Dynamical Constraints for Global 3D-Var Assimilation Using GEM-Strato

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Acknowledgments:
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[1]**ARMA**, [2]**ARQX**

Dynamical Constraints: Overview

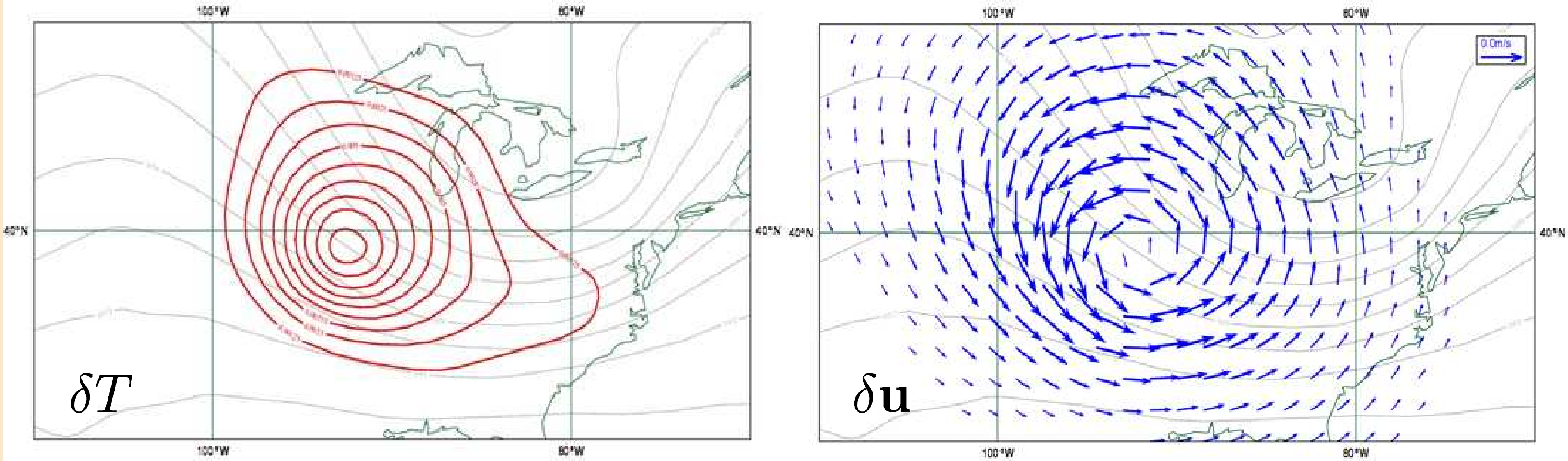
- **Constraints between mass and wind**
  - **e.g. Temperature T and streamfunction Ψ**
- **Constraints between rotational and irrotational wind**
  - **e.g. Streamfunction Ψ and velocity potential χ**

- **Dynamical constraints help to**
  - ***spread information from observed to unobserved variables***
  - ***minimize imbalance (supress spurious gravity waves)***

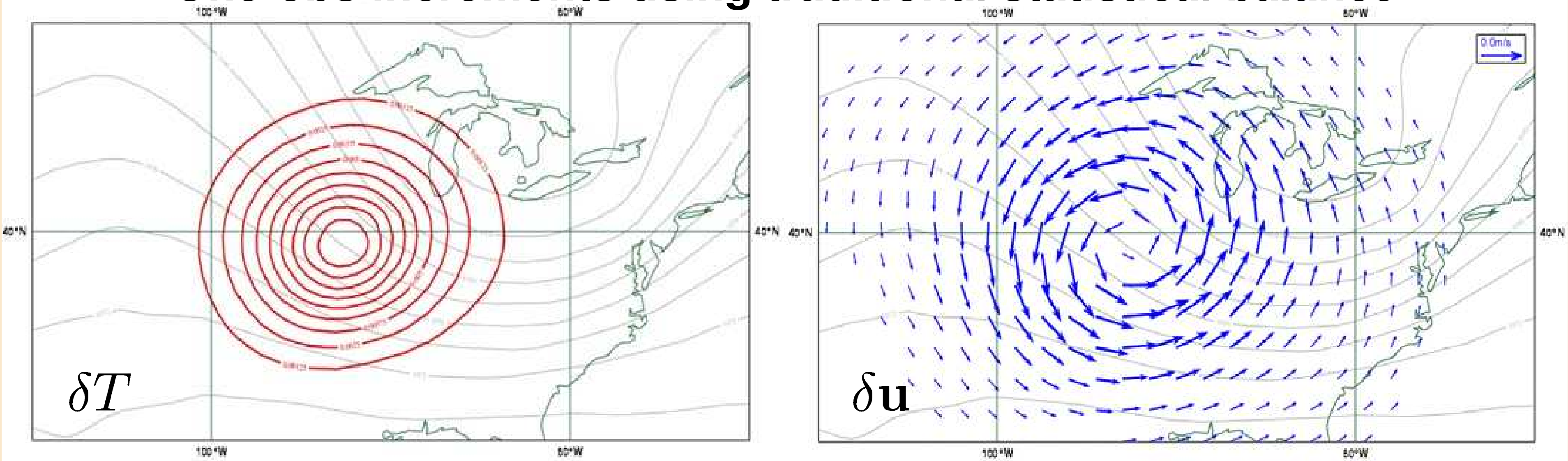
- **CMC 3D-Var formulation**
  - **is incremental (as most major NWP centres)**
  - **balanced temperature and velocity potential increments δT_{B}, $\delta\chi_{\mathsf{B}}$ derived from streamfunction increment $\delta\Psi$**
  - **minimization involves *unbalanced* increments**
  - **$\delta\Psi$, δT_{U}, $\delta\chi_{\mathsf{U}}$, etc. are uncorrelated with each other**

Motivation 1: Flow dependence of increments

One-obs increments using Charney and QG omega balances

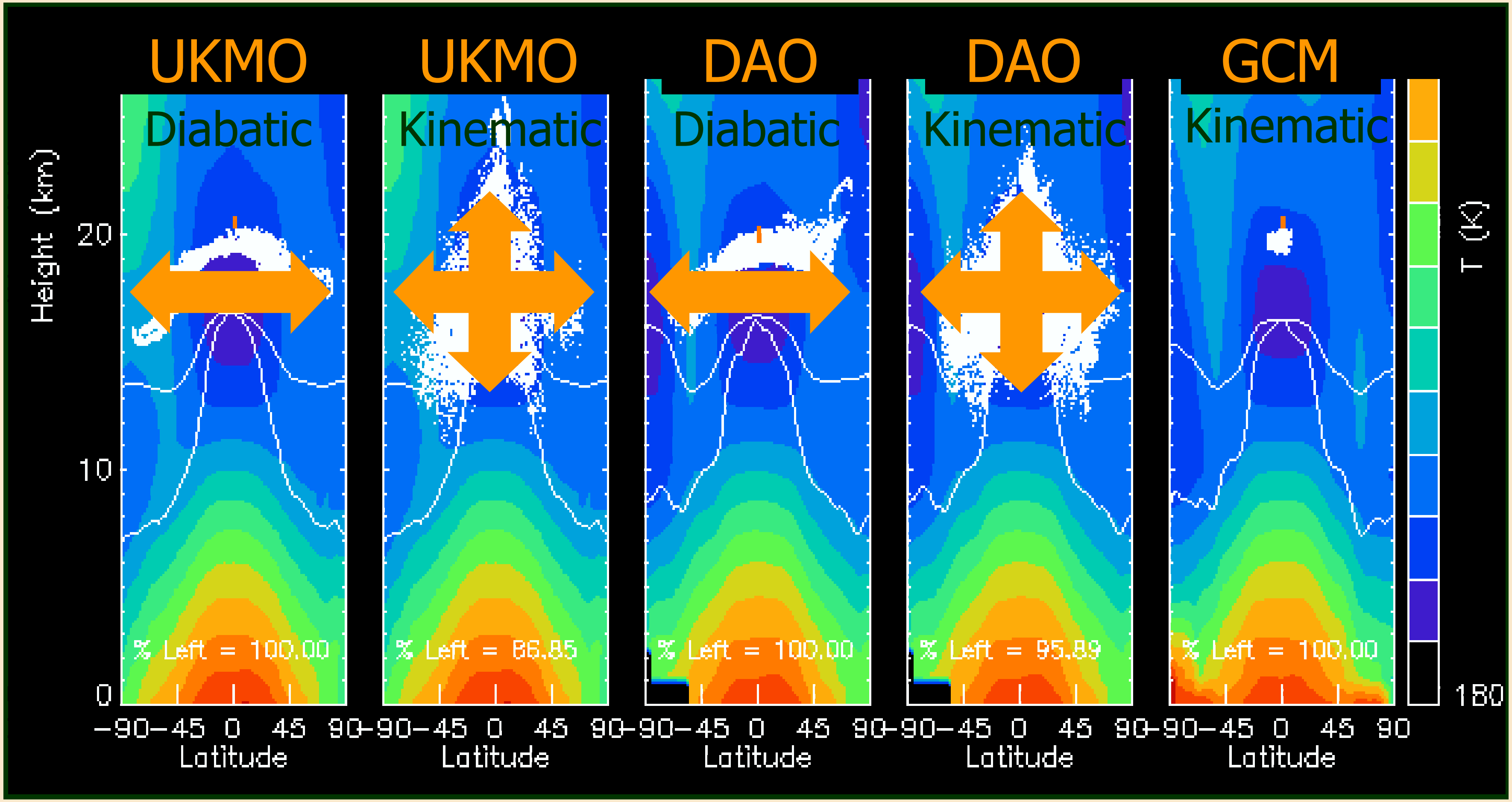


One-obs increments using traditional statistical balance



Fisher (ECMWF Tech Note, 2003)

Motivation 2: Anomalous dispersion



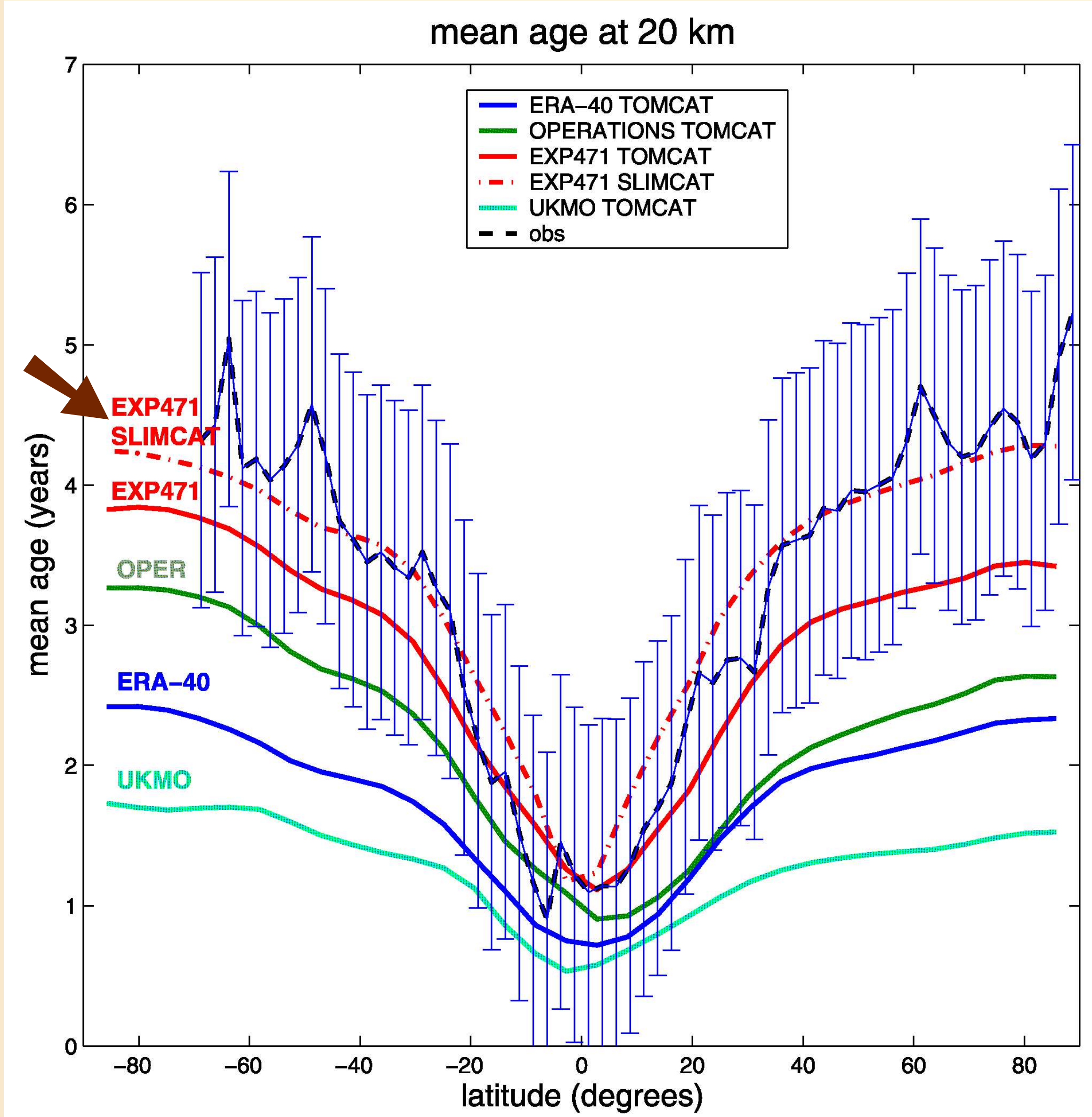
Schoeberl *et al* (JGR, 2003)

- **3D back-trajectory calculations after 50 days**
  - **Diabatic: w estimated using heating rates**
  - **Kinematic: w derived from divergence**

- ***Excessive mixing in analyses compared with free model***

Motivation 3: Age of air

- Age of air too low in most analysis-driven CTMs
- CTM age of air closest to obs using EXP471 winds
- *Improvement partially attributed to better balance*



Monge-Sanz *et al* (GRL, 2007)

Project Outline

- **Goal of study:**

To implement Charney and QG omega balances in CMC 3D-Var scheme

- **Step 1: Code and test solvers for full equations**
- **Step 2: Code and test solvers for TLM equations**
  - **Can utilize model 6-hour differences**
  - **Issues with dynamics in incremental context**
- **Step 3: Code and test adjoint models**
- **Step 4: Run 3D-Var with control and new balance constraints**

Experimental Setup

- **Forecast model: GEM-Strato (240x120, L80, lid at 0.1 hPa)**
- **Assimilation scheme: CMC 3D-Var FGAT**
- **Variances and correlations: similar to operations**

Traditional constraints (i.e. control)

- δT_{B} **and** $\delta \chi_{\mathsf{B}}$ **obtained from** $\delta \Psi$
- **Both constraints based on statistical regression**
- **Both constraints time-averaged** $\longrightarrow$ ***no flow-dependence***
- $\delta \Psi - \delta \chi_{\mathsf{B}}$ **constraint only active in lowest 8 levels**

Schematic of new constraints

$$\delta\Psi \rightarrow \left[\begin{array}{c} \textbf{Linear or} \\ \textbf{Charney} \end{array} \right] \rightarrow \delta\Phi_{\mathsf{B}} \rightarrow \left[\textbf{ Hydrostatic } \right] \rightarrow \delta T_{\mathsf{B}}$$

$$(\delta\Psi, \delta T_{\mathsf{B}}) \rightarrow \left[\textbf{ QG } \omega \right] \rightarrow \delta\omega_{\mathsf{B}} \rightarrow \left[\textbf{ Continuity } \right] \rightarrow \delta\chi_{\mathsf{B}}$$

Acronyms to keep in mind

SB - Statistical Balance (i.e. control)

LB - Linear Balance + Hydrostatic Balance

CB - Charney Balance + Hydrostatic Balance

QG - QG omega balance + Continuity Equation

Calculation of T_{B} and χ_{B} from Ψ

- **Charney (or linear) balance and hydrostatic balance yield T_{B}**

$$\nabla^2\Phi_{\mathsf{B}} = f\nabla^2\Psi - \beta u + 2J(u,v)$$

$$\frac{\partial\Phi_{\mathsf{B}}}{\partial p} = -R\frac{T_{\mathsf{B}}}{p}$$

- **QG ω equation and continuity equation yield χ_{B}**

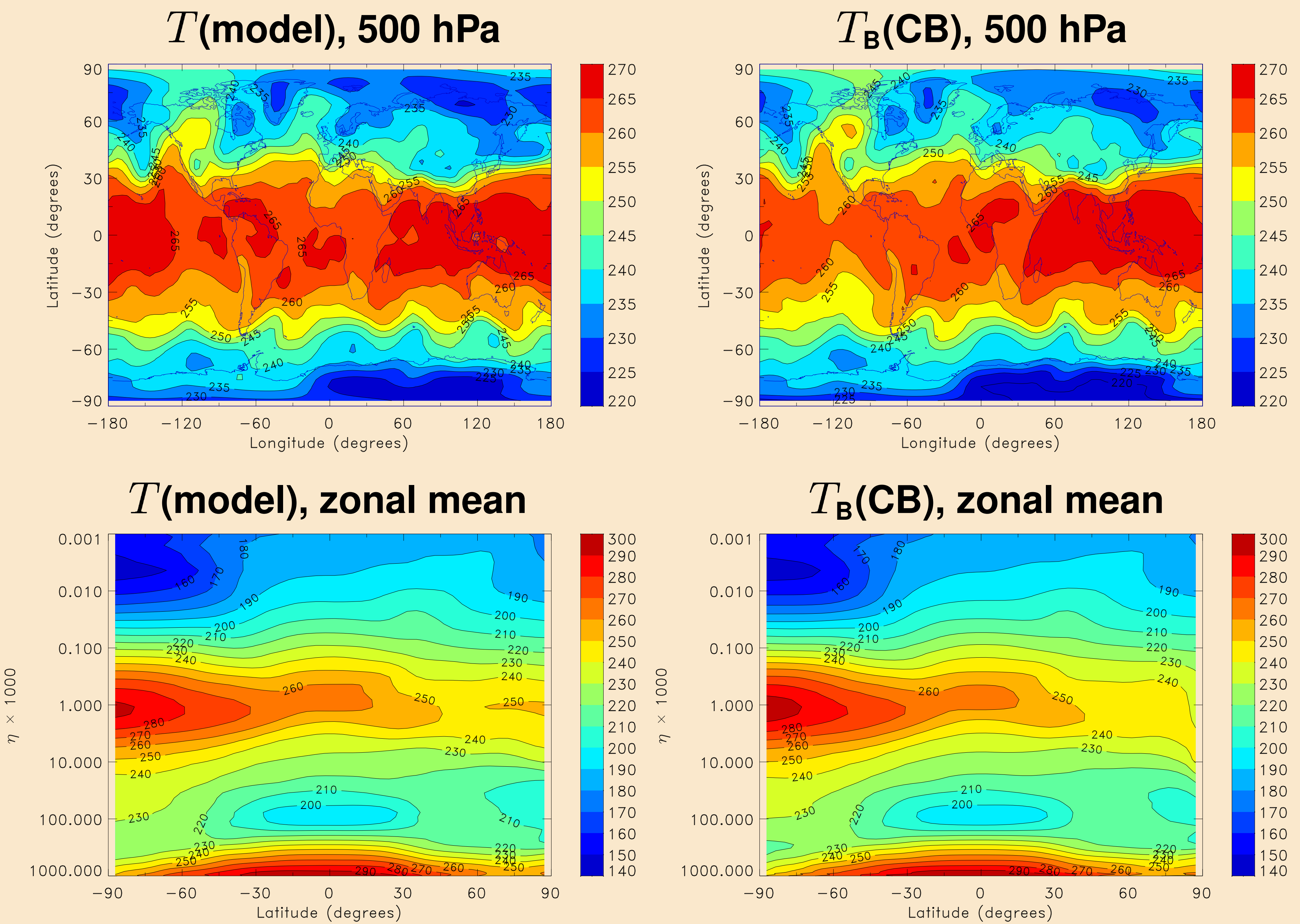
$$\left(\nabla^2 + \frac{f^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega_{\mathsf{B}} = \frac{f}{\sigma}\frac{\partial}{\partial p}[\mathbf{u}_\psi\cdot\nabla(f+\zeta)] + \frac{R}{\sigma p}\nabla^2(\mathbf{u}_\psi\cdot\nabla T_{\mathsf{B}})$$

$$\nabla^2\chi_{\mathsf{B}} + \frac{\partial\omega_{\mathsf{B}}}{\partial p} = 0$$

- **Equations are linearized about reference state in η coordinates**

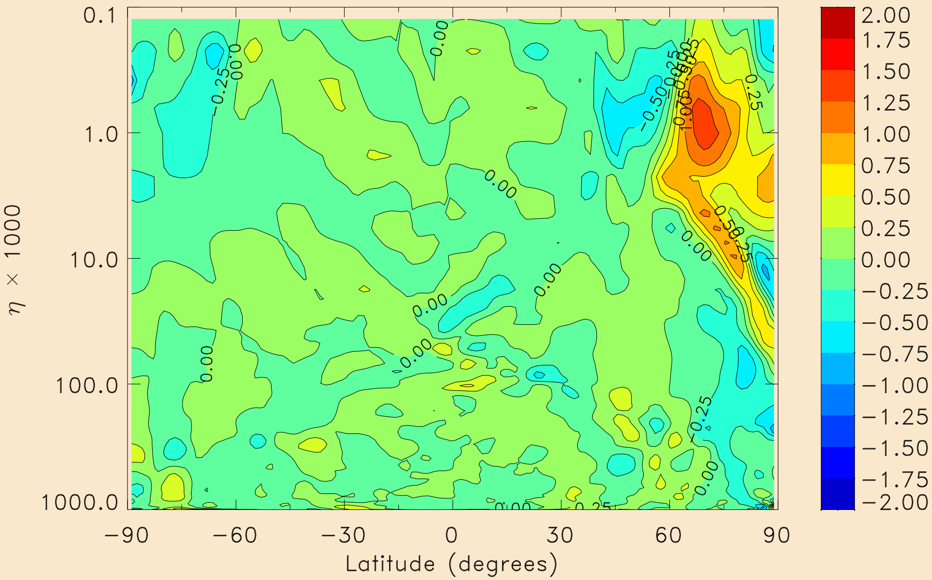
- **Linearized equations are solved at every analysis time**
  - $\delta\Psi$ ***assumed balanced, compute*** δT_{B} ***and*** $\delta\chi_{\mathsf{B}}$

Free model vs. Charney balance (full fields)

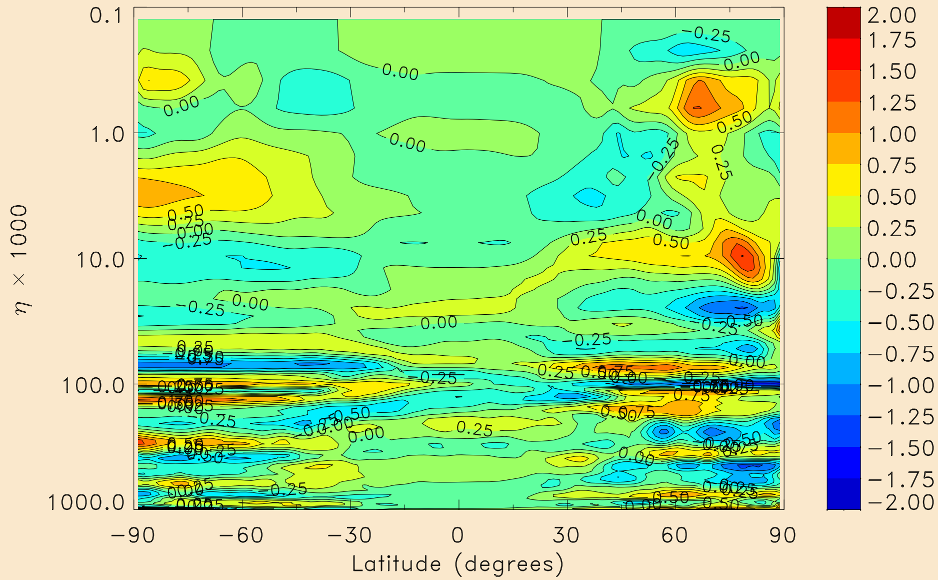


Free model vs. Charney balance (6-hour differences)

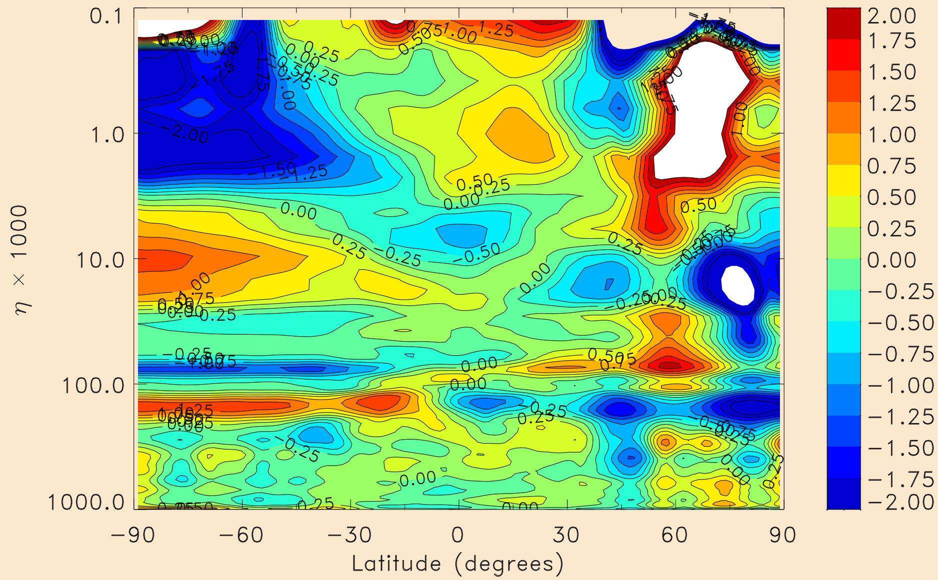
δT(model), zonal mean



δT_{B}(CB), zonal mean



$10^{-6} \times \delta\Psi$, zonal mean



- **strong vertical gradients in δT_{B}**
- **can be traced back to $\delta\Psi$**
- ***common in models (and 3D-Var)***

Source of problem and a potential solution

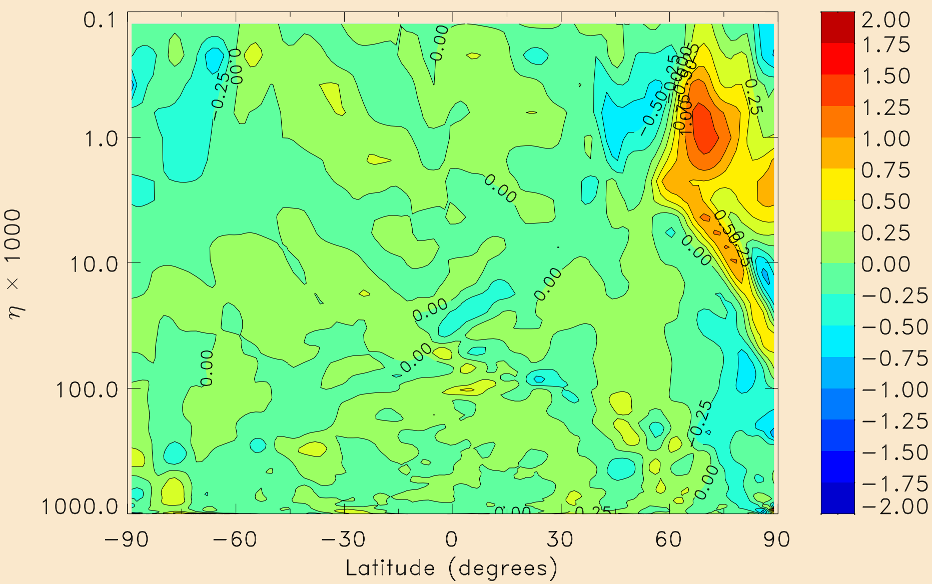
- **Small horizontal scales (large vertical scales)**
  - ◦ $\delta\Psi$ **evolution determines** δT **evolution**
  - ◦ **also appropriate in tropics**
- **Large horizontal scales (small vertical scales)**
  - ◦ δT **evolution determines** $\delta\Psi$ **evolution**

} **Lorenc *et al* 2003**

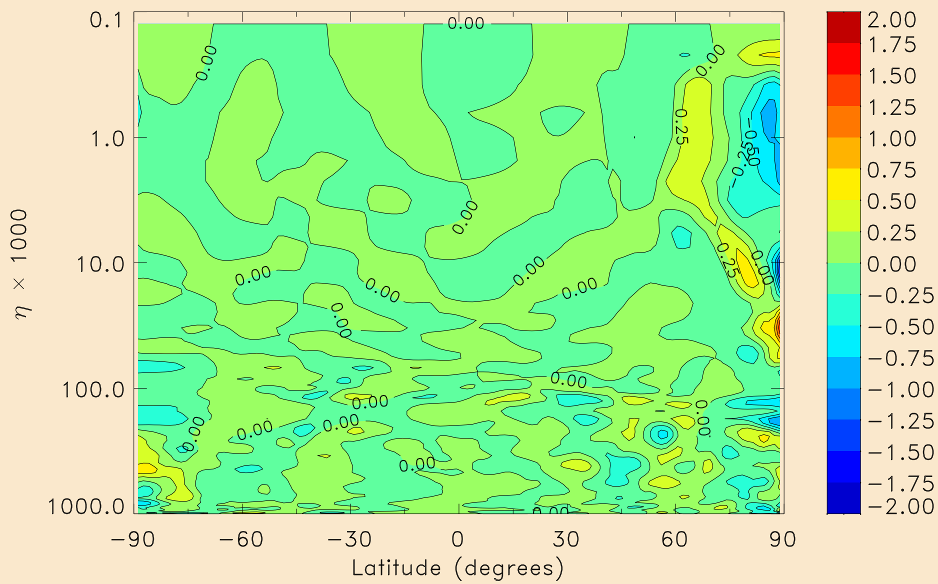
$\Longrightarrow$ $\delta\Psi$ **contains no useful information about** δT **on planetary scales**

- **Solution:** ***Filter out first 5 wavenumbers from*** $\delta\Psi$ ***before calculation***

δT**(model), zonal mean**

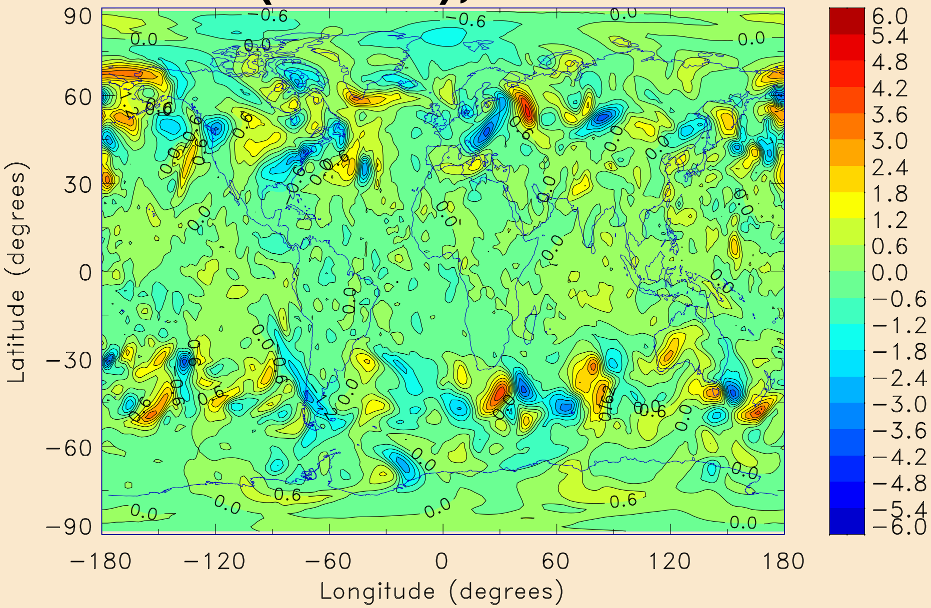


$\delta T_{\mathbf{B}}$**(CB), zonal mean**

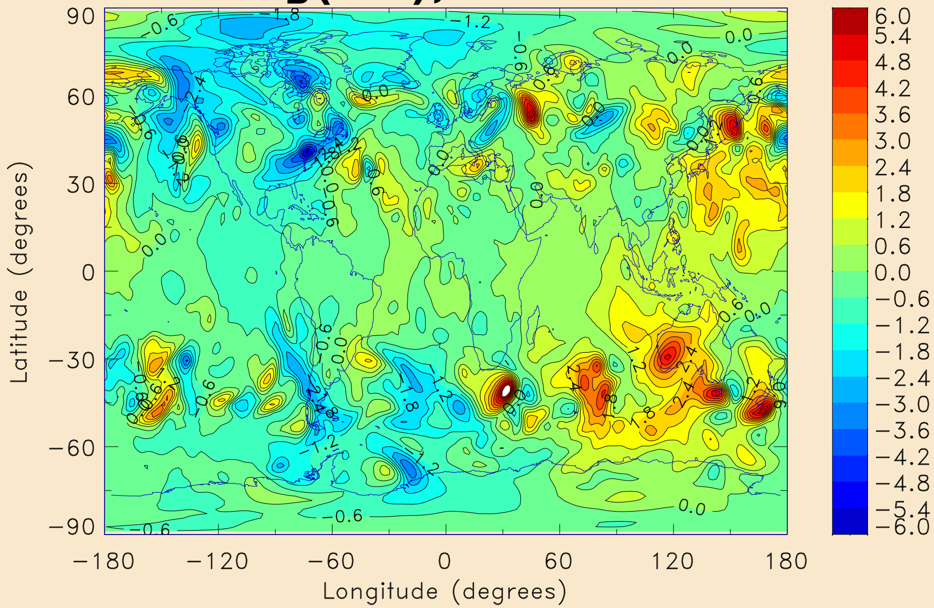


Free model vs. filtered Charney balance (6-hour differences)

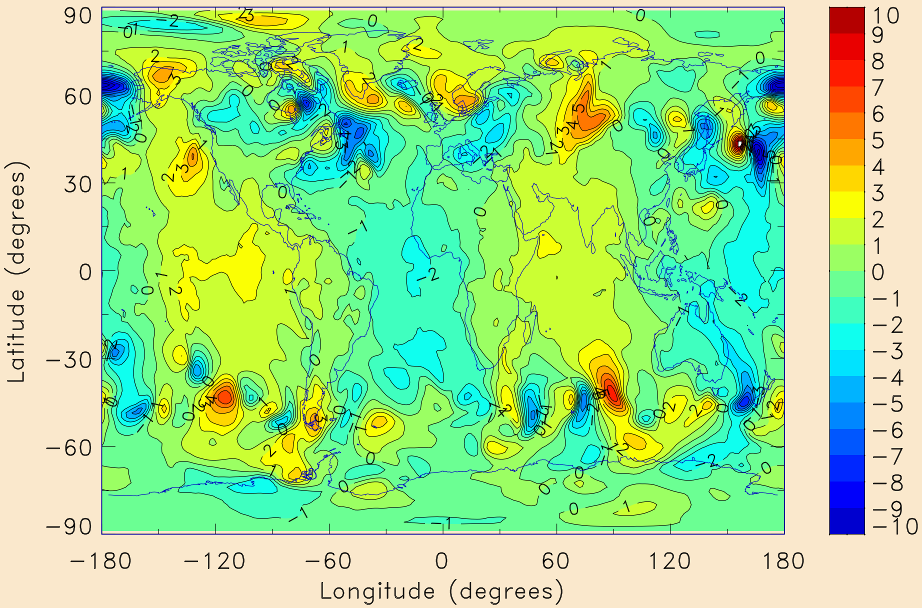
δT(model), 500 hPa



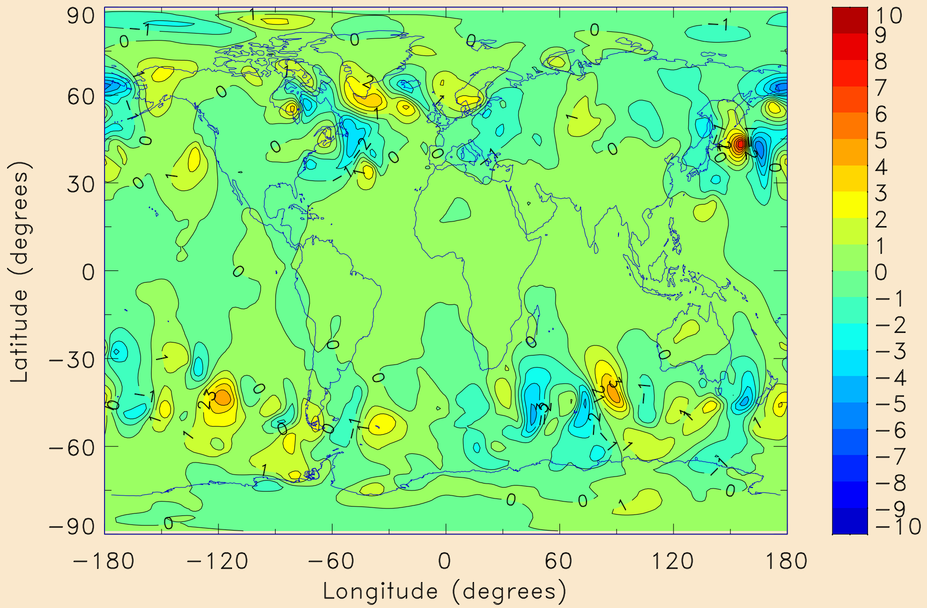
$\delta T_{\mathbf{B}}$(CB), 500 hPa



δp_s(model)



$(\delta p_s)_{\mathbf{B}}$ (CB)



Linear vs. Charney Balance: 6-hour differences

January-mean Std. Dev. of

$$\delta T_{\mathsf{U}} = \delta T - \delta T_{\mathsf{B}}$$

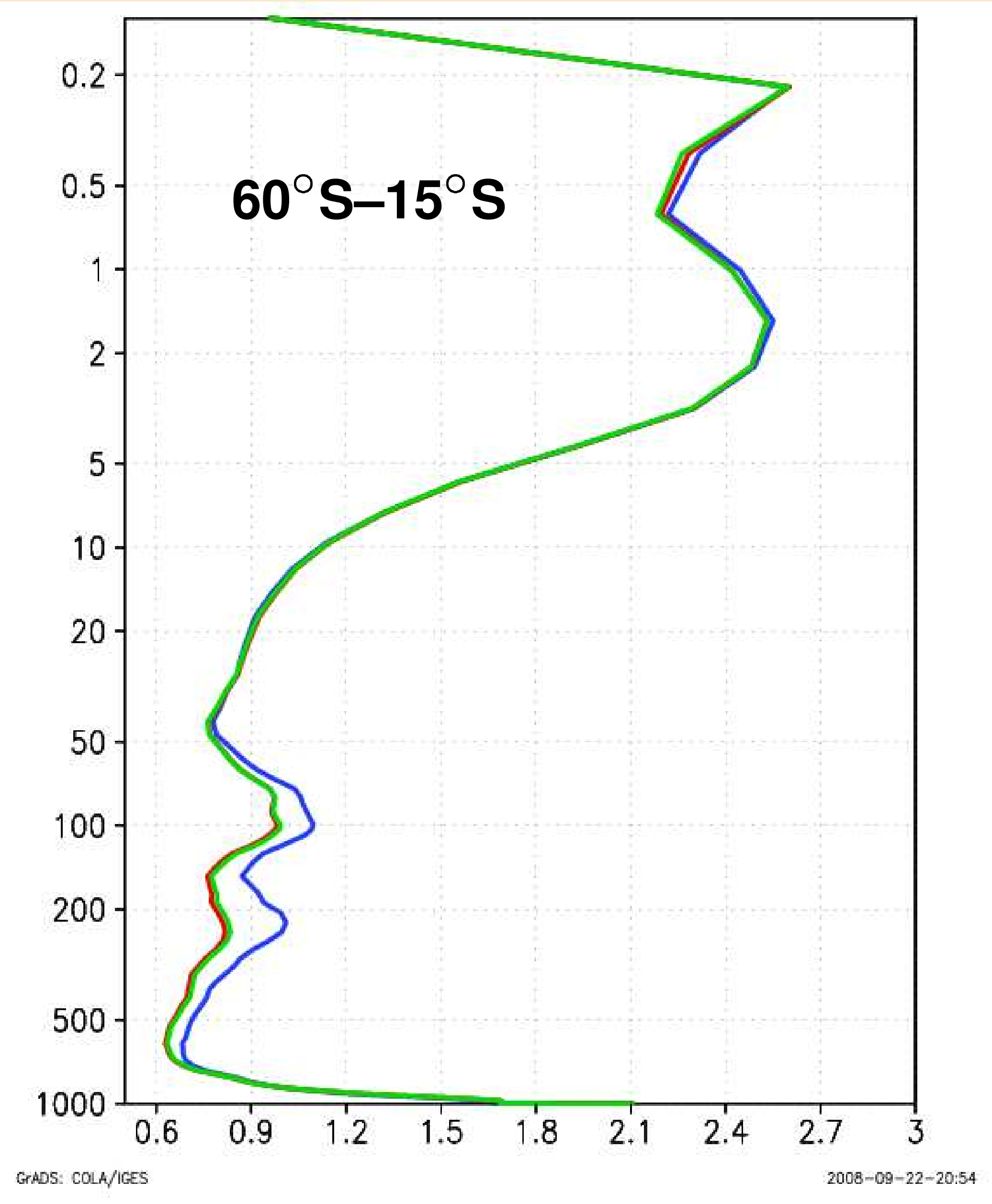
where

δT **= model 6-hour difference**

δT_{B} **= Linear Balance (LB)**

δT_{B} **= Charney Balance (CB)**

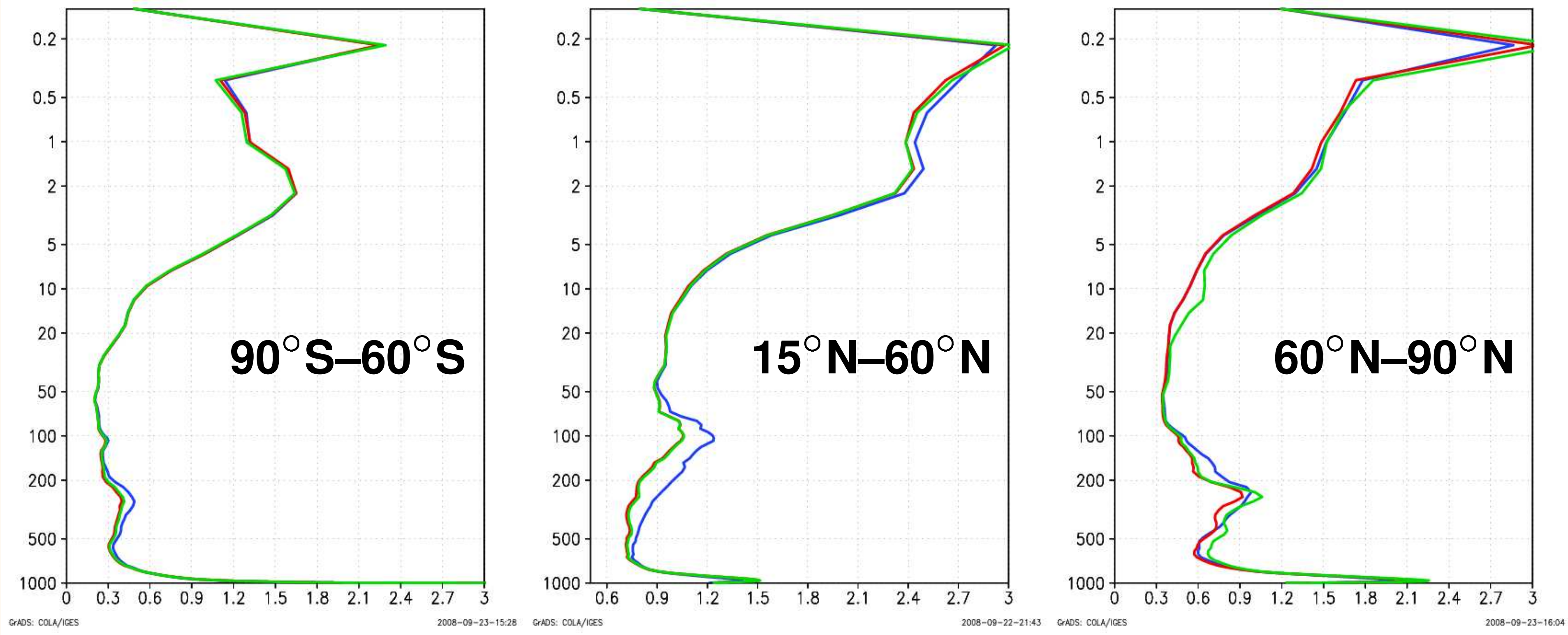
δT_{B} **= Charney Balance + sphericity terms**



Note, for full fields,

$$\nabla^2\Phi = \nabla\cdot(f\nabla\Psi) + 2J(u,v) - \frac{1}{a}\left(1 + \tan\phi\frac{\partial}{\partial\phi}\right)(u^2+v^2)$$

Linear vs. Charney Balance, continued...



- Colors: Linear (LB), Charney (CB), Charney + spherical terms

- CB better than LB in midlatitudes, troposphere
- Not much difference elsewhere
- Spherical terms (Houghton 1968) make no significant contribution

Implementation of Constraints in 3D-Var

- **Tangent-linear code tested and validated offline**
- **Adjoint code tested offline using Adjoint Test**

$$(Lx)^T y = x^T (L^T y) \qquad \textbf{for all} \qquad (x, y)$$

- **Tangent-linear and Adjoint codes imported into 3D-Var**
  - **Scheme tested using Gradient Test**

$$J(x_0 + \delta x) = J(x_0) + \delta x^T \nabla_x J(x_0) + O(\delta x^2)$$

Let

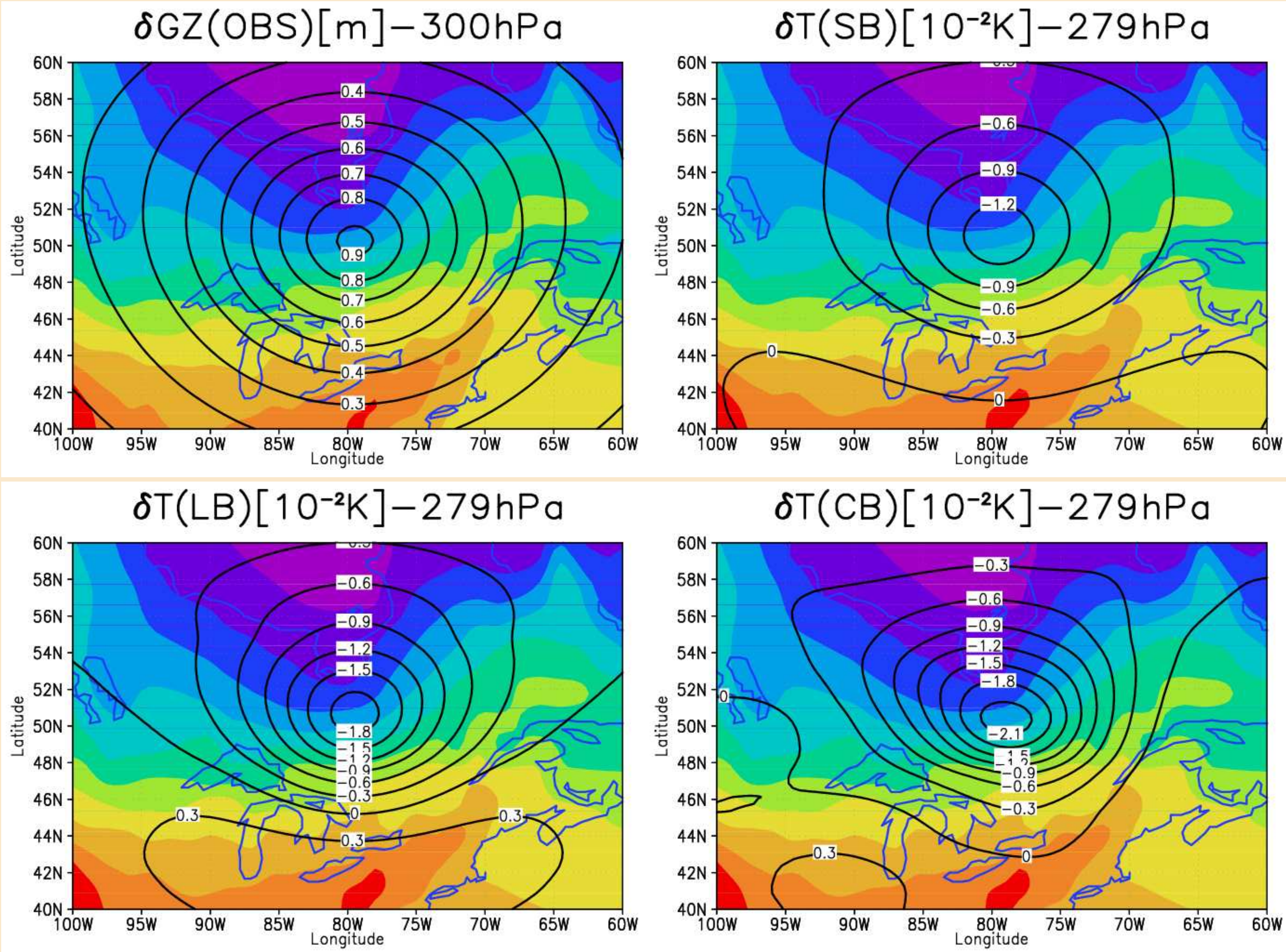
$$\delta x = -\alpha \nabla_x J(x_0)$$

Then as $\alpha \longrightarrow 0$

$$\frac{J(x_0 + \delta x) - J(x_0)}{-\alpha ||\nabla_x J(x_0)||^2} \longrightarrow 1 \qquad \textbf{from below}$$

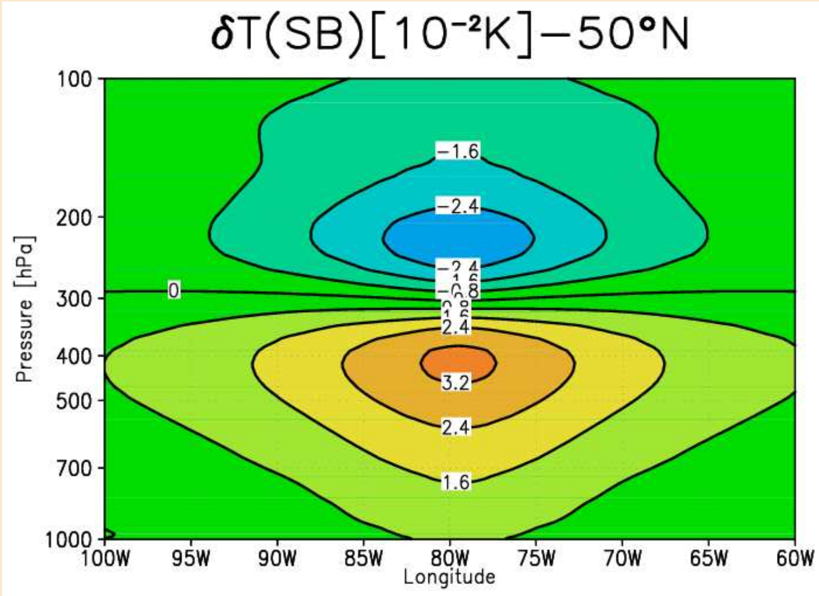
IT CAME FROM PLANET ADJOINT...

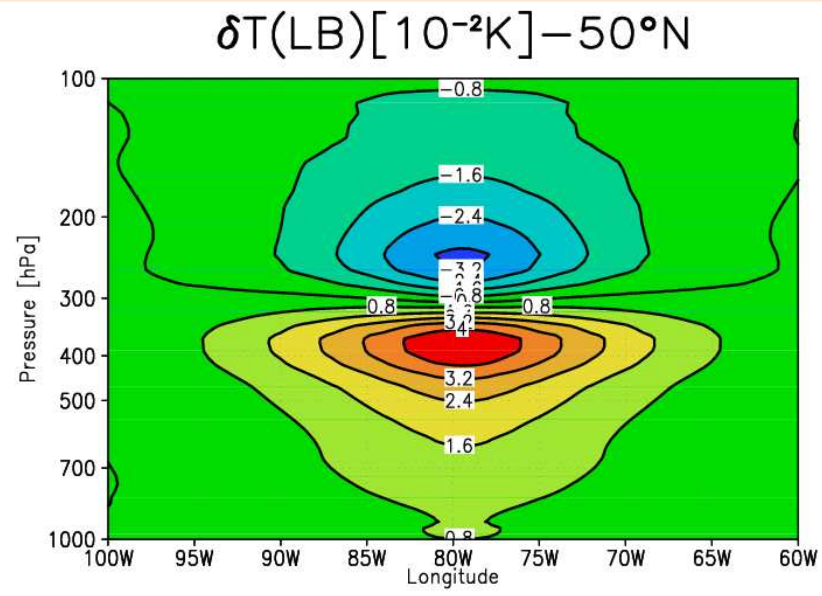
One-obs experiment (obs in GZ at 80°W, 50°N, 300hPa)

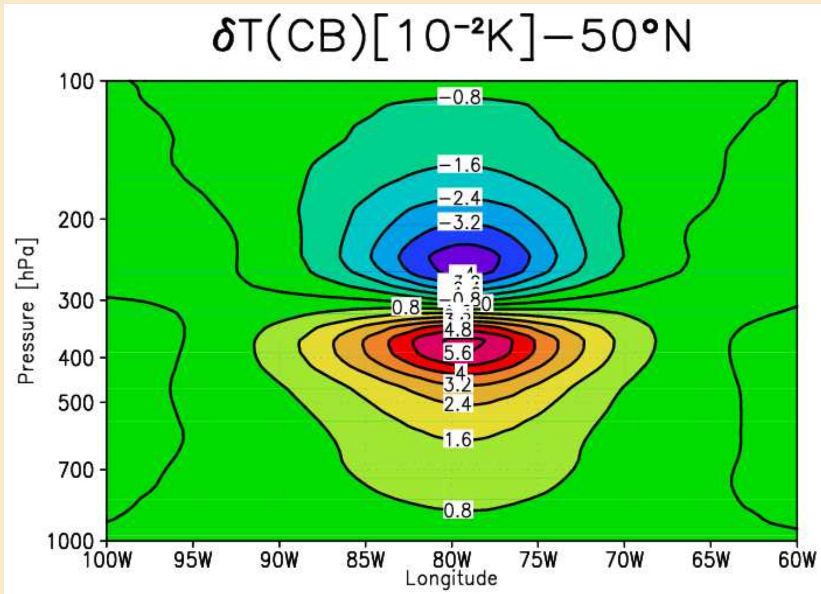


- ***Flow-dependent δT response for CB constraint*** **(not SB or LB)**
- **Color shading shows background flow (geopotential)**

One-obs: Increment vertical structure

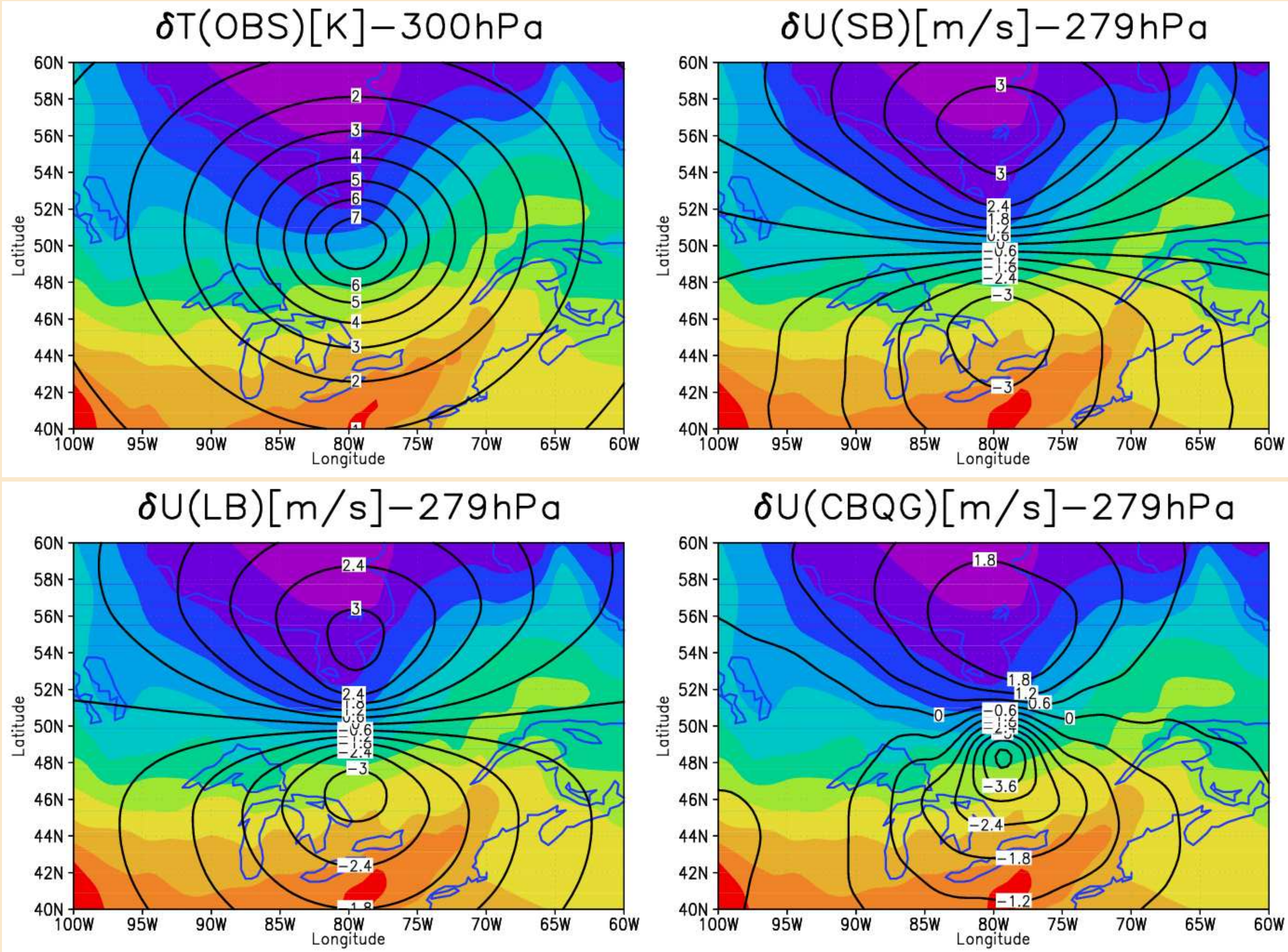






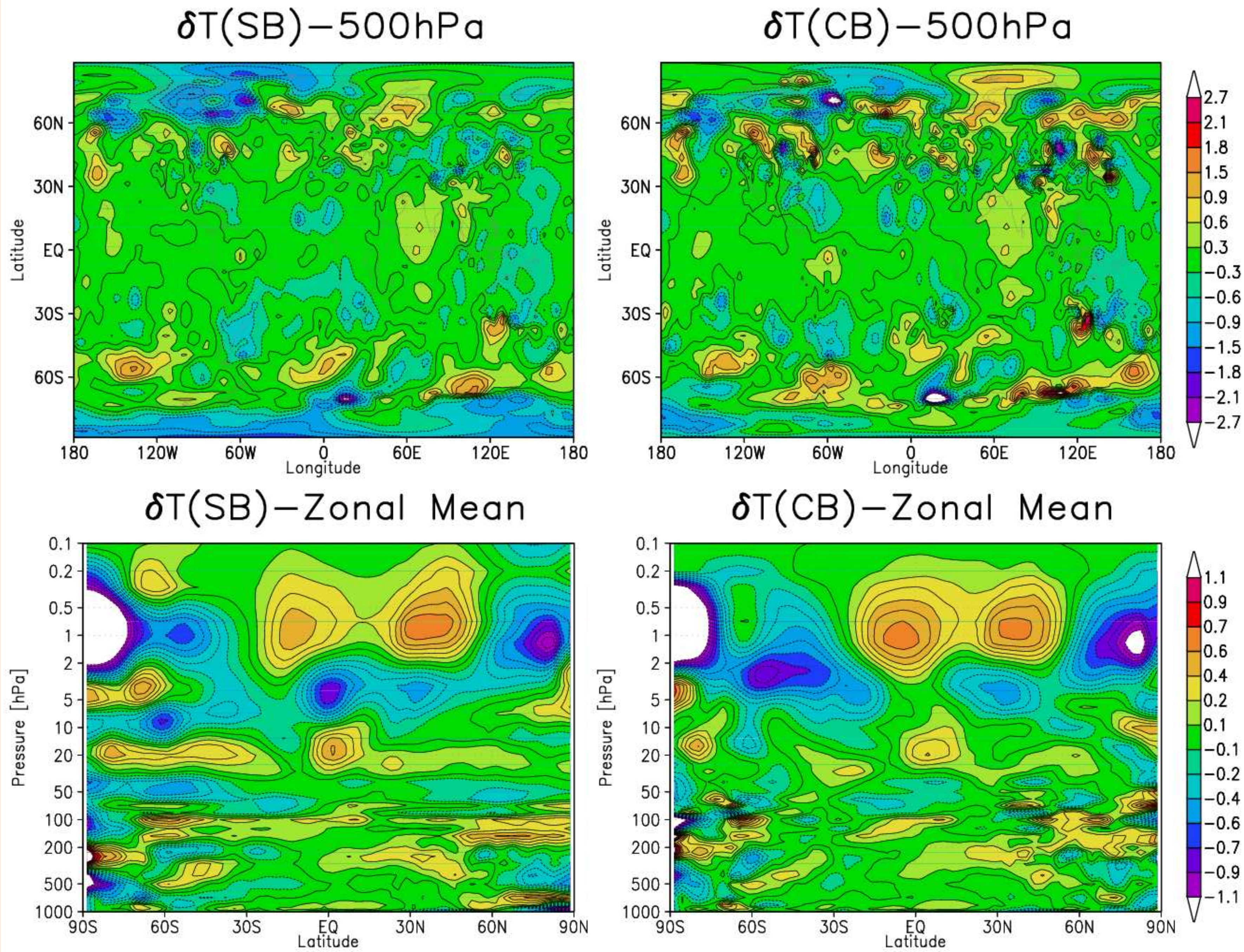
- **Response somewhat larger and more focused with LB, CB constraint**

One-obs experiment (obs in TT at 80°W, 50°N, 300hPa)

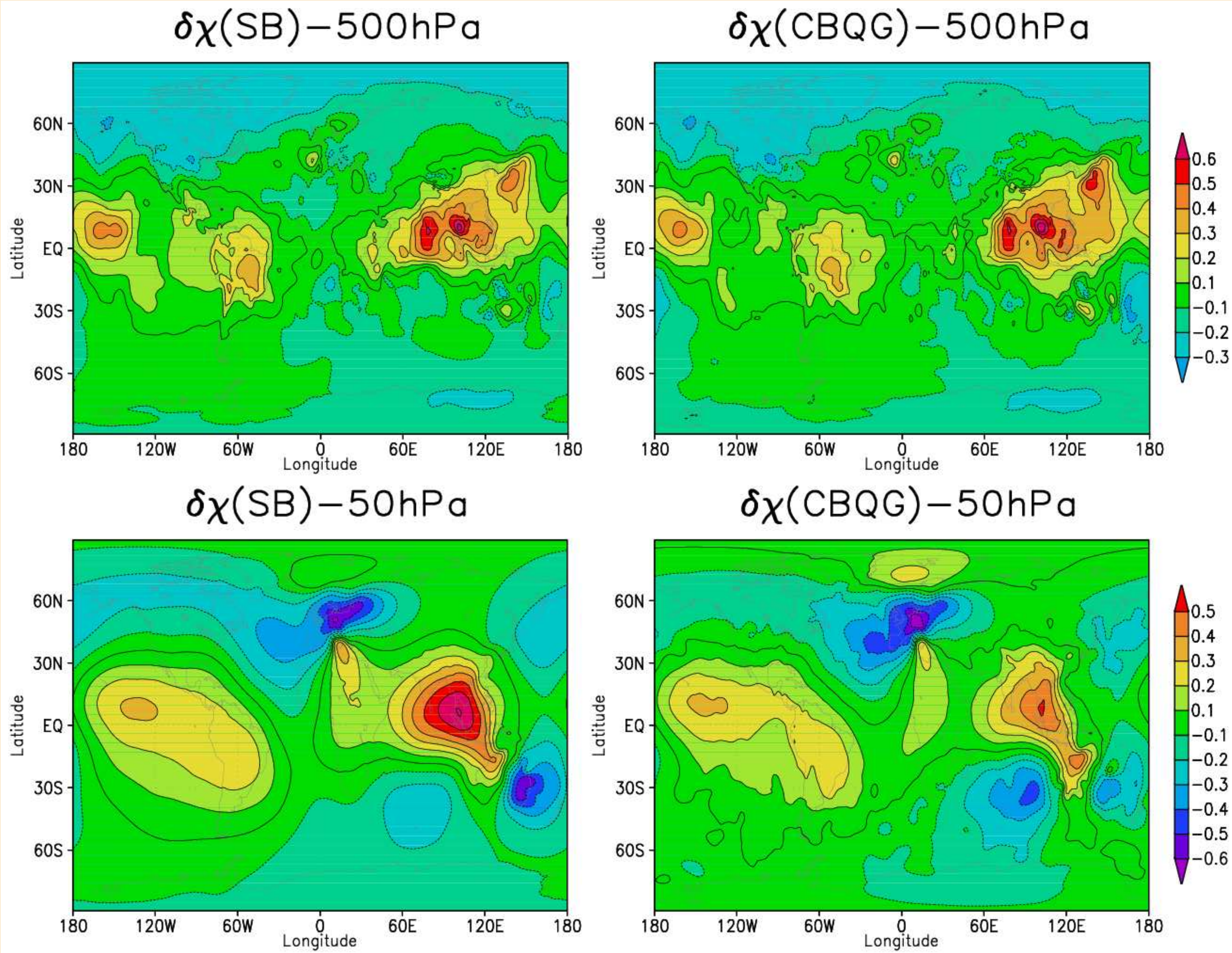


- ***Flow-dependent* δU *response for CBQG constraint*** **(not SB or LB)**
- **Color shading shows background flow (geopotential)**

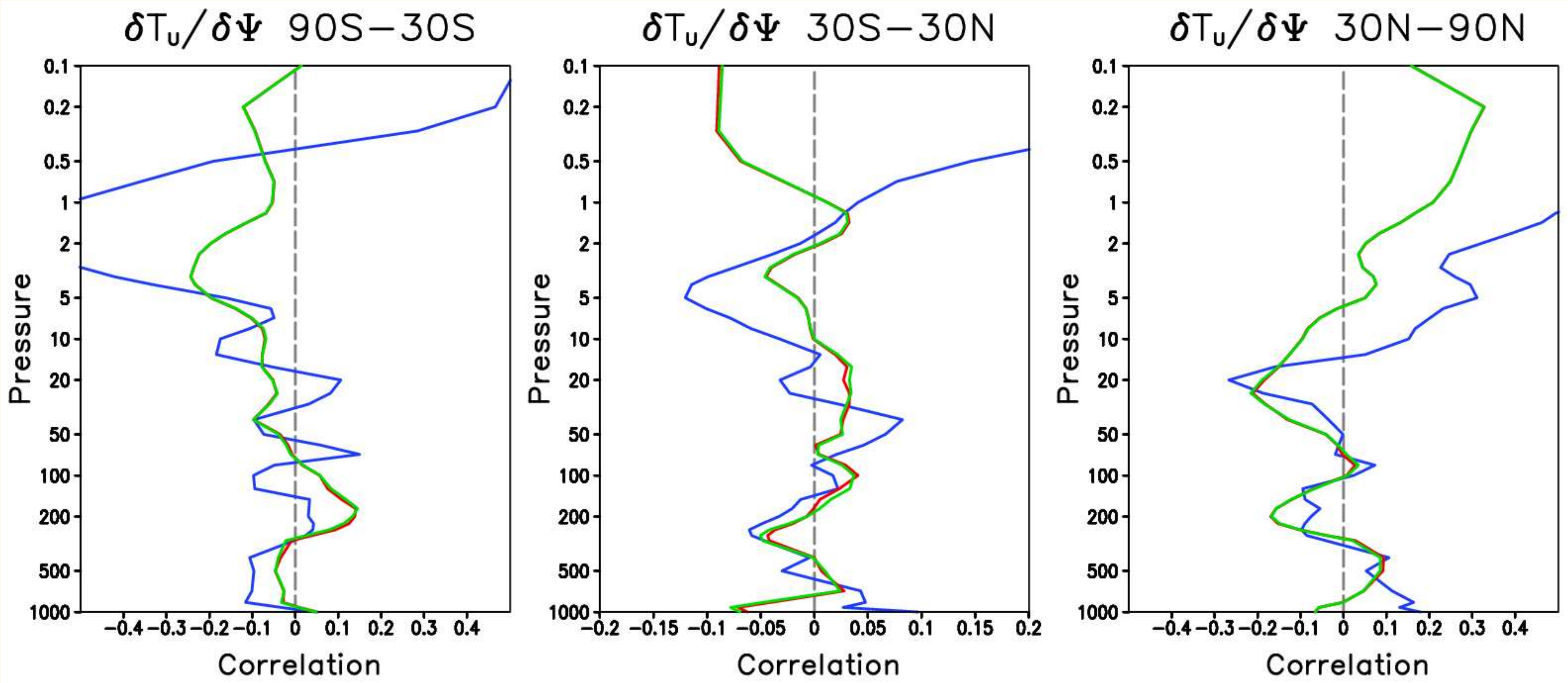
Snapshots of δT increments from 3D-Var (500 hPa and zonal mean)



Snapshots of $\delta\chi$ increments from 3D-Var (500 hPa and 50 hPa)



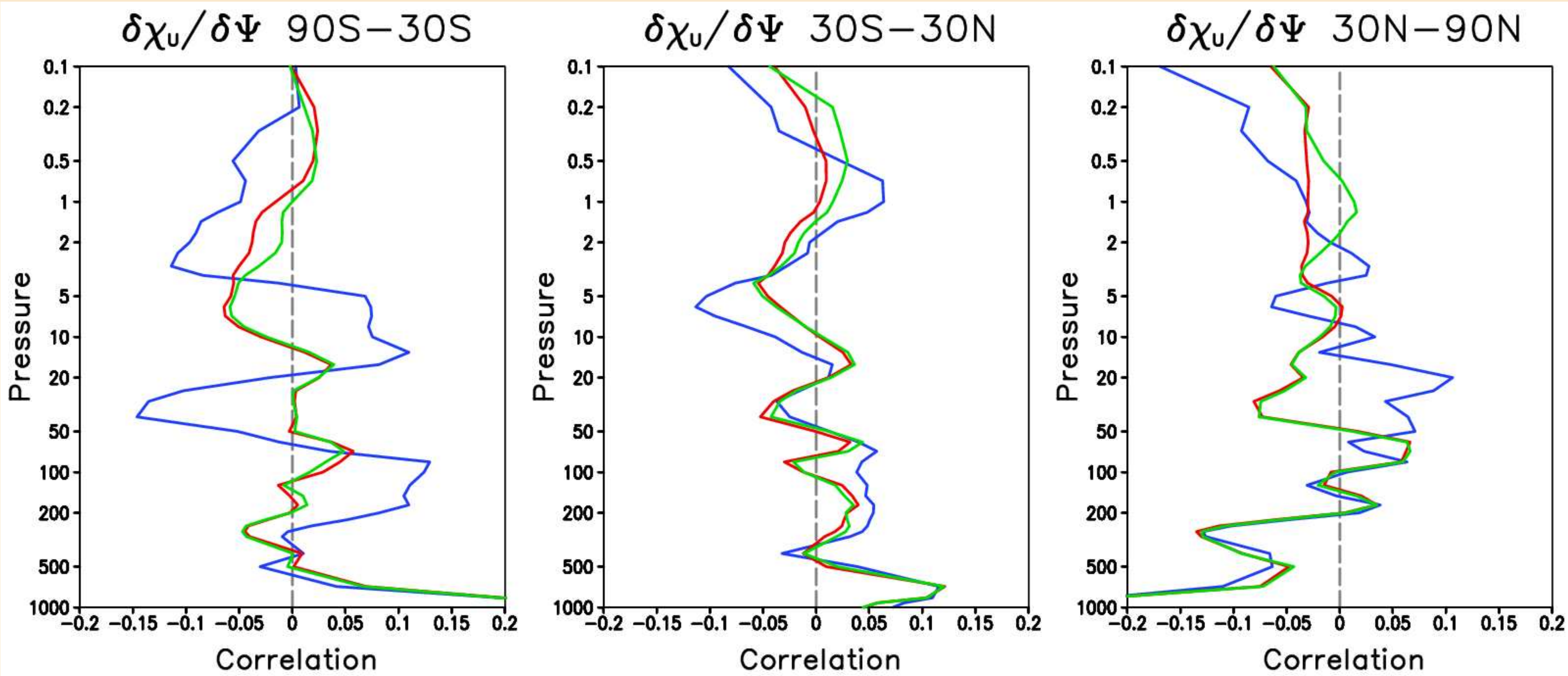
January-mean $\delta\Psi$, δT_u correlations in 3 latitude bands



- **BLUE**: SB (control) operator
- **RED**: CB (Charney) operator
- **GREEN**: CBQG (Charney + QG omega)

- **CB and CBQG very similar**
- **Some improvement (less correlation) with CB than with SB**

January-mean $\delta\Psi$, $\delta\chi_u$ correlations in 3 latitude bands



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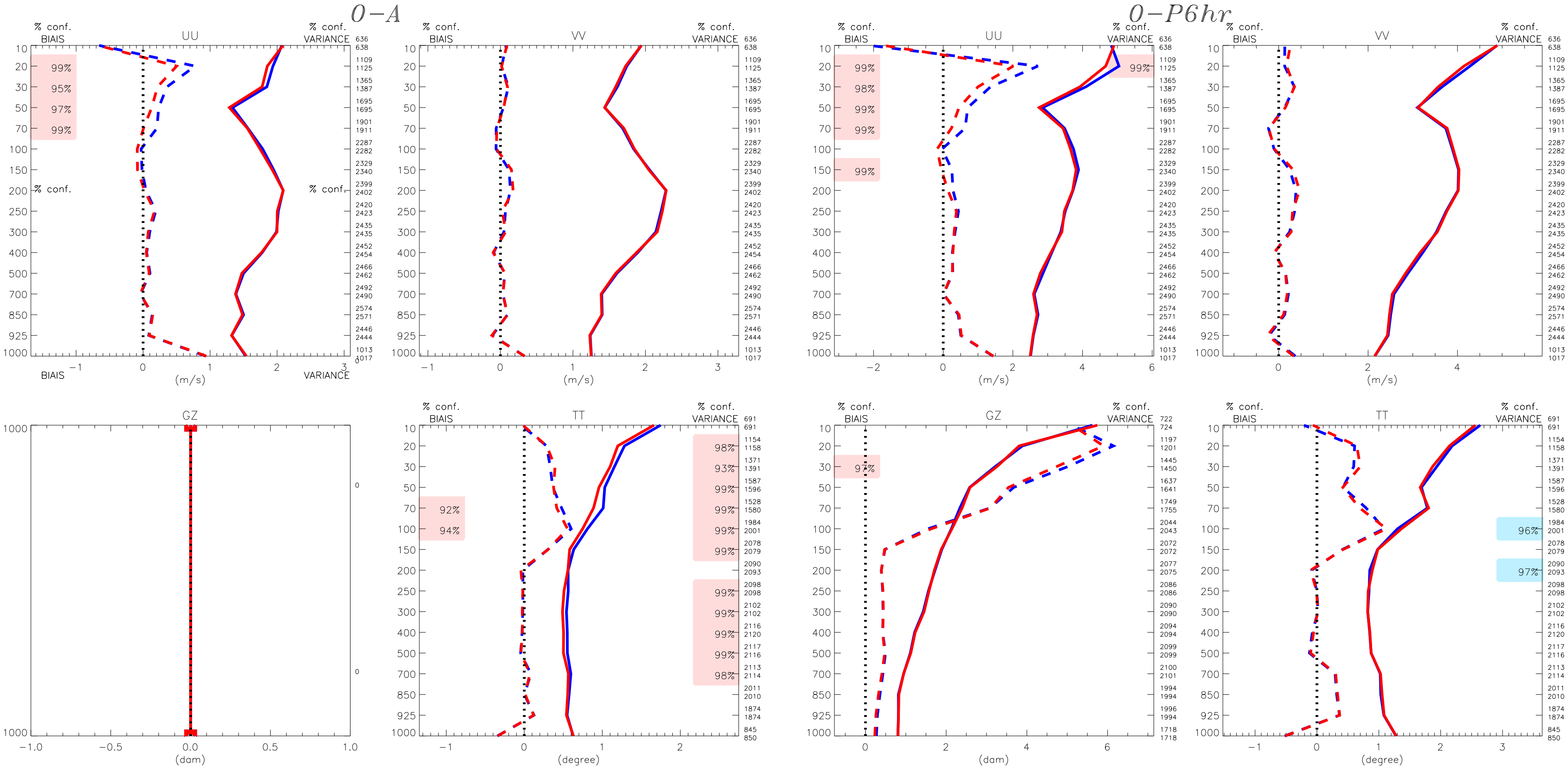
January-mean scores (tropics) for control and new balances



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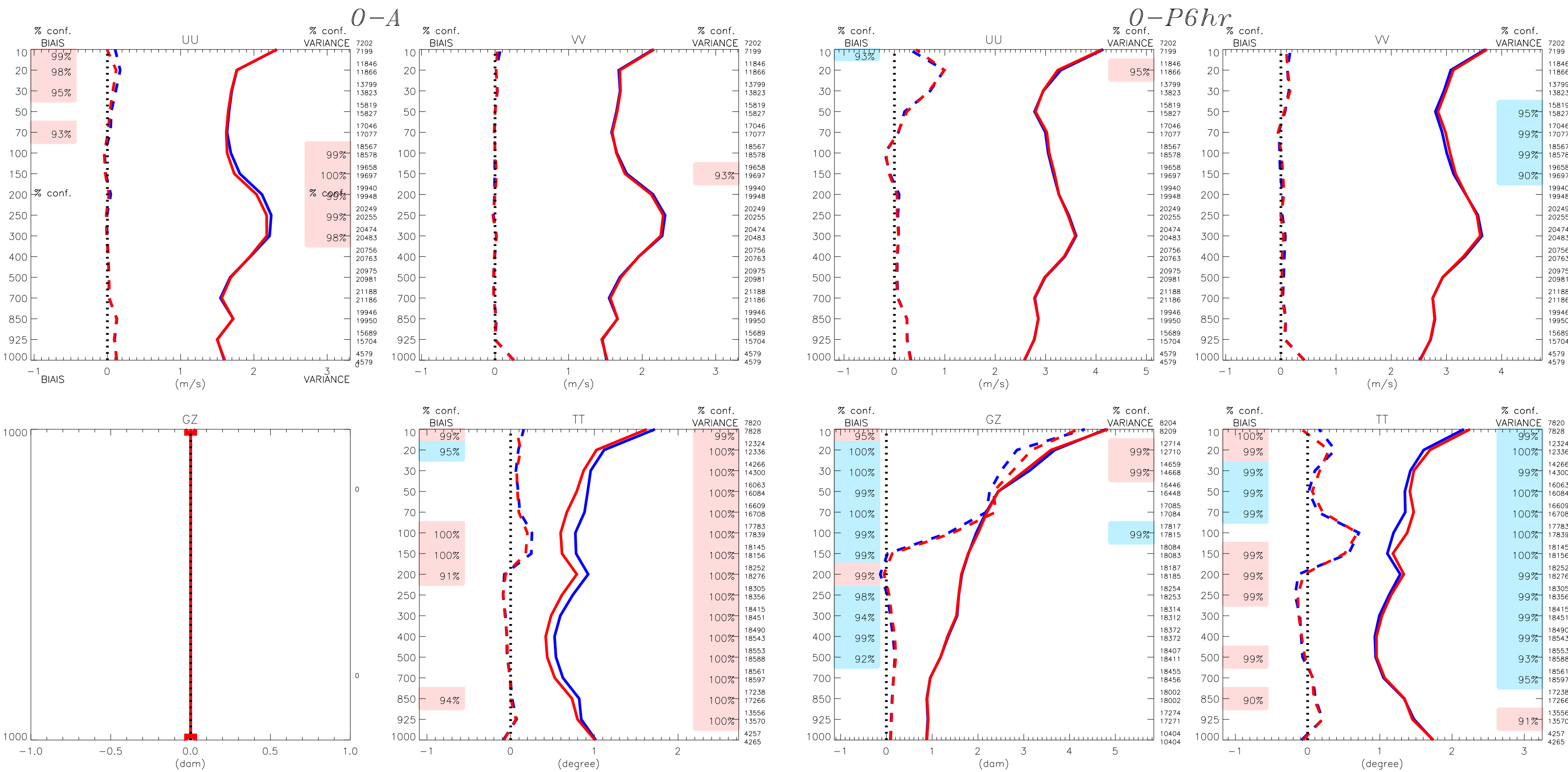
- ***Some improvement with CB both in O-A and O-P***

January-mean scores (global) for control and new balances



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- ***Significant* improvement in O-A TT std. dev. at all levels**
- **Deterioration in O-P TT std. dev. above 200, no change below.**

Efficiency of Constraints within 3D-Var

January cycles

	Control	Charney	Charney + QG
Number of iterations	105	68	68
Number of simulations	112	74	73
3D-Var duration (minutes)	19	24	56

September cycles

	Control	Charney	Charney + QG
Number of iterations	88	60	60
Number of simulations	94	65	65
3D-Var duration (minutes)	18	22	52

- **Note:** *New constraints not optimized yet*

Deriving new background variances

- **Derivation of consistent variances/correlations not trivial**
  - **want stats to be consistent with each balance**
  - **should respect scalings applied to control stats**
  - **should use the same total variance in all cycles**

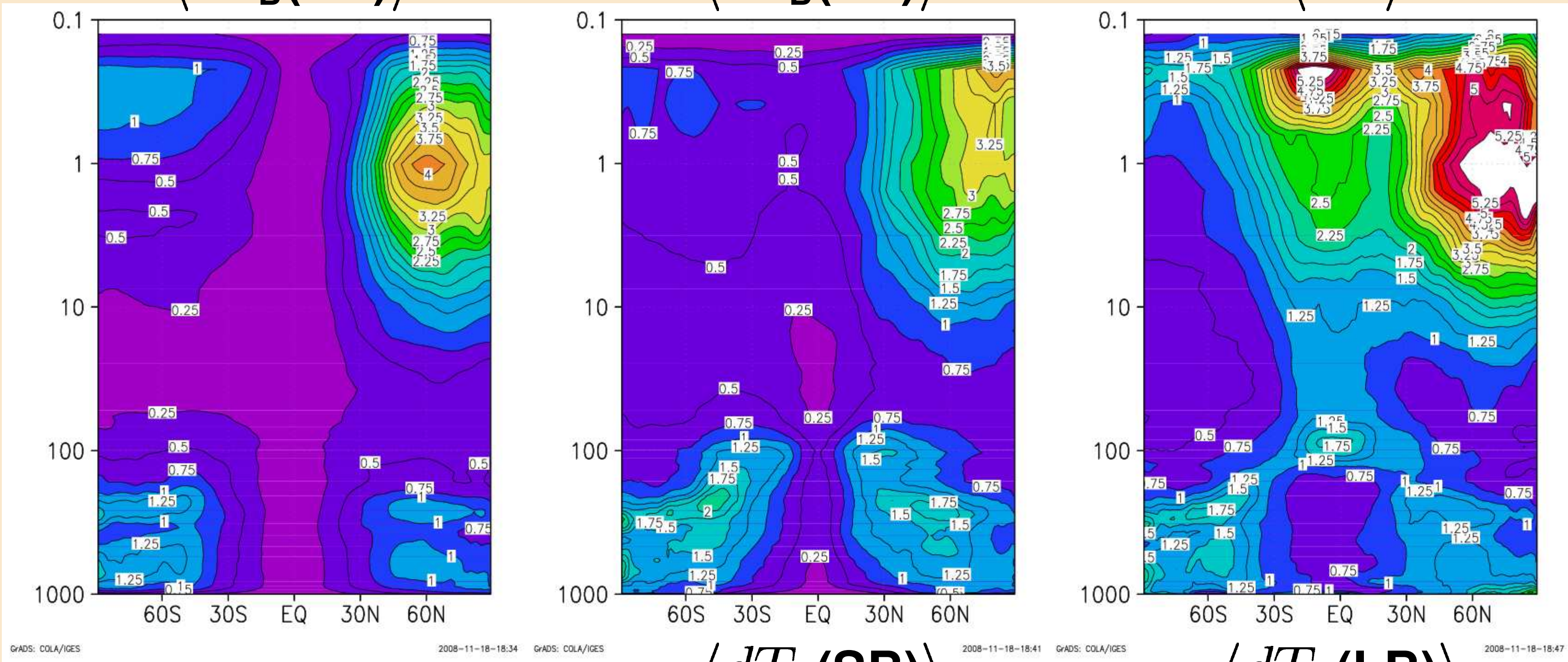
- **Idea: Use 24-48 hour forecast differences $d\Psi$, dT, etc.**

- **Compute total, balanced, unbalanced variance using SB**
  - $\langle dT \rangle^2$, $\langle dT_{\mathbf{B}}(\mathbf{SB}) \rangle^2$, $\langle dT_{\mathbf{U}}(\mathbf{SB}) \rangle^2 = \langle dT \rangle^2 - \langle dT_{\mathbf{B}}(\mathbf{SB}) \rangle^2$

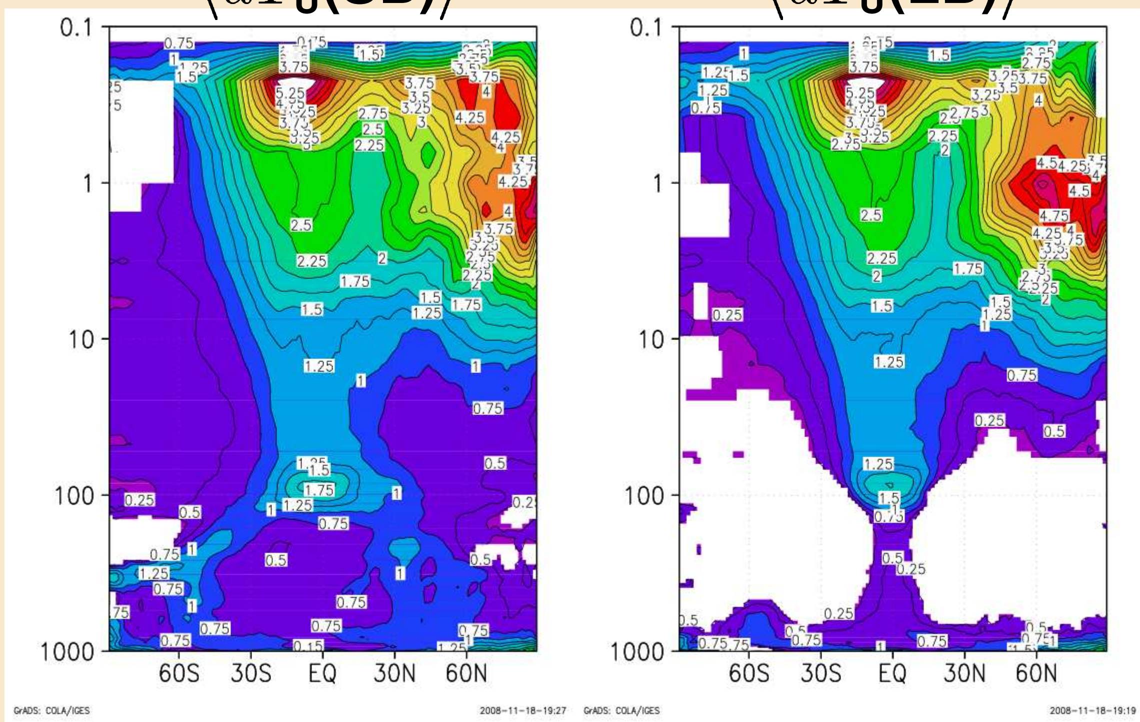
- **Compute balanced, unbalanced variance using LB**
  - $\langle dT_{\mathbf{B}}(\mathbf{LB}) \rangle^2$, $\langle dT_{\mathbf{U}}(\mathbf{LB}) \rangle^2 = \langle dT \rangle^2 - \langle dT_{\mathbf{B}}(\mathbf{LB}) \rangle^2$

- ***Produce new variances by scaling control variances***
  - $\langle \delta T_{\mathbf{U}}(\mathbf{LB}) \rangle = \dfrac{\langle dT_{\mathbf{U}}(\mathbf{LB}) \rangle}{\langle dT_{\mathbf{U}}(\mathbf{SB}) \rangle} \langle \delta T_{\mathbf{U}}(\mathbf{SB}) \rangle$

$\langle dT_{\mathsf{B}}(\textbf{SB})\rangle$ $\langle dT_{\mathsf{B}}(\textbf{LB})\rangle$ $\langle dT\rangle$



$\langle dT_{\mathsf{U}}(\textbf{SB})\rangle$ $\langle dT_{\mathsf{U}}(\textbf{LB})\rangle$



- **Raw stats derived from 24-48 hour forecast differences over January**

- **Note:**

$\langle * \rangle =$ **Std. Dev.**

$$\langle dT_{\mathsf{U}}\rangle = \sqrt{\langle dT\rangle^2 - \langle dT_{\mathsf{B}}\rangle^2}$$

Conclusions

- **New $\Psi - T$, $\Psi - \chi$ constraints were implemented in 3D-Var**
  - **based on Charney Balance and QG ω equation**
- **New constraints provide *flow-dependent increments***
- **Resulting increments are reasonable physically**
  - **Spurious vertical δT correlations still a concern**

- **New constraints *improve O-A and O-P scores in tropics***
- **O-A scores improve in extra-tropics**
  - **O-P scores deteriorate above 200 hPa, no change below**
- **Number of iterations decreases but execution time increases**
- **In global problem Charney Balance makes dominant contribution**
  - **QG ω contribution relatively small *in adiabatic case***

Future directions

- **Introduce new statistics consistent with each balance**
- **Introduce diabatic forcing into QG ω equation**
- **Consider scale-dependent or PV-based control variables?**

Thank you!

Merci!

Incremental Formulation: Overview

- **Analysis increment δx transformed *via***

$$\delta x = \begin{bmatrix} \delta\Psi \\ \delta\chi \\ \delta T, \delta p_s \\ \delta \ln q \end{bmatrix} = \begin{bmatrix} \mathbf{I} & 0 & 0 & 0 \\ \mathbf{E} & \mathbf{I} & 0 & 0 \\ \mathbf{N} & 0 & \mathbf{I} & 0 \\ 0 & 0 & 0 & \mathbf{I} \end{bmatrix} \begin{bmatrix} \delta\Psi_{\mathsf{U}} \\ \delta\chi_{\mathsf{U}} \\ \delta T_{\mathsf{U}}, (\delta p_s)_{\mathsf{U}} \\ (\delta \ln q)_{\mathsf{U}} \end{bmatrix} = \mathbf{K}\delta x_{\mathsf{U}}$$

- **Then "background component" of cost function is**

$$J_b(\delta x) = \delta x^T \mathbf{B}^{-1} \delta x := \delta\hat{x}^T \delta\hat{x}$$

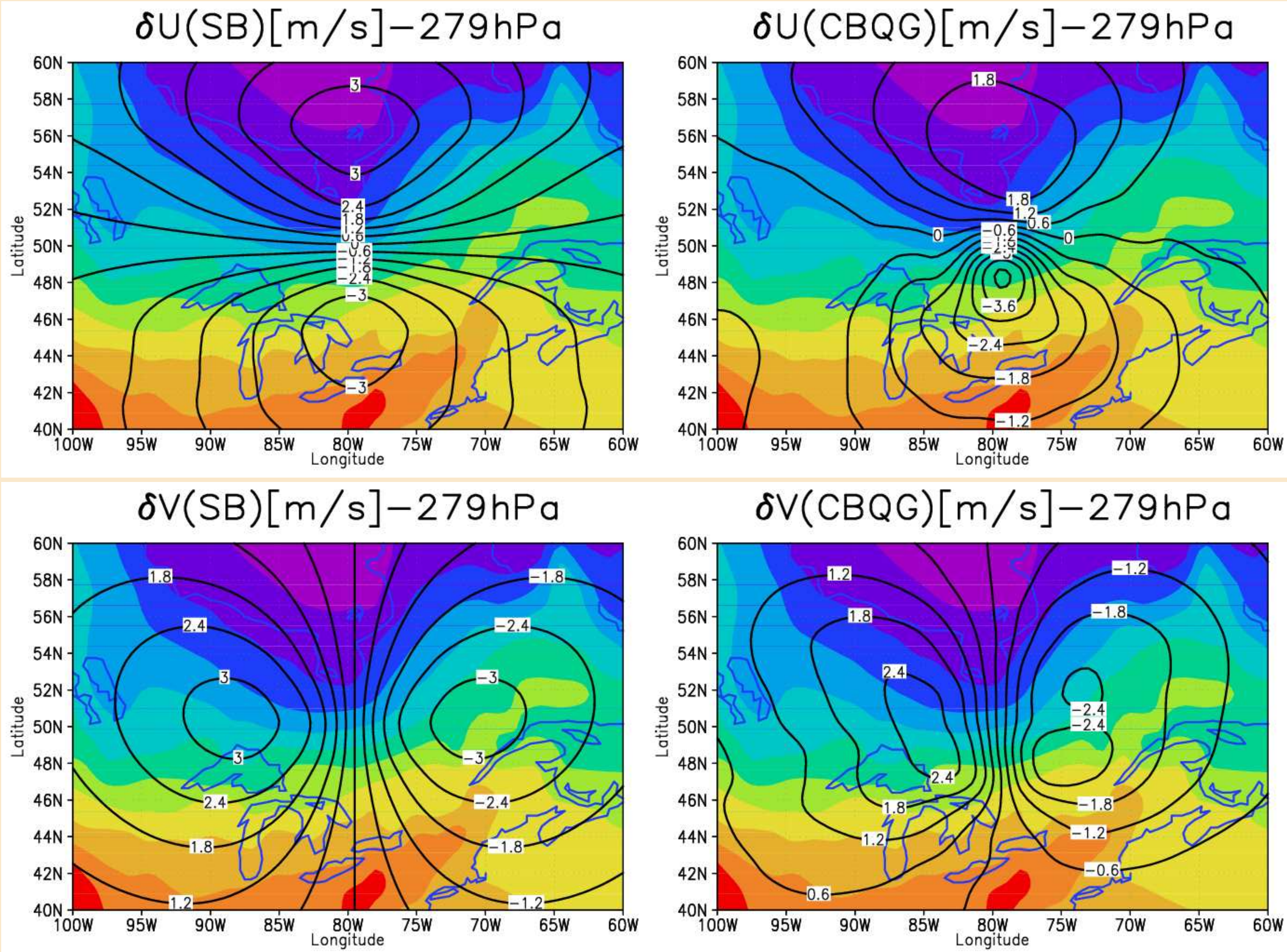
where

$$\mathbf{B} = \mathbf{K}\mathbf{\Sigma}_{\mathsf{U}}\mathbf{S}^{-1}\mathbf{E}_{\mathsf{U}}\mathbf{\Lambda}_{\mathsf{U}}\mathbf{E}_{\mathsf{U}}^T\mathbf{S}^{-T}\mathbf{\Sigma}_{\mathsf{U}}\mathbf{K}^T$$

- **After minimization, recover δx**

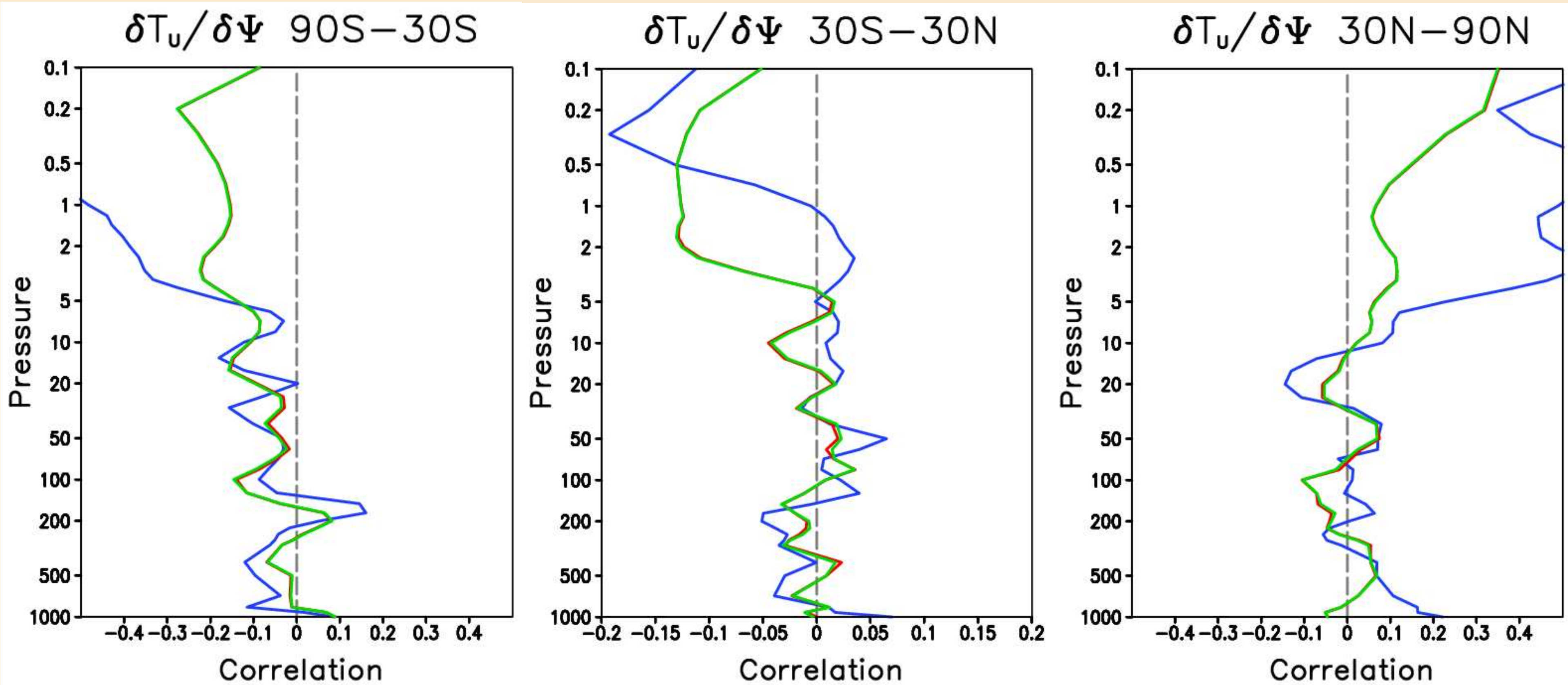
$$\delta x = \mathbf{K} x_{\mathsf{U}} = \mathbf{K}\mathbf{\Sigma}_{\mathsf{U}}\mathbf{S}^{-1}\mathbf{E}_{\mathsf{U}}\mathbf{\Lambda}_{\mathsf{U}}^{\frac{1}{2}}\delta\hat{x}$$

One-obs experiment (obs in TT at 80°W, 50°N, 300hPa)



- ***Flow-dependent*** $\delta U, \delta V$ ***response for CBQG constraint*** **(not SB)**
- **Color shading shows background flow (geopotential)**

September-mean $\delta\Psi$, δT_u correlations in 3 latitude bands



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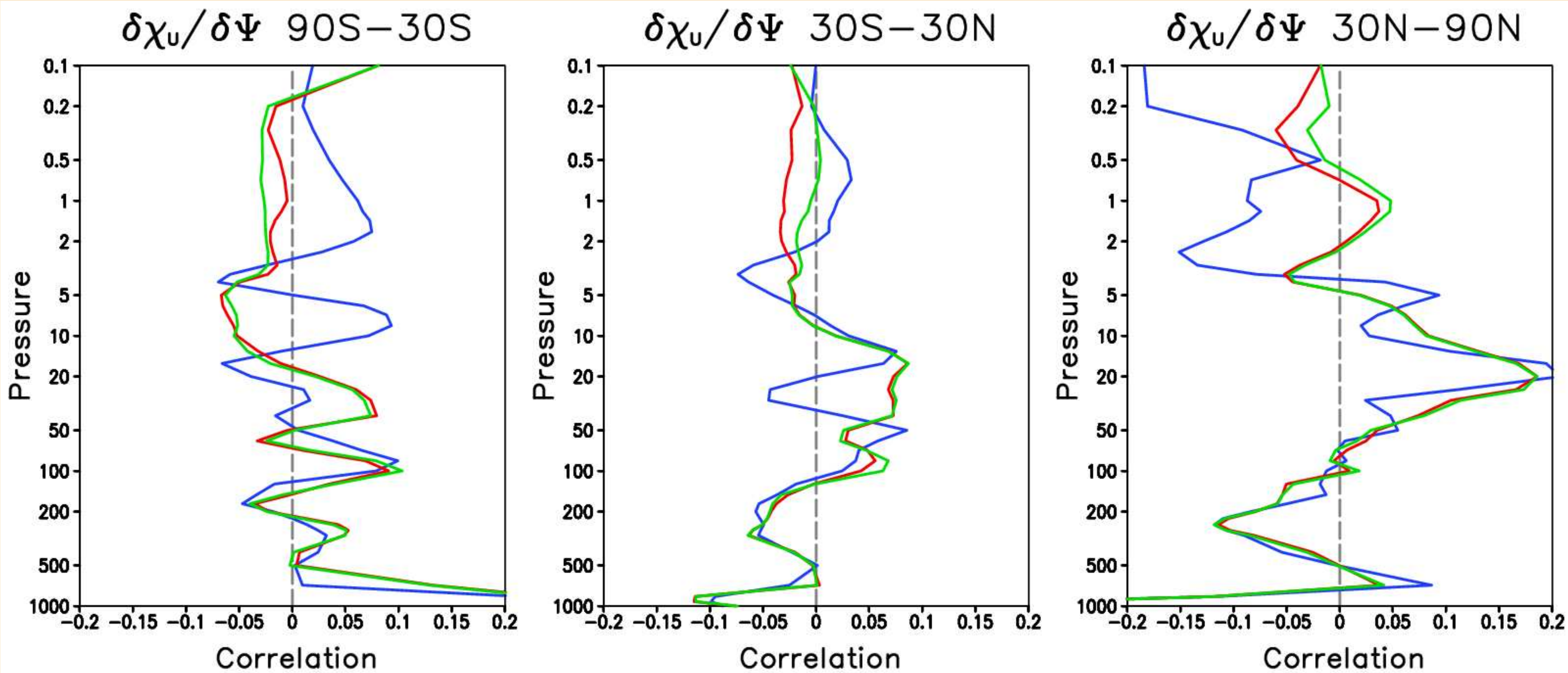

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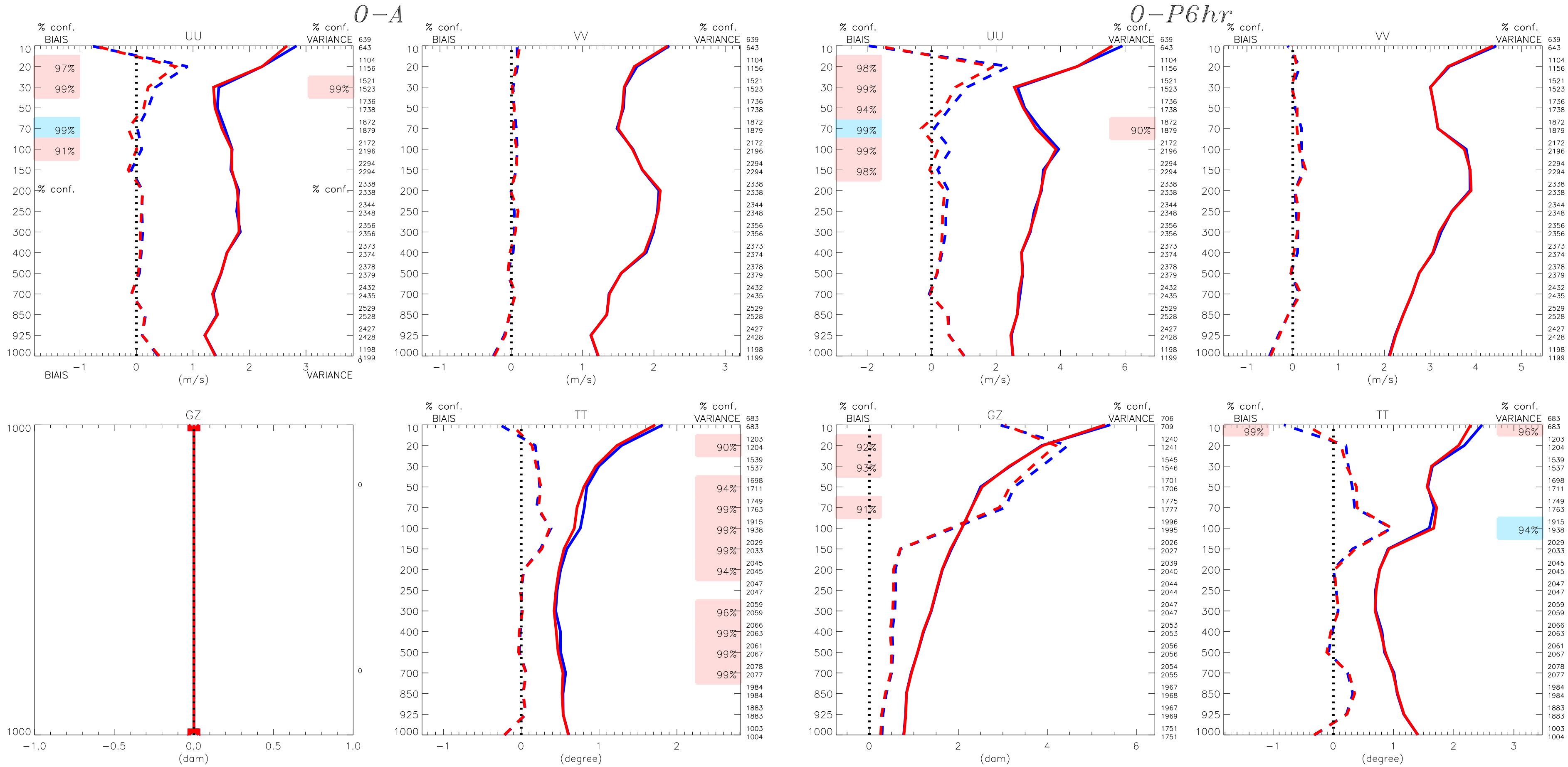
September-mean scores (tropics) for control and new balances



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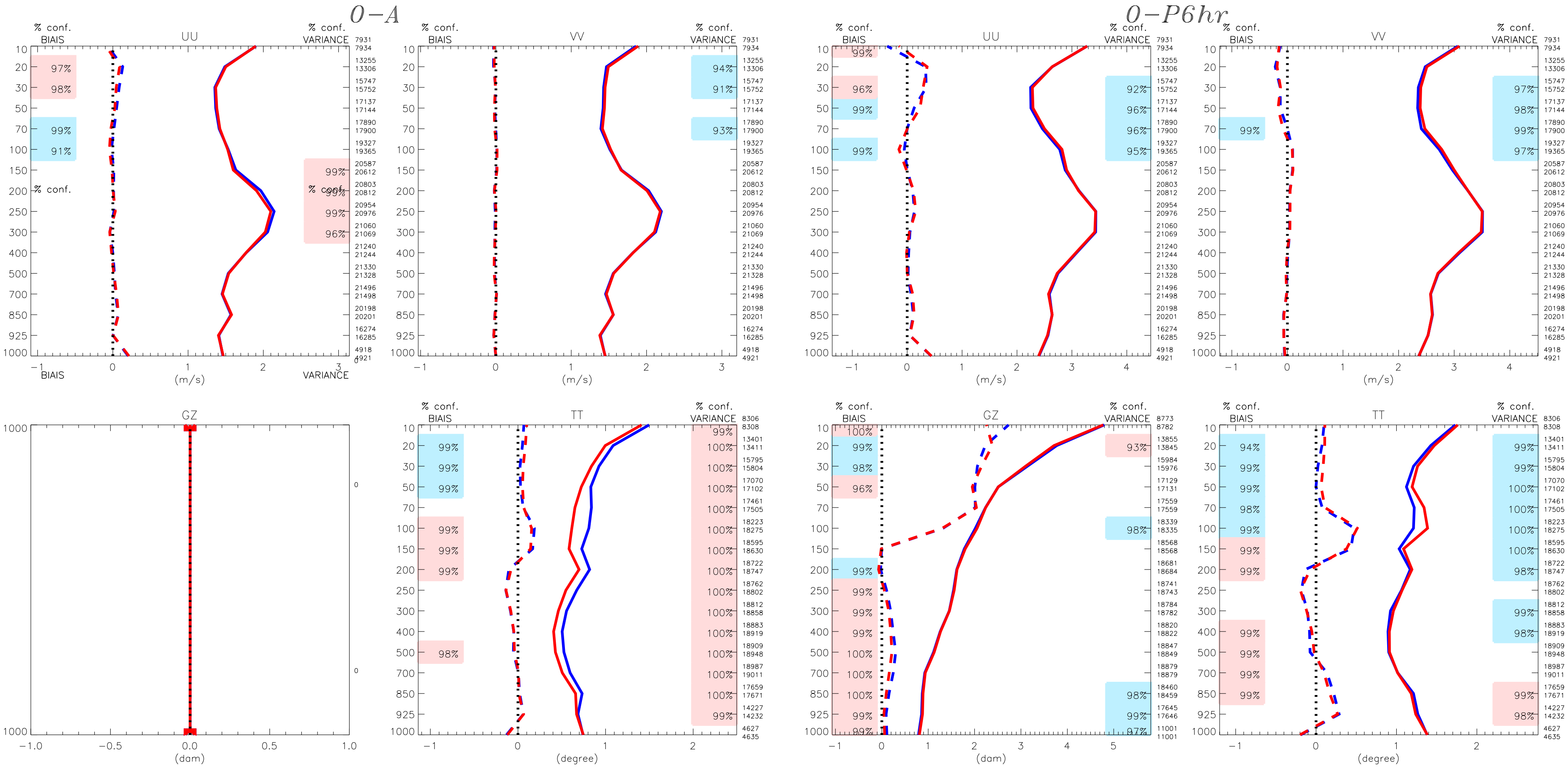
September-mean scores (global) for control and new balances



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