 \times 10^6 \text{ m}^{-4} \]
\[ \mu = 0 \]
\[ N_r(D) = N_{0r} D^{\mu_r} e^{-\lambda_r D} \]

* Wacker and Lüpkes (2009)
BULK MICROPHYSICS SCHEMES

Sedimentation: 1-MOMENT scheme

Initial Conditions:
\[ \rho q = 0.5 \, \text{g m}^{-3} \]
\[ N_0 = 8 \times 10^6 \, \text{m}^{-4} \]
\[ \mu = 0 \]
\[ N_r(D) = N_0 r D^\mu e^{-\lambda r D} \]
BULK MICROPHYSICS SCHEMES

Sedimentation: 1-MOMENT scheme

\[
M_0 = \langle N_{Tr} \rangle \\
M_3 = \langle \rho q_r \rangle \\
M_6 = \langle Z_r \rangle \\
D_m = \left[ \frac{M_3}{M_0} \right]^{\frac{1}{3}}
\]

Initial Conditions:
- \( \rho q = 0.5 \text{ g m}^{-3} \)
- \( N_0 = 8 \times 10^6 \text{ m}^{-4} \)
- \( \mu = 0 \)
- \( N_r(D) = N_{0r} D^{\mu_r} e^{-\lambda_r D} \)
BULK MICROPHYSICS SCHEMES

Initial Conditions:
\[ \rho q = 0.5 \, \text{g m}^{-3} \]
\[ N_0 = 8 \times 10^6 \, \text{m}^{-4} \]
\[ \mu = 0 \]
\[ N_r(D) = N_0 r D^\mu e^{-\lambda_r D} \]
Analytic bin model calculation: (1D column)

\[ M_0 \quad (N_{Tr}) \quad M_3 \quad (\rho q_i) \quad M_6 \quad (Z_r) \quad D_m = \left[ \frac{M_3}{M_0} \right]^{\frac{1}{3}} \]
Evaluation approach: **COMPARE PROFILES** of prognostic moments

1-MOMENT

2-MOMENT

Useful information, BUT …
... other moments are important for microphysical growth rates.

e.g. continuous collection of cloud water ($CL_{cx}$):

$$\frac{dq_x}{dt}|_{CL} = \int_{0}^{\infty} \frac{dm(D)}{dt}|_{CL} N(D) dD$$

$$\frac{dm(D)}{dt}|_{CL} = \frac{\pi D^2}{4} V(D) E_{xc} \rho q_c = \left(\frac{\pi}{4} E_{xc} \rho q_c\right) D^{2+b_x}$$

$$\frac{dq_x}{dt}|_{CL} = \left(\frac{\pi}{4} E_{xc} \rho q_c\right) \int_{0}^{\infty} D^{2+b_x} N(D) dD$$

$$\left[ M_x(p) \equiv \int_{0}^{\infty} D^p N_x(D) dD \right]$$

The $p^{th}$ moment of $N_x(D)$

... etc. for other processes.

Most processes depend on moments between $M_0$ and $M_{3+b}$.
Comparisons of profiles of a given moment: $M_0$

For a given time:

- sedimentation profiles are plotted (for both analytic and bulk models)
- errors (differences, normalized against the initial value) are computed
Error plots for a range of computed moments: $M_0 - M_7$
(for a given time)

Normalized Errors are
POSITIVE / NEGATIVE
“Standard”* 1-MOMENT Scheme:

\[ N_x(D) = N_{0x} D^\mu_x e^{-\lambda_x D} \]

Prognostic \( M_3 (q) \)

Fixed \( N_0 \)

Fixed \( \mu = 0 \)
“Standard”* 1-MOMENT Scheme:

\[ N_x(D) = N_0 D^{\mu_x} e^{-\lambda_x D} \]

Prognostic \( M_3(q) \)

Fixed \( N_0 \)

Fixed \( \mu = 0 \)
“Standard” 1-MOMENT Scheme:

$t = 0 \text{ s}$

$t = 400 \text{ s}$

$t = 800 \text{ s}$

$t = 1200 \text{ s}$

$t = 1600 \text{ s}$
“Standard” Bulk Schemes

1-moment:

2-moment:

3-moment:

$N_x(D) = N_0 D^\mu e^{-\lambda_x D}$

Prognostic $M_3(q)$

Fixed $N_0$

Fixed $\mu = 0$

Prognostic $M_0(N_T), M_3(q)$

Fixed $\mu = 0$

Prognostic $M_0(N_T), M_3(q), M_6(Z)$

$t = 600$ s
Alternative Choices of Prognostic Moments:

1-MOMENT schemes
Alternative Choices of Prognostic Moments: 1-MOMENT schemes
Alternative Choices of Prognostic Moments: $M_0 - M_3$
Alternative Choices of Prognostic Moments: 2-MOMENT schemes
Alternative Choices of Prognostic Moments: 3-MOMENT schemes
Alternative Choices of Prognostic Moments:

3-MOMENT schemes
Alternative treatment of the shape parameter: 2-MOM schemes $M_0 - M_3$

$\mu = 0$
Alternative treatment of the shape parameter: 2-MOM schemes

\[ N_x(D) = N_0 D^\mu e^{-\lambda D} \]

Different pairs of moments, \( k \) and \( j \)

Rate of size-sorting is proportional to ratio \( V_k/V_j \).

This ratio is a function of \( \mu \); therefore, the value of \( \mu \) controls the rate of size-sorting.

NOTE: \( \mu \) is a measure of the relative spectral dispersion.
Alternative treatment of the shape parameter: 2-MOM schemes $M_0-M_3$

Rate of size-sorting is proportional to ratio $V_k/V_j$.

This ratio is a function of $\mu$; therefore, the value of $\mu$ controls the rate of size-sorting.

NOTE: $\mu$ is a measure of the relative spectral dispersion.
Alternative treatment of the shape parameter:

- 2-MOM schemes
- $M_0 - M_3$

Seifert (2008)

$\mu = f(M_0, M_3)$

Milbrandt and McTaggart-Cowan (2010)

$\mu = f(D_m)$

Milbrandt and Yau (2005a)

$V_{M3}/V_{M0} = f(M_0, M_3)$

(Generalized for any $M_j, M_k$ combo)
CONCLUSION – Part 2

1. Minimizing the sedimentation-induced errors in “computed” moments is important

2. Errors can be shifted to different ranges of moments by choosing different prognostic moment(s)

3. 3-moment schemes are generally superior to 2-moment schemes in terms of reducing sedimentation-induced errors

4. Existing 2-moment schemes can be dramatically improved by controlling excessive size sorting that results with a fixed DSD dispersion (shape parameter, $\mu$)

1. Prognostic Snow Density
2. Sedimentation-Induced Errors
3. Comparison of 2-Moment Schemes
PREMISE:

• 1-moment BMSs suffer from the need to specify DSD parameters; 2-moment BMSs predict DSD more feely

• 2-moment BMSs can better represent certain processes (e.g. sedimentation, self-collection, drop breakup)

• **Implication:** Increasing complexity of a BMS tends towards truth

MOTIVATING QUESTIONS:

• Do similar 2-moment schemes produce similar results?
• What are the major sensitivities in 2-moment BMSs?
METHODOLGY:

• Use similar 2-moment BMSs in a common modeling framework
• Conduct simulations (with each scheme) and compare results
• Identify, through sensitivity tests, the reasons for any major differences
**METHODOLOGY:**

- **BMSs:** Morrison* (MOR) and Milbrandt-Yau** (MY)
- **Model:** WRF (v3.1)
- **Case:** Idealized supercell (1-km, initial warm/moist bubble)

<table>
<thead>
<tr>
<th>MOR</th>
<th>MY</th>
</tr>
</thead>
<tbody>
<tr>
<td>+ 2-moment (all categories*)</td>
<td>+ 2-moment (all categories*)</td>
</tr>
<tr>
<td>+ cloud, rain, ice, snow, graupel</td>
<td>+ cloud, rain, ice, snow, graupel, hail</td>
</tr>
<tr>
<td>+ fixed shape parameters (0)</td>
<td>+ fixed shape parameters (0)</td>
</tr>
<tr>
<td>+ $N_c = 250 \text{ cm}^{-3}$</td>
<td>+ $N_c = 250 \text{ cm}^{-3}$</td>
</tr>
</tbody>
</table>

- similar fall velocity parameters
- similar warm rain coalescence parameterizations
- similar ice initiation
- different raindrop breakup parameterizations

*** As tested in this study ***


** Milbrandt and Yau (2005), *J. Atmos. Sci.*
BASELINE (CONTROL) SIMULATIONS

Radar Reflectivity

Morrison: 2-moment

Milbrandt-Yau: 2-moment

$z = 0.25 \text{ km}$

$z = 11.6 \text{ km}$

$t = 60 \text{ min}$
BASELINE (CONTROL) SIMULATIONS

**Evaporative cooling rates**

**Cold Pool Strength***(θ′)*

**Morrison:**
2-moment

**Milbrandt-Yau**
2-moment
BASELINE (CONTROL) SIMULATIONS

Morrison:

Milbrandt-Yau:

Vertical Velocity

\( z = 0.8 \text{ km} \)

\( z = 4.7 \text{ km} \)

\( t = 40 \text{ min} \)

\( t = 60 \text{ min} \)

\( t = 120 \text{ min} \)
### Sensitivity Experiments: 1. Graupel vs. Hail

**Morrison:**

<table>
<thead>
<tr>
<th>Category x = graupel</th>
<th>$\rho_g = 400 \text{ kg m}^{-3}$</th>
<th>$V_g \sim 1 - 3 \text{ m s}^{-1}$</th>
<th>medium-density GRAUPEL</th>
</tr>
</thead>
<tbody>
<tr>
<td>OR</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Category x = hail</td>
<td>$\rho_g = 900 \text{ kg m}^{-3}$</td>
<td>$V_g \sim 10 - 40 \text{ m s}^{-1}$</td>
<td>high-density HAIL</td>
</tr>
</tbody>
</table>

→ with a switch to toggle between types

**Milbrandt-Yau:**

<table>
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<td>AND</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Category x = hail</td>
<td>$\rho_h = 900 \text{ kg m}^{-3}$</td>
<td>$V_h \sim 10 - 40 \text{ m s}^{-1}$</td>
<td>high-density HAIL</td>
</tr>
</tbody>
</table>

→ with switches to shut OFF either category
SENSITIVITY EXPERIMENTS:
1. GRAUPEL vs. HAIL

*Cold Pool Strength* ($\theta'$)

Morrison:
GRAUPEL - only

Milbrandt-Yau:
GRAUPEL - only

Radar Reflectivity

$z = 0.25 \text{ km}$
SENSITIVITY EXPERIMENTS:
1. GRAUPEL vs. HAIL

Morrison:
HAIL – only (BASELINE)

Milbrandt-Yau:
HAIL - only

Radar Reflectivity

Cold Pool Strength* ($\theta'$)

$z = 0.25 \text{ km}$
CONVERSION of GRAUPEL to HAIL

- When a frozen particle growing by accretion first reaches the Shumann-Ludlam limit (SLL), it is termed a hailstone (Young, 1993).

- The size of a particle at the SLL is a function of the ambient $T$, $LWC$, and $IWC$:

$$D_{SLL} = 0.01 \exp\left(\frac{-T_c}{1 \times 10^4 \rho (q_c + q_r) - 1.3 \times 10^3 \rho q_i + 10^{-3}}\right)$$

This portion of GRAUPEL is undergoing wet growth and should therefore convert to HAIL.

Strictly, the incomplete gamma distribution ($D_{SLL} \to \infty$) should be evaluated.

Currently in Milbrandt-Yau scheme:

$$CN_{gh} = \frac{D_{mg}}{2D_{SLL}} (CL_{cg} + CL_{rg} + CL_{ig})$$
SENSITIVITY EXPERIMENTS:
2. PARAMETERIZATION OF DROP BREAKUP

Raindrop breakup is parameterized by:

1. Imposing a drop size-limiter (maximum $D_{r\_mean}$)

**MORRISON:** $D_{r\_max} = 0.9$ mm

**MILBRANDT-YAU:** $D_{r\_max} = 5.0$ mm

2. Reduction of collection efficiency in rain self-collection equation

$$N_yC_L_{yx} = -\frac{\pi}{4} \int_0^\infty \int_0^\infty |V_x(D_x) - V_y(D_y)|^2 E(x, y) N_y(D_y) N_x(D_x) dD_y dD_x$$

**MORRISON:** none

**MILBRANDT-YAU:** Ziegler (1985)
SENSITIVITY EXPERIMENTS:  
2. PARAMETERIZATION OF DROP BREAKUP

Morrison: 2-moment

Milbrandt-Yau: 2-moment

Cold Pool Strength* (min. $\theta'$)

All runs with HAIL-only
CONCLUSION – Part 3

1. The simulation of deep convection can be sensitive to the parameterization of graupel/hail in a 2-moment BMS

2. The simulation of deep convection can be very sensitive to the parameterization of raindrop breakup (depending on how the ice-phase results in big/little drops)

3. Increasing complexity in a BMS does not necessarily lead to convergence

For more details:  Morrison and Milbrandt (2010)  
Mon. Wea. Rev. (accepted)
CONCLUSION – Part 3

1. The simulation of deep convection can be sensitive to the parameterization of graupel/hail in a 2-moment BMS

2. The simulation of deep convection can be very sensitive to the parameterization of raindrop breakup (depending on how the ice-phase results in big/little drops)

3. Increasing complexity in a BMS does not necessarily lead to convergence

Continued research: (collaboration with NCAR)

→ To examine the sensitivity of the parameterization of specific processes in 2-moment schemes – though sensitivity studies and comparison to observation – towards understanding the behavior of these schemes and of the microphysics of storm systems
Current Research and Development

1. Upgrade of 2-moment (M-Y) scheme for HRDPS
   • diagnostic $\mu_r$ and $\mu_h$
   • new parameterization for hail initiation

2. Development of simplified version for RDPS
   • reduction to essential categories and processes
   • time-splitting for microphysics

3. Development* of version for GDPS
   • cloud (and precipitation) fraction
   * current research of Frederick Chosson (McGill University)
THANK YOU

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Ron McTaggart-Cowan (RPN)
Hugh Morrison (NCAR)
Milbrandt-Yau Multi-Moment Scheme

- Six hydrometeor categories:
  - 2 liquid: *cloud* and *rain*
  - 4 frozen: *ice*, *snow*, *graupel* and *hail*
  - Each size spectrum described by a 3-parameter gamma distribution function
    → Full version has 17 prognostic variables

\[ N_x(D) = N_{0x} D^{\alpha_x} e^{-\lambda_x D} \]

- ~ 50 distinct microphysical processes

- **Diagnostic-\( \alpha_x \)** relations added for 2-moment version

- **Predictive equations for** \( Z_x \) **added for 3-moment version**

* Milbrandt and Yau (2005a,b) *J. Atmos. Sci.*
Milbrandt-Yau* Multi-Moment Scheme

Applications of Scheme: (since 2005)

- Implementation of 1-moment version for GEM-LAM-2.5 system
- Implementation of 3-moment version into ARPS (U of Oklahoma)
- 2-moment version used for 2010 Vancouver Olympics (1-km LAM)
- 2-moment version implemented into official WRF_v3.2
- 2-moment version to be implemented into HRDPS

* Milbrandt and Yau (2005a,b) J. Atmos. Sci.