The vertical in GEM

By André Plante, CMC, May 7th 2010

Ip1, sigma, eta, hyb, P0, PT, HY, !!

PX, Rcoef, A, B, GZ

r.hy2pres, hyb_to_pres

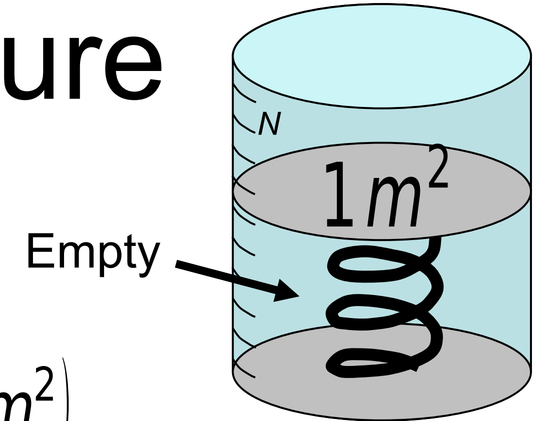
Outline

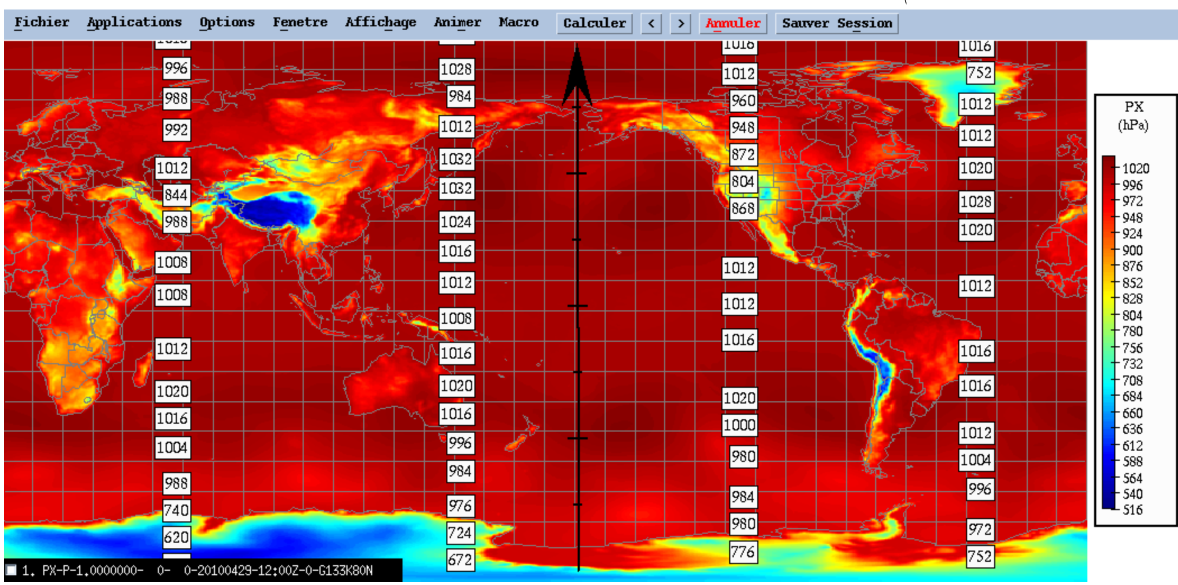
- The hydrostatic pressure
- Model coordinate in sigma, eta, hybrid-GEM3 and hybrid-GEM4
- Encode all this in RPN files
- Computing the hydrostatic pressure in scripts and in programs

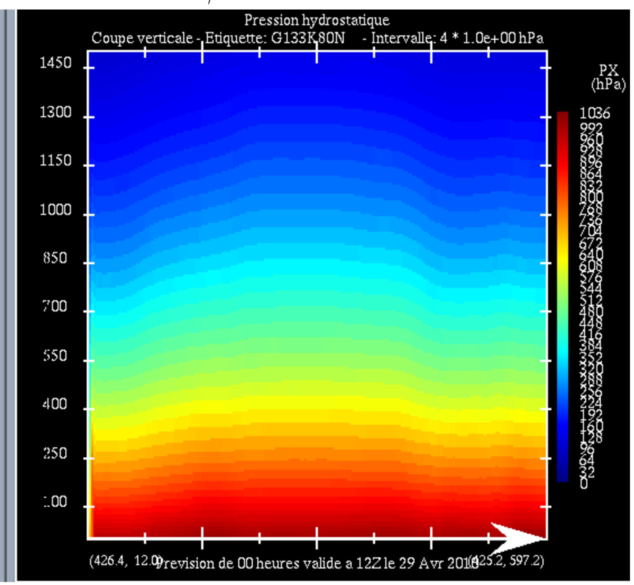
The Hydrostatic pressure

The hydrostatic pressure $\pi \quad \left[N/m^2 \right]$ is the weight of the air in an atmosphere at rest over a square meter.

nomvar P0 and PX $\quad 100\,kPa \left(¿\, 150 \text{ persons}/m^2 \right)$





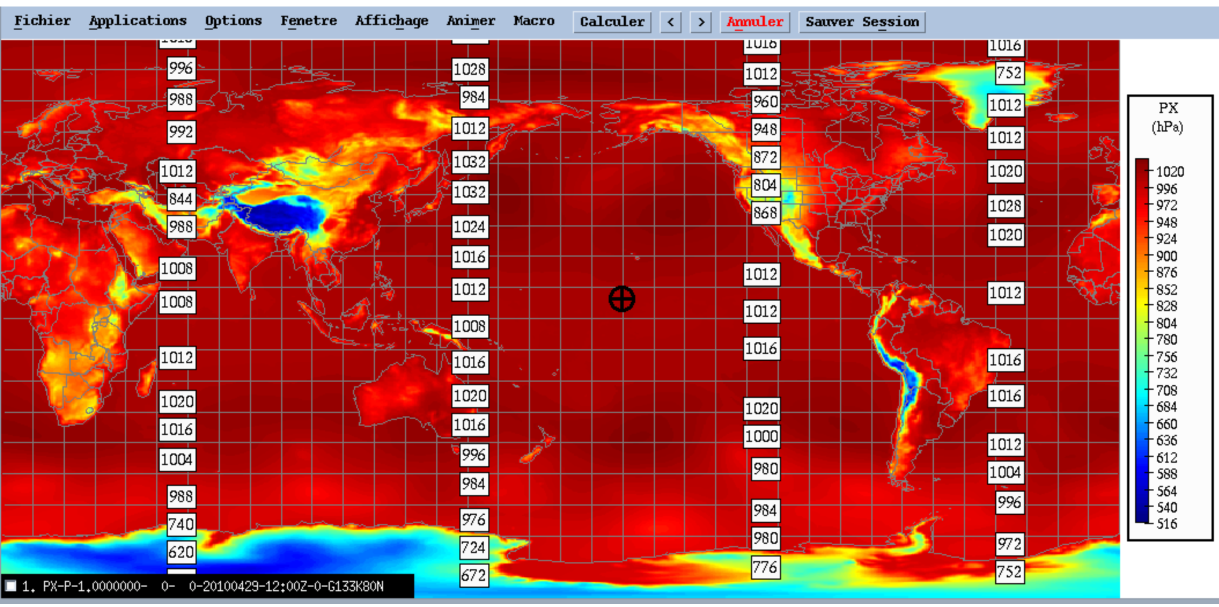


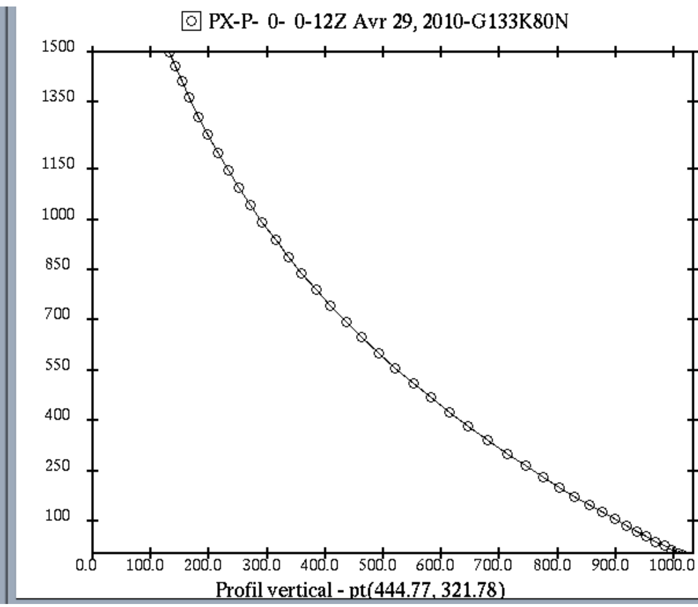
The atmosphere is very close the be hydrostatic except for some small scale systems < 10km. The global (35km) and regional (15km) models are hydrostatic but the LAMs (2.5km) are non hydrostatic.

The Hydrostatic pressure is monotonic with height z, therefore it can replace z in the model equations.

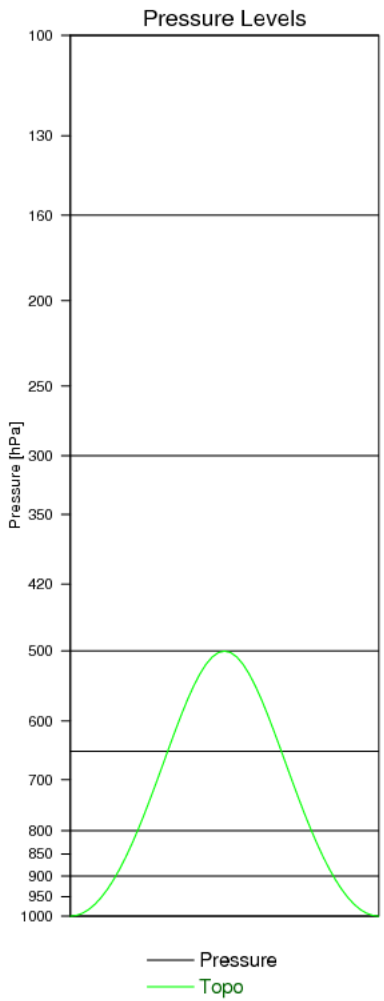
This change of independent variable makes the hydrostatic model equations simpler. For non hydrostatic model, the advantage doesn't hold anymore, e.g. MC2 is in height coordinate.

$$\frac{\partial \pi}{\partial z} = -g\rho$$





Model levels in pressure



•Simple equation formulation
•Simple output
•Grid points below topo

Günther Zängl, Monthly Weather Review 2003:

•The step-coordinate formulation employed in the National Centers for Environmental Prediction (NCEP) Eta Model (Mesinger et al. 1988) turned out to be unsuitable for high-resolution simulations of airflow over mountains (Gallus and Klemp 2000).

•A very promising method to overcome these problems was proposed by Steppeler et al. (2002). They use so-called shaved cells to impose a smooth forcing of the vertical wind component where the coordinate surfaces intersect the ground. Yet, coupling the physics packages to this type of model is tedious and still in progress. In particular, the highly variable distance of the lowermost model level from the ground requires substantial modifications of the boundary layer parameterizations.

Phillips 1957

A COORDINATE SYSTEM HAVING SOME SPECIAL ADVANTAGES FOR NUMERICAL FORECASTING

By N. A. Phillips

Massachusetts Institute of Technology[1]

(Manuscript received 29 October 1956)

The coordinate system used to date in numerical forecasting schemes has been the x, y, p, t-system introduced by Sutcliffe and Godart [4] and also by Eliassen [3]. This system, in common with the ordinary x, y, z, t-system, has certain computational disadvantages in the vicinity of mountains, because the lower limit of the atmosphere is not a coordinate surface. The purpose of this brief note is to describe a modified coordinate system in which the ground is always a coordinate surface.

It is obtained by replacing the vertical coordinate p in the x, y, p, t-system by the independent variable $\sigma = p/\pi$, where $\pi = \pi(x, y, t)$ is the pressure at ground level. σ ranges monotonically from zero at the top of the atmosphere to unity at the ground. In describing the relation between this x, y, σ, t-system and the usual x, y, p, t-system, we will use a subscript p to indicate a derivative along a pressure surface. Differentiation in the new x, y, σ, t-system will have no subscripts.

The following relation holds, where ξ can be x, y, or t:

$$\left(\frac{\partial}{\partial \xi}\right)_p = \frac{\partial}{\partial \xi} - \frac{\sigma}{\pi}\frac{\partial \pi}{\partial \xi}\frac{\partial}{\partial \sigma}.$$

The horizontal equations of motion then become

$$\frac{du}{dt} = fv - \frac{\partial \phi}{\partial x} + \frac{\sigma}{\pi}\frac{\partial \phi}{\partial \sigma}\frac{\partial \pi}{\partial x} + F_x, \quad (1)$$

and

$$\frac{dv}{dt} = -fu - \frac{\partial \phi}{\partial y} + \frac{\sigma}{\pi}\frac{\partial \phi}{\partial \sigma}\frac{\partial \pi}{\partial y} + F_y, \quad (2)$$

where F_x and F_y are the horizontal components of the frictional force per unit mass, $u = dx/dt$, $v = dy/dt$, f is the Coriolis parameter, and ϕ is the geopotential. As is customary in most meteorological work, the Coriolis terms proportional to the cosine of the latitude have been neglected. Equations (1) and (2) differ from those in the x, y, p, t-system only by the inclusion of the terms in $\partial\phi/\partial\sigma$.

The operator d/dt is given by

$$\frac{d}{dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + \dot{\sigma}\frac{\partial}{\partial \sigma}, \quad (3)$$

where $\dot{\sigma} = d\sigma/dt$.

The hydrostatic equation is obtained from the relation

$$\frac{\partial}{\partial p} = \frac{\partial \sigma}{\partial p}\frac{\partial}{\partial \sigma} = \frac{1}{\pi}\frac{\partial}{\partial \sigma},$$

and becomes, simply,

$$\partial\phi/\partial\sigma = -RT/\sigma. \quad (4)$$

Here R is the gas constant, and T is the absolute temperature. The coefficient $\pi^{-1}\sigma(\partial\phi/\partial\sigma)$ appearing in (1) and (2) can thus be replaced by $-RT/\pi$. Since ϕ is known at $\sigma = 1$ (at the ground), a knowledge of $T(\sigma)$ will give $\phi(\sigma)$ from (4) by integration.

The equation of continuity in the x, y, p, t-system is

$$\nabla_p \cdot v + \partial\omega/\partial p = 0,$$

where $\omega = dp/dt$, and v is the horizontal velocity. Introducing the relation $dp/dt = \pi\dot{\sigma} + \sigma(d\pi/dt)$, we obtain the continuity equation in the new system:

$$\nabla \cdot \pi v + \pi\,\partial\dot{\sigma}/\partial\sigma + \partial\pi/\partial t = 0. \quad (5)$$

Since $\dot{\sigma}$ is zero at the top of the atmosphere ($\sigma = 0$), integration of (5) with respect to σ gives

$$\pi\dot{\sigma} = -\int_0^\sigma \nabla\cdot\pi v\, d\sigma - \sigma\frac{\partial \pi}{\partial t}. \quad (6)$$

Extension of the integration all the way to the ground ($\sigma = 1$) gives the formula for $\partial\pi/\partial t$:

$$\frac{\partial \pi}{\partial t} = -\int_0^1 \nabla\cdot\pi v\, d\sigma, \quad (7)$$

since $\dot{\sigma} = 0$ at the ground.

[1] The research reported here has been sponsored by the Geophysics Research Directorate, Air Force Cambridge Research Center, and by the Office of Naval Research under Contract Nonr-1841(18).

The first law of thermodynamics can be written as

$$\frac{d\ln\theta}{dt} = \frac{1}{c_pT}\dot{Q}, \quad (8)$$

where θ is the potential temperature, c_p the specific heat at constant pressure, and $\dot{Q}$ is the non-adiabatic rate of heating per unit mass. When $\dot{Q}$ is proportional to dp/dt, as in the pseudo-adiabatic condensation process, dp/dt can be computed from the equation

$$\frac{dp}{dt} = \sigma v\cdot\nabla\pi - \int_0^\sigma \nabla\cdot\pi v\, d\sigma. \quad (9)$$

Finally, the potential temperature θ is related to σ, π and T by the equation

$$\ln\theta = \ln T - \kappa(\ln\pi + \ln\sigma) + \kappa\ln P, \quad (10)$$

where $\kappa = R/c_p$ and P is the standard pressure (normally 1000 mb) at which θ is defined.

Equations (1) to (10), in the dependent variables v, ϕ, θ, T and π, would seem to have their greatest advantage in making a numerical forecast with the "primitive" equations of motion. Although they could undoubtedly also be used in formulating a system which incorporates either the quasi-geostrophic or the quasi-nondivergent assumption [1; 2], the somewhat more complicated forms of the pressure-force term in (1) and (2), and of the continuity equation (5), naturally result in more complicated vorticity and divergence equations. However, the new system does have the following very real advantages in the numerical process:

1. Vertical advection terms, *e.g.*, $\dot{\sigma}\,\partial u/\partial\sigma$, are identically zero at the top and bottom of the atmosphere.
2. The ground is a coordinate surface, so that the effect of orography can be introduced without leading to either (*a*) uncentered horizontal differences in the vicinity of mountains or, alternatively, (*b*) the assumption that the hypothetical flow patterns obtained by reduction to sea level actually exist.

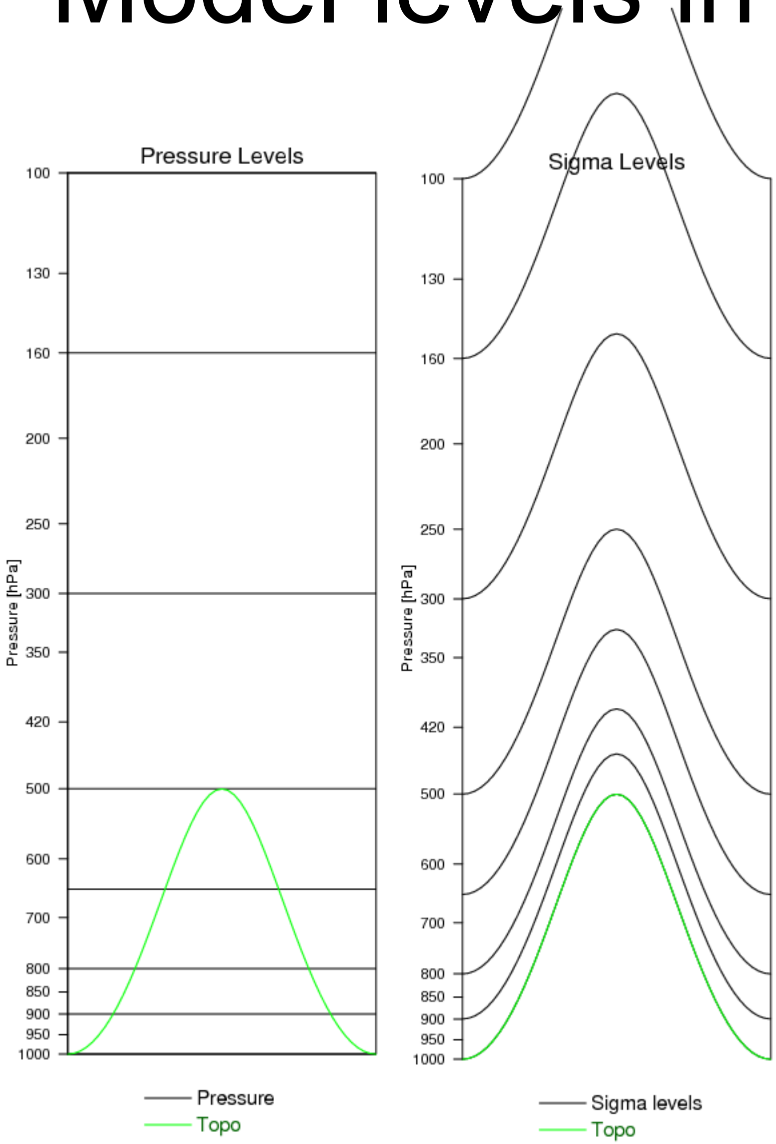
The observations defining the initial state of the atmosphere in this system would, of course, have to be interpolated so as to apply at the various σ-levels used in the finite-difference forecast scheme rather than at the conventional standard pressure levels. Since the actual method used for this would probably depend on the forecast equations to be employed, and since the various possibilities for performing this interpolation are quite obvious, this aspect of the problem will not be discussed here.

REFERENCES

1. Bolin, B., 1956: An improved barotropic model and some aspects of using the balance equation for three-dimensional flow. *Tellus*, **8**, 16–75.
2. Charney, J., 1948: On the scale of atmospheric motions. *Geofys. Publ.*, **17**, No. 2, 17 pp.
3. Eliassen, A., 1949: The quasi-static equations of motion with pressure as independent variable. *Geofys. Publ.*, **17**, No. 3, 44 pp.
4. Sutcliffe, R., 1947: A contribution to the problem of development. *Quart. J. r. meteor. Soc.*, **73**, 370–383.

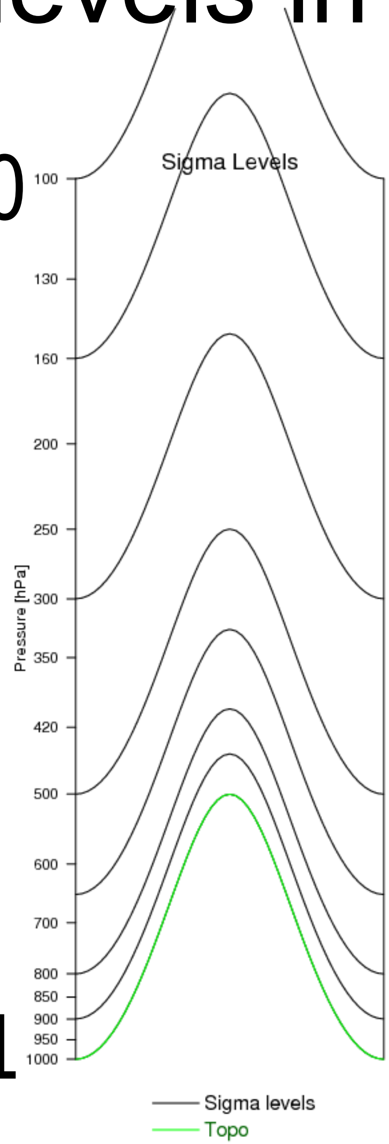
$$\sigma = \frac{\pi}{\pi_s}$$

Model levels in $\sigma = \frac{\pi}{\pi_s}$ (Phillips 1957)



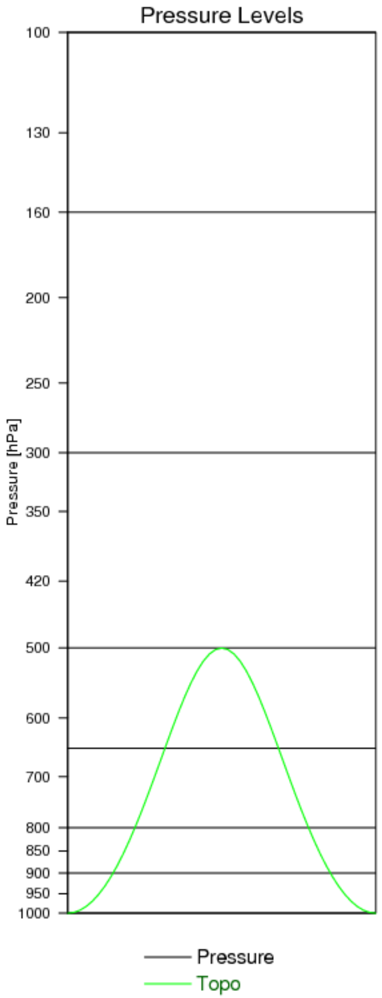
Model levels in $\sigma = \frac{\pi}{\pi_s}$ (Phillips 1957)

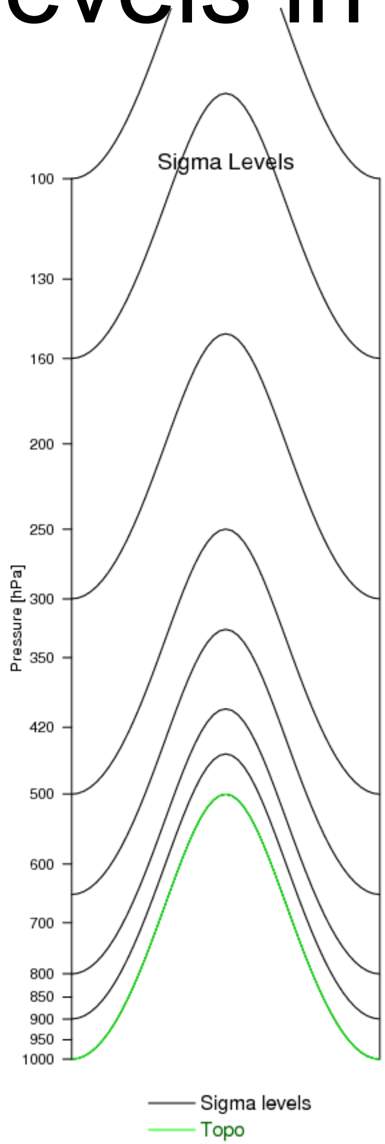
$$\sigma_T = \frac{\pi_T}{\pi_S} \neq 0$$



$$\sigma_S = \frac{\pi_S}{\pi_S} = 1$$

Model levels in $\sigma = \dfrac{\pi}{\pi_s}$ (Phillips 1957)





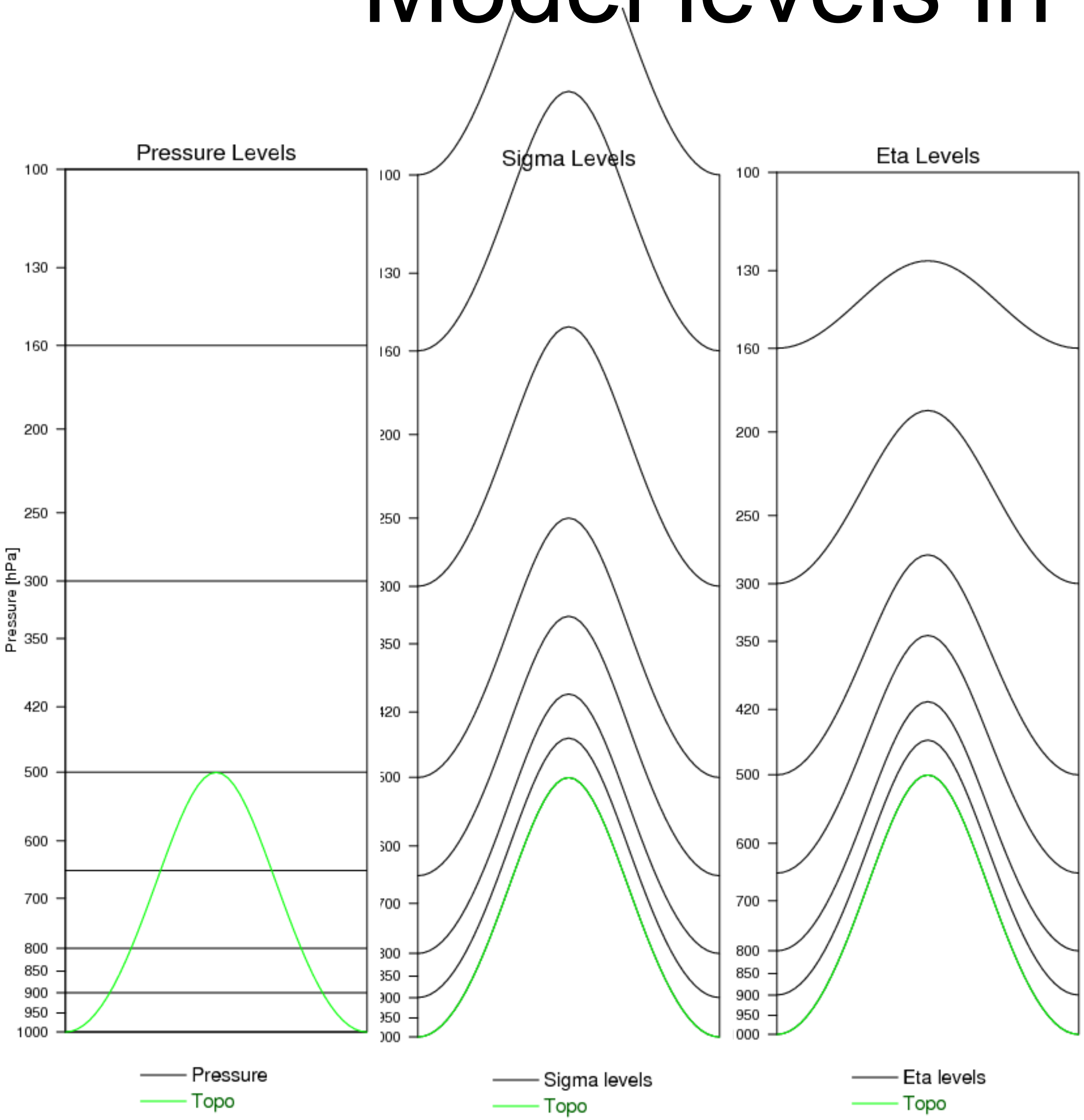
Günther Zängl, Monthly Weather Review 2003 :

•Easy to couple with boundary layer parameterizations because of its almost homogeneous vertical resolution near the surface
•Implementing the lower boundary condition is straightforward

•Imbalances in the discretization of the horizontal pressure gradient may lead to spurious motions over mountains
•Horizontal diffusion hard to implement

Top not at constant pressure makes it hard to implement the upper boundary conditions

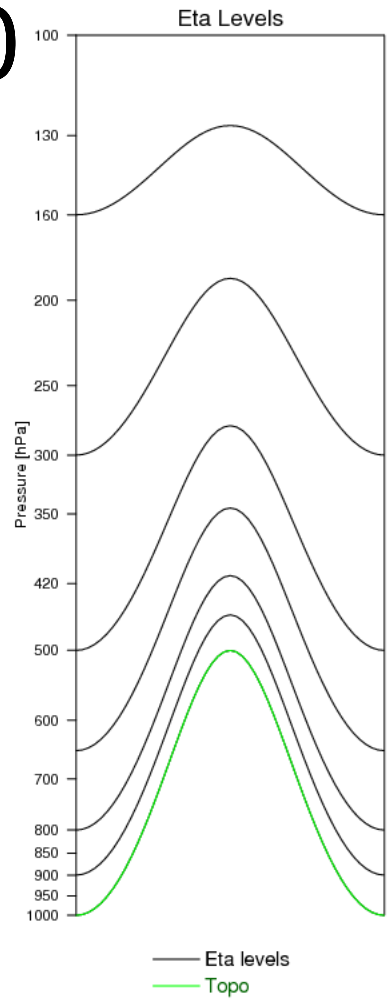
Model levels in $\eta = \frac{\pi - \pi_T}{\pi_S - \pi_T}$



This is a minor generalization of the original Phillips (1957) coordinate for models with a nonvanishing ptop (Mintz 1965).

Model levels in $\eta = \frac{\pi - \pi_T}{\pi_S - \pi_T}$

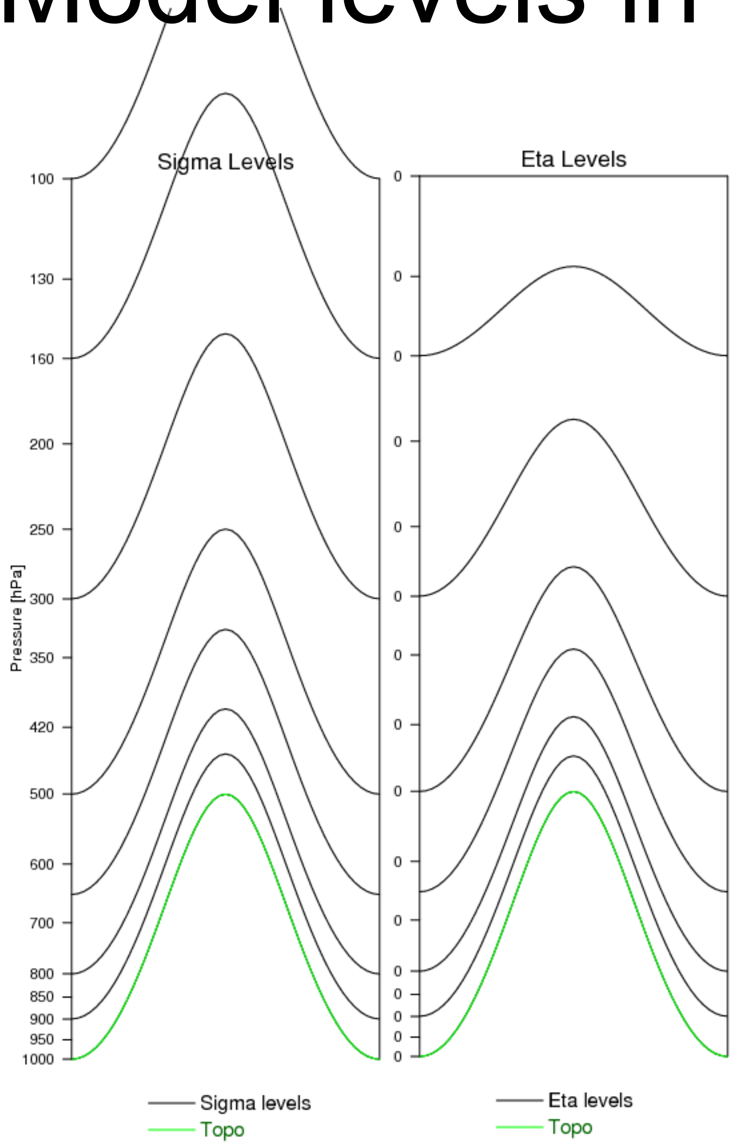
$$\eta_T = \frac{\pi_T - \pi_T}{\pi_S - \pi_T} = 0$$



This is a minor generalization of the original Phillips (1957) coordinate for models with a nonvanishing ptop (Mintz 1965).

$$\eta_S = \frac{\pi_S - \pi_T}{\pi_S - \pi_T} = 1$$

Model levels in $\eta = \dfrac{\pi - \pi_T}{\pi_S - \pi_T}$



This is a minor generalization of the original Phillips (1957) coordinate for models with a nonvanishing ptop (Mintz 1965).

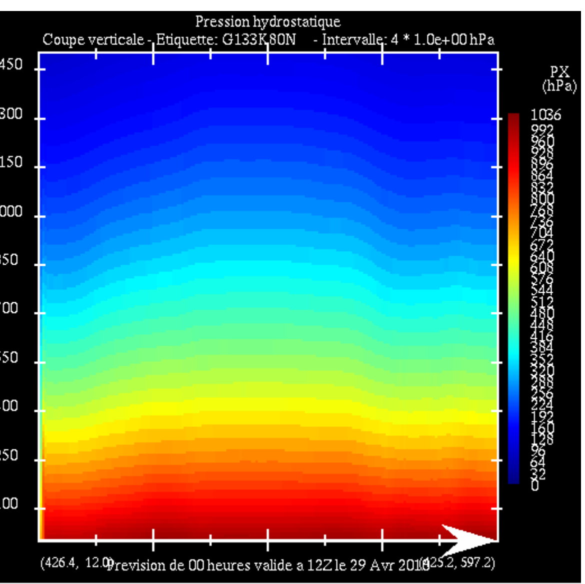
- Same advantages as the sigma but with a flat top in pressure.
- Flattens faster than sigma but remains bumpy at high levels.

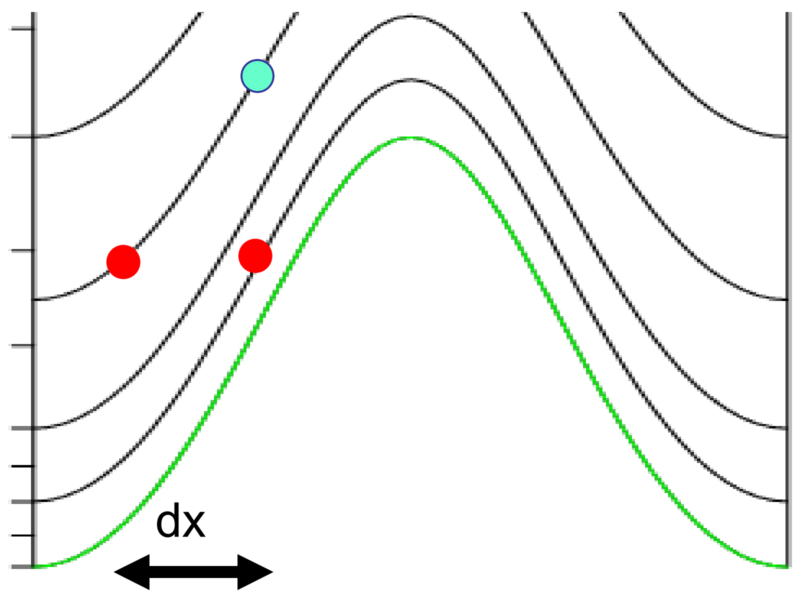
Imbalances in the discretization of the horizontal pressure gradient may lead to spurious motions over mountains

$$\frac{d\mathbf{V}_h}{dt} + f\mathbf{k}\mathrm{x}\mathbf{V}_h + RT\nabla_z \ln p = \mathbf{F}_z$$

$$\frac{d\mathbf{V}_h}{dt} + f\mathbf{k}\mathrm{x}\mathbf{V}_h + RT\nabla_\eta \ln p + (1+\mu)\nabla_\eta \phi = \mathbf{F}_h$$

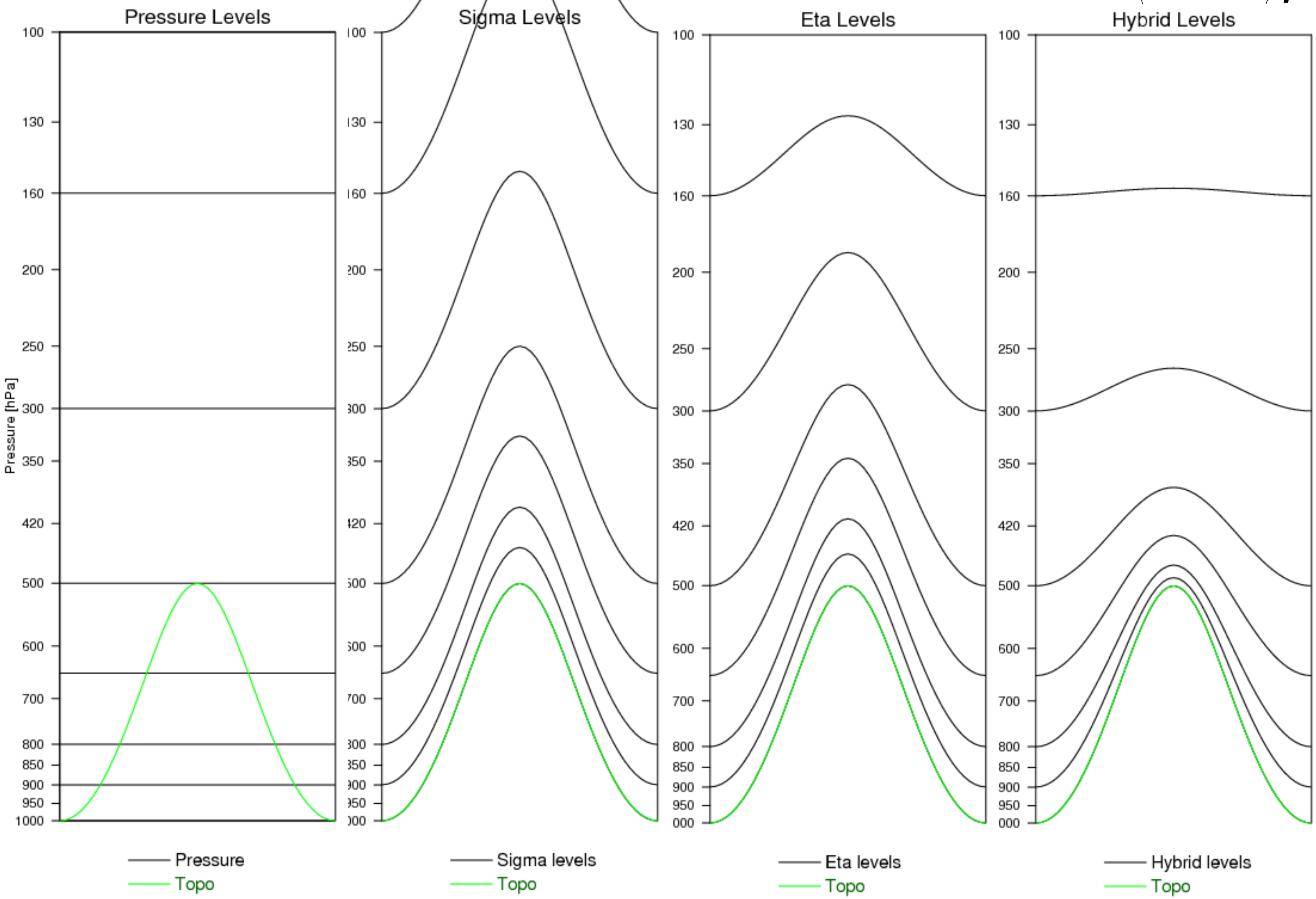
Two big terms of opposite sign





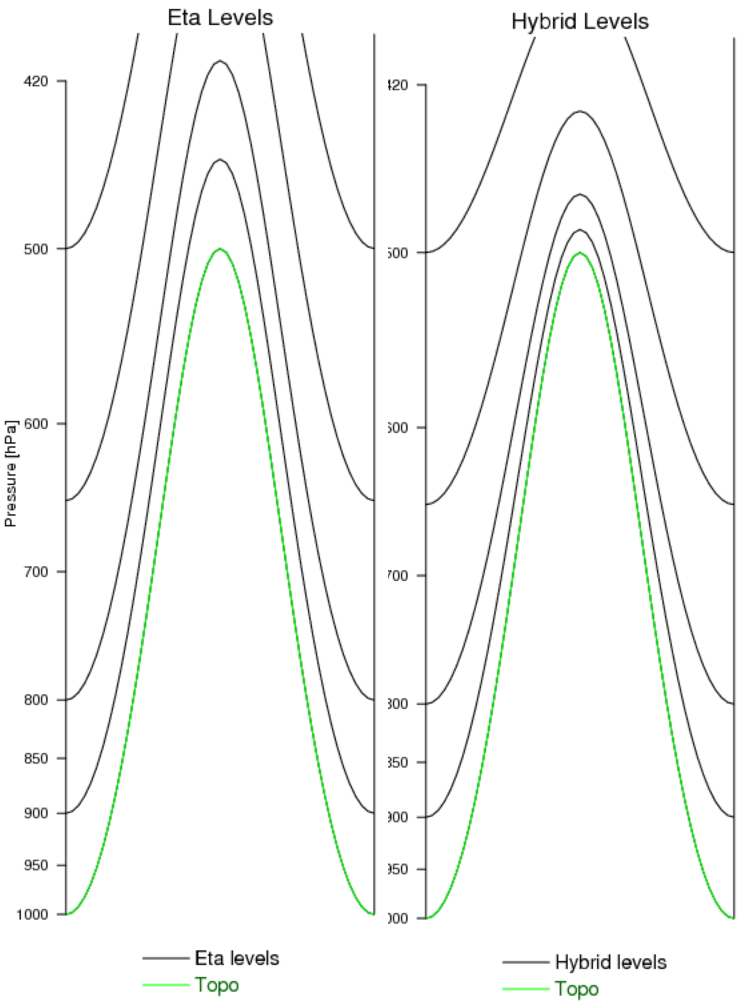
Hybrid (sigma-pressure) Model Levels

$$\sigma = \frac{\pi}{\pi_S} \Rightarrow \pi = \sigma \pi_S \qquad \pi = \underbrace{A}_{(h-B)p_{ref}} + B\pi_S$$



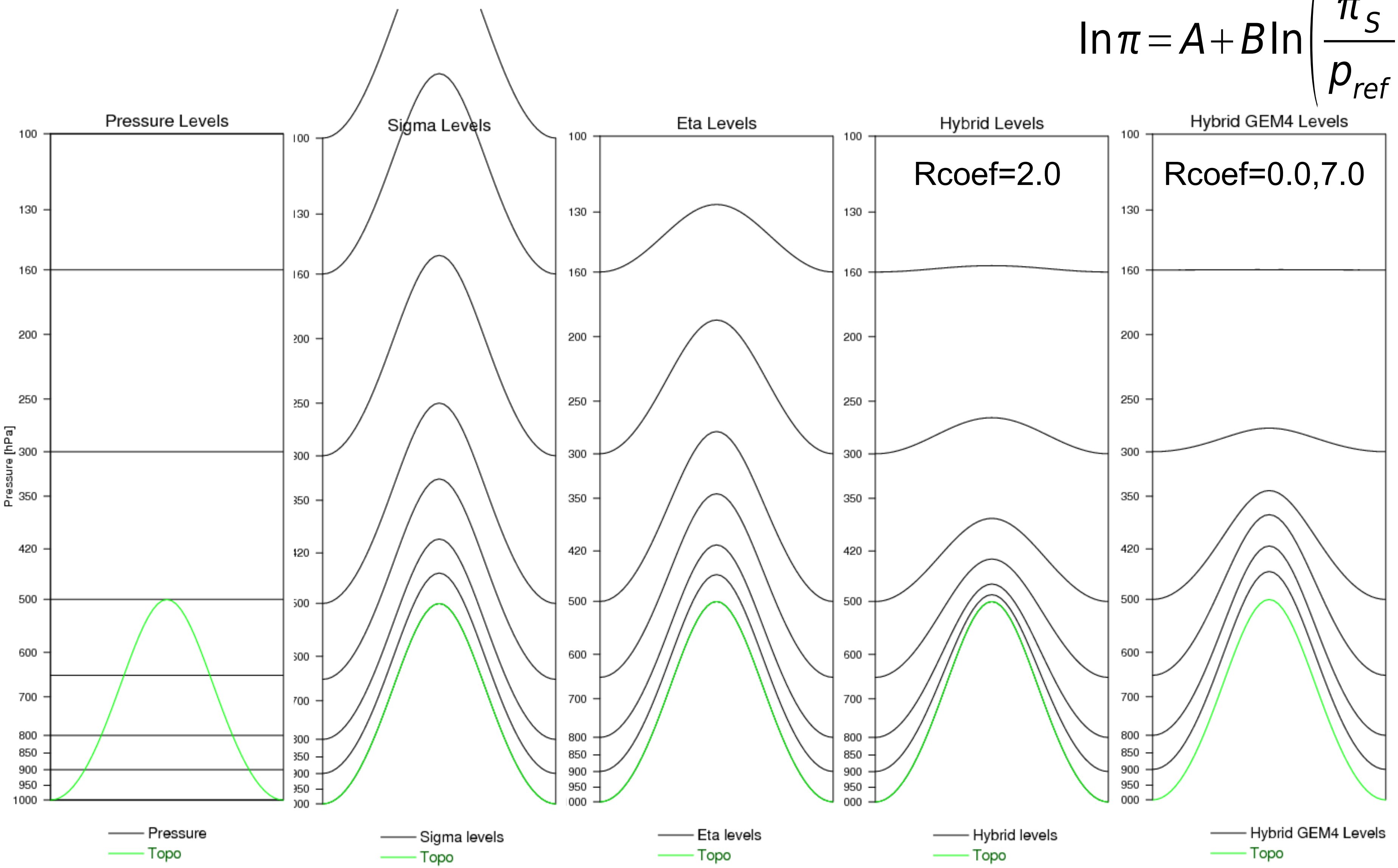
- Flattens faster than eta but flattens to much near the surface (Mt Logan problem)

Low Level Zoom

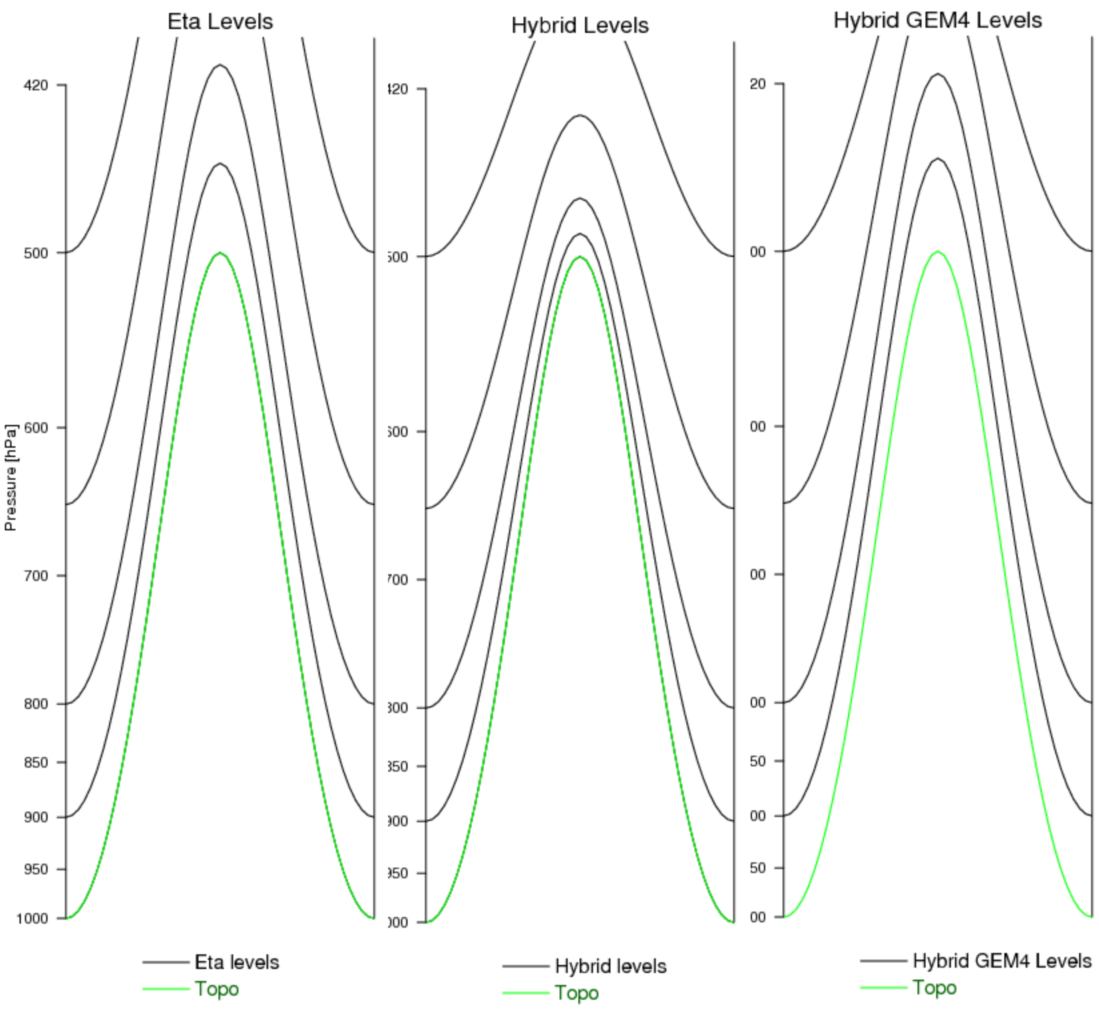


GEM4 Hybrid Model Levels

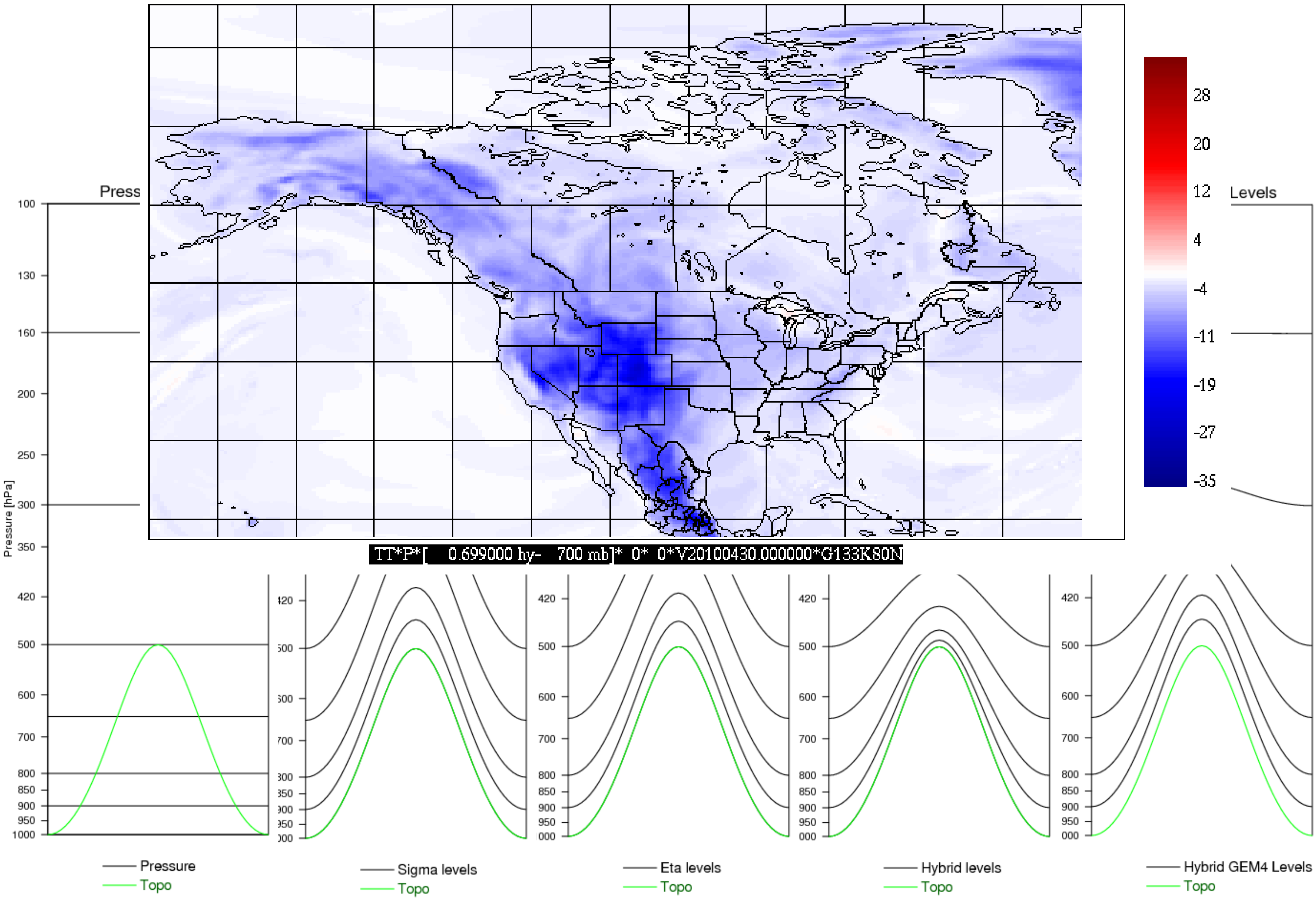
$$\ln \pi = A + B \ln\left(\frac{\pi_S}{p_{ref}}\right)$$



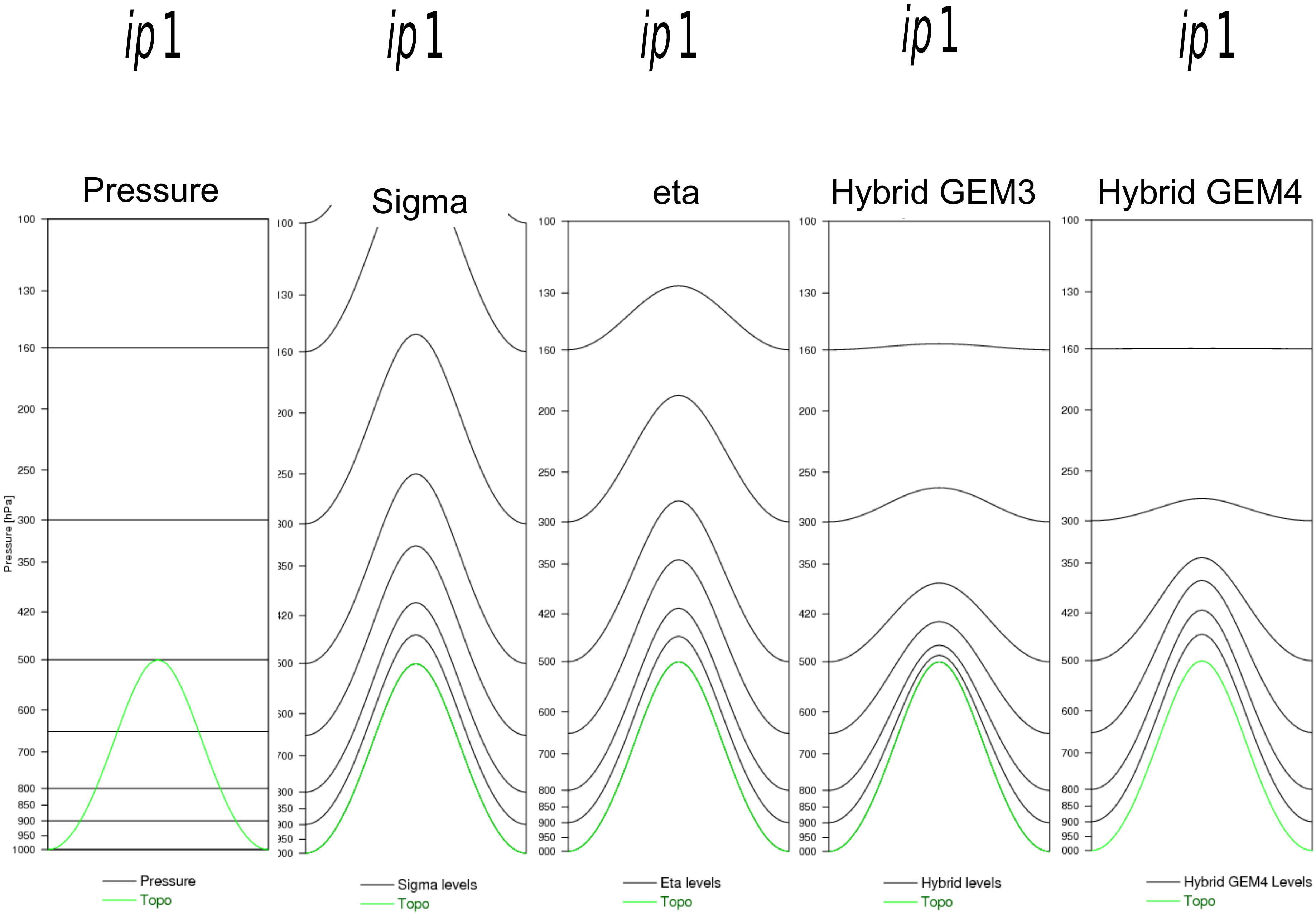
Low Level Zoom



Encoding all this in RPN files



Encoding all this in RPN files

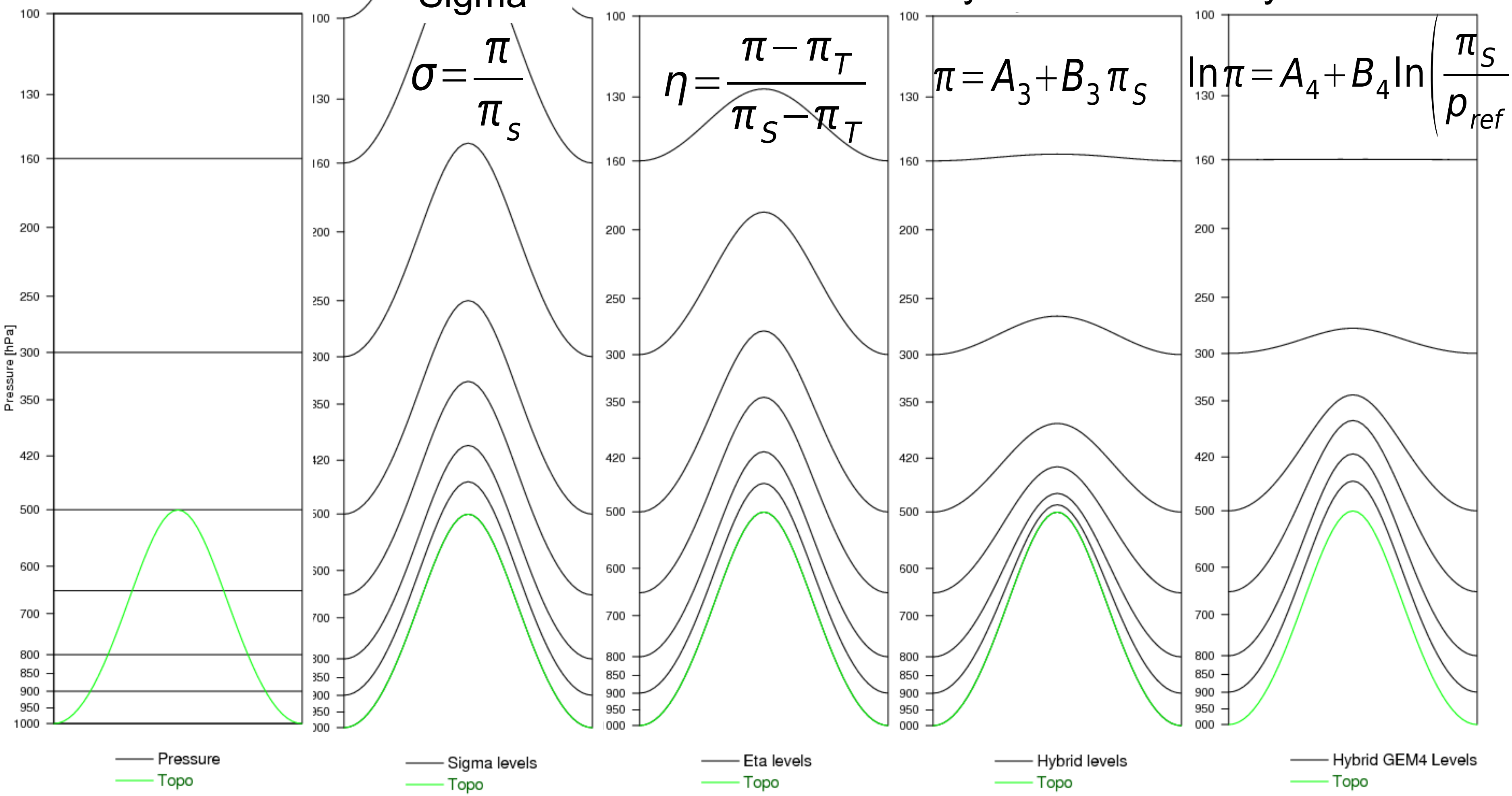


r.ip1

```
        r.ip1
Usage : r.ip1 [-nk] ip1code
Result: value [level_type or kind]
Usage : r.ip1 [-no] [--]value kind
Result: ip1code(newstyle or oldstyle)
Formats :
options : -n to add end of line char
        : -k to get code for kind
        : -o to get ip1code in oldstyle
        : -- to indicate value is negative
kind    : level_type
0       : m  [metres] (height with respect to sea level)
1       : sg [sigma] (0.0->1.0)
2       : mb [mbars] (pressure in millibars)
3       :    [others] (arbitrary code)
4       : M  [metres] (height with respect to ground level)
5       : hy [hybrid] (0.0->1.0)
6       : th [theta]
```

Encoding all this in RPN files

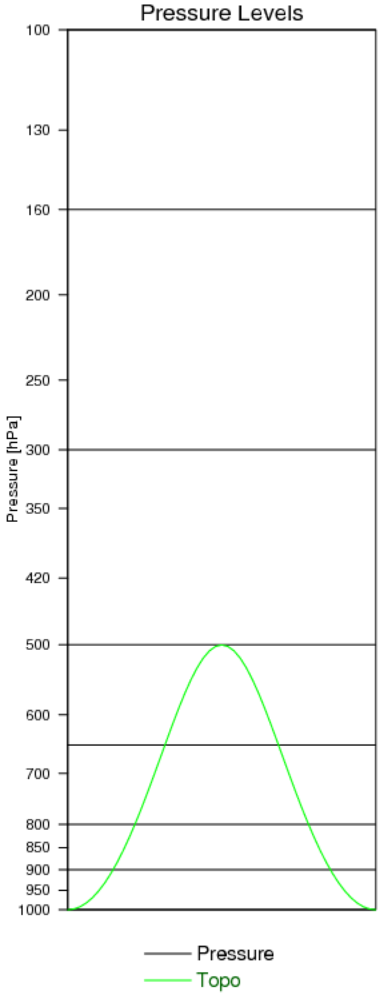
Pressure	Sigma	eta	Hybrid GEM3	Hybrid GEM4
$ip1$	$ip1$	$ip1$	$ip1$	$ip1$
Kind=2	Kind=1	Kind=1	Kind=5	Kind=5
	P0	P0,PT	P0,HY	P0,!!
	$\sigma=\frac{\pi}{\pi_s}$	$\eta=\frac{\pi-\pi_T}{\pi_S-\pi_T}$	$\pi=A_3+B_3\pi_S$	$\ln\pi=A_4+B_4\ln\left(\frac{\pi_S}{p_{ref}}\right)$



Computing the hydrostatic pressure

Pressure Levels (r.ip1, convip)

Pressure



$ip1$

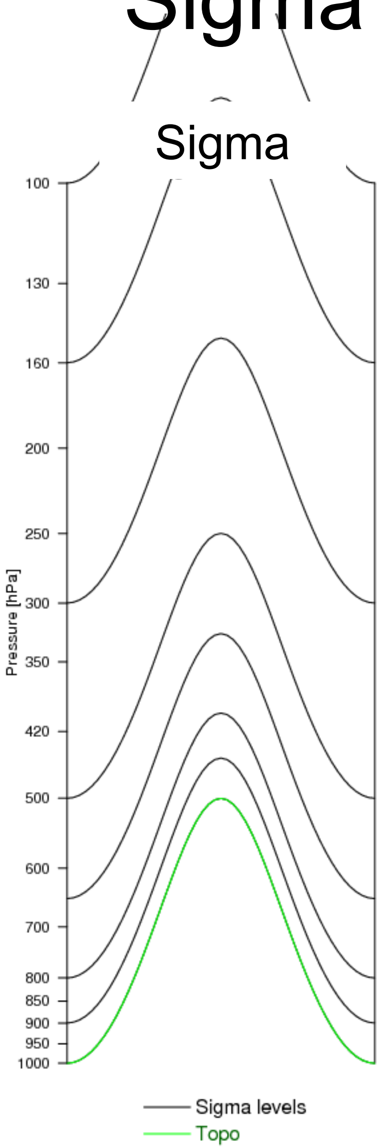
Kind=2

r.ip1 -k 500
500.000000 2
r.ip1 500
500 mb

$$\pi = convip(ip1)$$

Computing the hydrostatic pressure

Sigma Levels (use r.hy2pres , hyb_to_pres)



$ip1$

Kind=1

P0

```
r.ip1 -k 11950
0.995000 1
 r.ip1 11950
0.9950 sg
```

$$\sigma = convip(ip1)$$

$$\sigma = \frac{\pi}{\pi_s} \quad \Rightarrow \quad \pi = \sigma \pi_s$$

$$\begin{bmatrix} . & . & . \\ . & . & . \\ . & . & . \end{bmatrix} = \begin{bmatrix} . \\ . \\ . \end{bmatrix} \times \begin{bmatrix} . & . & . \end{bmatrix}$$

$$\pi \quad = \quad \sigma \times \quad \pi_s$$

~~$\sigma = \frac{(ip1 - 2000)}{10000}$~~

r.ip1 -on 1.0 1

12000 it works!

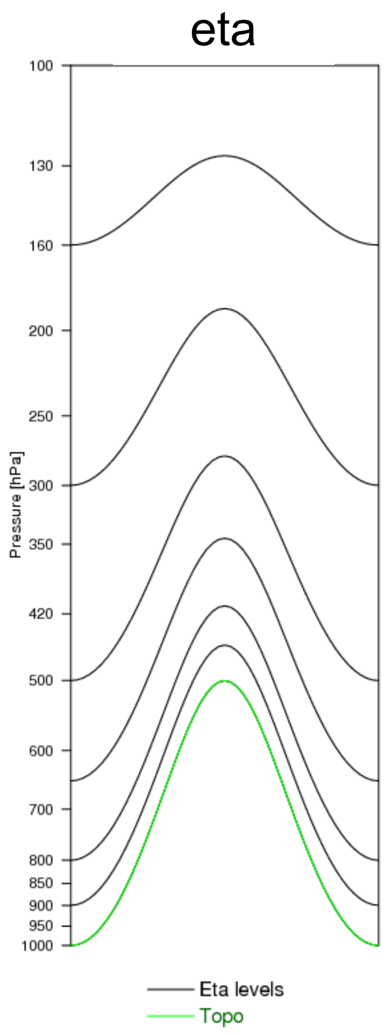
r.ip1 -n 1.0 1

26314400 oups

PX not in file to save space!

Computing the hydrostatic pressure

Eta Levels (use r.hy2pres, hyb_to_pres)



$ip1$

Kind=1

P0,PT

r.ip1 -k 11950
0.995000 1
r.ip1 11950
0.9950 sg

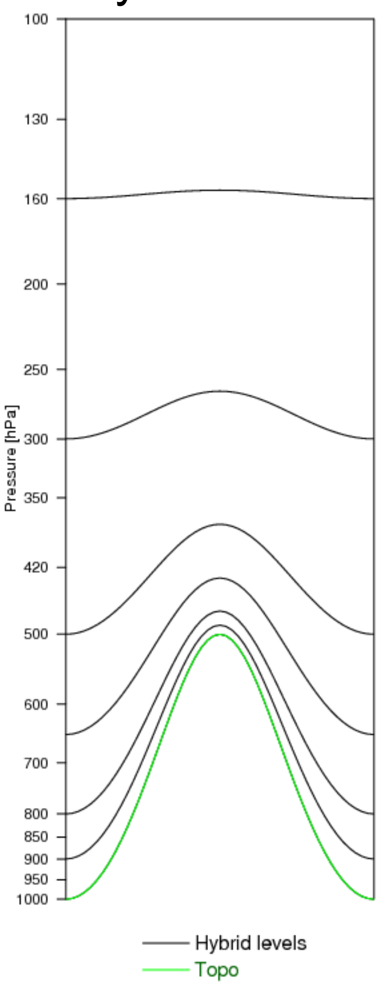
$$\eta = convip(ip1)$$

$$\eta = \frac{\pi - \pi_T}{\pi_s - \pi_T} \quad \Rightarrow \pi = \eta(\pi_s - \pi_T) + \pi_T$$

Computing the hydrostatic pressure

Hybrid Levels GEM3 (use r.hy2pres, hyb_to_pres)

Hybrid GEM3



$ip1$

Kind=5

P0,HY

```
r.ip1 -k 93423264
1.000000 5
r.ip1 93423264
1 hy
```

$$h = convip(ip1)$$

HY

$$h_T = \frac{p_T}{p_{ref}}$$

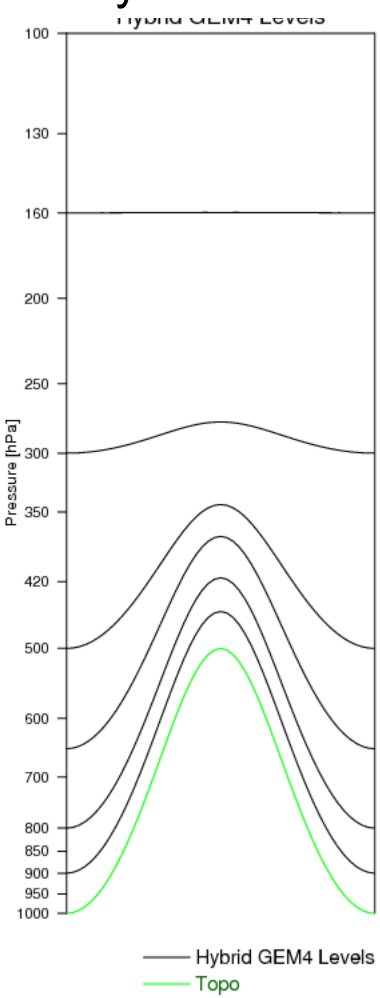
$$\pi = A + B\pi_S$$

$$B = \left(\frac{h - h_T}{1 - h_T}\right)^r, \quad A = (h - B)\, p_{ref}$$

Computing the hydrostatic pressure

Hybrid Levels GEM4 (use r.hy2pres eventually)

Hybrid GEM4



Kind=5
P0,!!

```
r.ip1 -k 93423264
1.000000 5
r.ip1 93423264
1 hy
```

Kind=5	Version=2	Skip=2
ptop	pref	rcoef1
rcoef2	P0	empty
ip1	A	B
…	…	…

!! Record for GEM4.1.2+

$$ip1$$

$$\pi_S = P0$$

$$\ln \pi = A + B \ln(\pi_S / p_{ref})$$

$$\zeta_T = \ln p_T, \qquad \zeta_S = \ln p_{ref}$$

$$A = \zeta = + \zeta_S \ln(H)$$

$$B = \left(\frac{\zeta - \zeta_T}{\zeta_S - \zeta_T} \right)^r$$

$$0 < r = r_{max} - (r_{max} - r_{min}) \left(\frac{\zeta - \zeta_T}{\zeta_S - \zeta_T} \right) < 30$$

Output example from GEM4
nk=3, hyb=0.08, 0.3, 0.6

There are NK UU + one 10m diag UU

hyb	GZ	PX	UU & VV	WT1	TT	Tracers (HU,QC...)
top						
0.0282843	X	X		X	X	X
0.0800000 (in namelist)	X	X	X			
0.154919	X	X		X	X	X
0.300000 (in namelist)	X	X	X			
0.424264	X	X		X	X	X
0.600000 (in namelist)	X	X	X			
0.774597	X	X		X	X	X
1.00000	X (topo)	X (equals P0)	X (diag 10 m)		X (diag 2 m)	X (copy of levels above)

The horizontal momentum levels are specified in gem_settings.nml

Output example from GEM4

3 levels model, hyb=0.08, 0.3, 0.6

There are NK+1 TT + one 2m diag TT

hyb	GZ	PX	UU & VV	WT1	TT	Tracers (HU,QC...)
top						
0.0282843	X	X		X	X	X
0.0800000 (in namelist)	X	X	X			
0.154919	X	X		X	X	X
0.300000 (in namelist)	X	X	X			
0.424264	X	X		X	X	X
0.600000 (in namelist)	X	X	X			
0.774597	X	X		X	X	X
1.00000	X (topo)	X (equals P0)	X (diag 10 m)		X (diag 2 m)	X (copy of levels above)

The thermodynamic levels are not specified in the gem_settings.nml, they are computed by the model

Output example from GEM4

3 levels model, hyb=0.08, 0.3, 0.6

There are NK+1 TR + one copy at surface

hyb	GZ	PX	UU & VV	WT1	TT	Tracers (HU,QC...)
top						
0.0282843	X	X		X	X	X
0.0800000 (in namelist)	X	X	X			
0.154919	X	X		X	X	X
0.300000 (in namelist)	X	X	X			
0.424264	X	X		X	X	X
0.600000 (in namelist)	X	X	X			
0.774597	X	X		X	X	X
1.00000	X (topo)	X (equals P0)	X (diag 10 m)		X (diag 2 m)	X (copy of levels above)

Output example from GEM4

3 levels model, hyb=0.08, 0.3, 0.6

Metric variables are defined on all levels but top
Therefore there are 2nk+2 GZ and PX

hyb	GZ	PX	UU & VV	WT1	TT	Tracers (HU,QC...)
top						
0.0282843	X	X		X	X	X
0.0800000 (in namelist)	X	X	X			
0.154919	X	X		X	X	X
0.300000 (in namelist)	X	X	X			
0.424264	X	X		X	X	X
0.600000 (in namelist)	X	X	X			
0.774597	X	X		X	X	X
1.00000	X (topo)	X (equals P0)	X (diag 10 m)		X (diag 2 m)	X (copy of levels above)

GZ at hyb=1.0 is the model topo
PX at hyb=1.0 is P0 the surface pressure

Making a profile of TT and UU

1) Get all TT records and their ip1
2) Sort the record by hyb value, not ip1, use convip
3) Get matching PX
4) Make TT profile
5) Get all UU records and their ip1
6) Sort the record by hyb value, not ip1, use convip
7) Get matching PX
8) Make UU profile

With GEM4, never assume that two variables are on same levels, make a test in your code. Converted code should work with GEM3

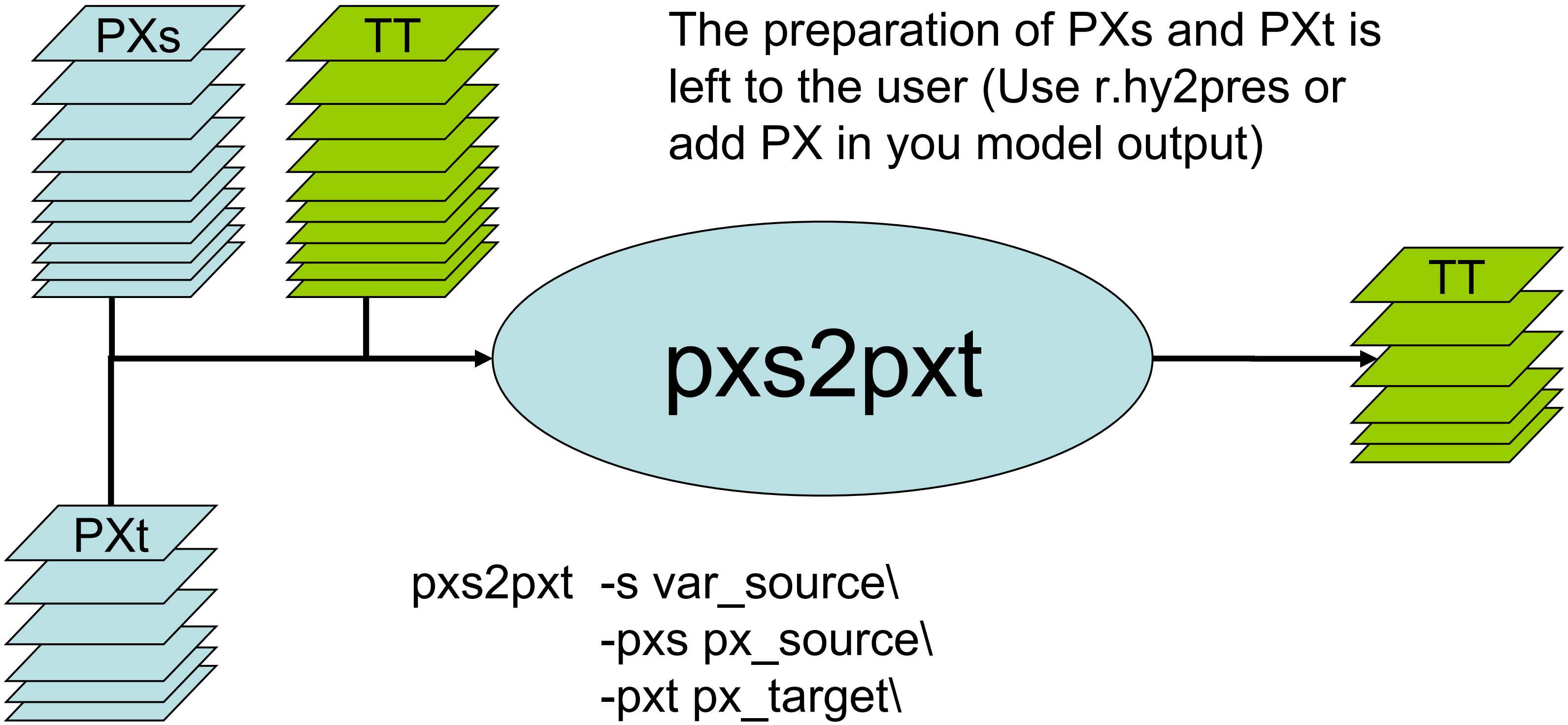
Computing the pressure in programs

- If possible use r.hy2pres in script and read PX in your program, or have model write PX
- Use armnlib function HYB_TO_PRE

A simple universal Interpolator

pxs2pxt
(PX Source To PX Target)

The preparation of PXs and PXt is left to the user (Use r.hy2pres or add PX in you model output)



Or a list of pressure levels

```
pxs2pxt  -s var_source\
         -pxs px_source\
         -pxt px_target\
         -d var_target\
         -var CUB_VT CUBP_HU CUB_UU CUB_VV
```

Search for **pxs2pxt** on the wiki

Conclusions

I want to	In scripts	In programs
Calculate PX	r.hy2pres	hyb_to_pres
Convert ip1	r.ip1	convip
Interpolate to a pressure cube	pxs2pxt	?