A Multi-Moment Bulk Microphysics Scheme and the Explicit Simulation of Hail

Jason Milbrandt

and

Peter Yau

McGill

November 5, 2004

MOTIVATION

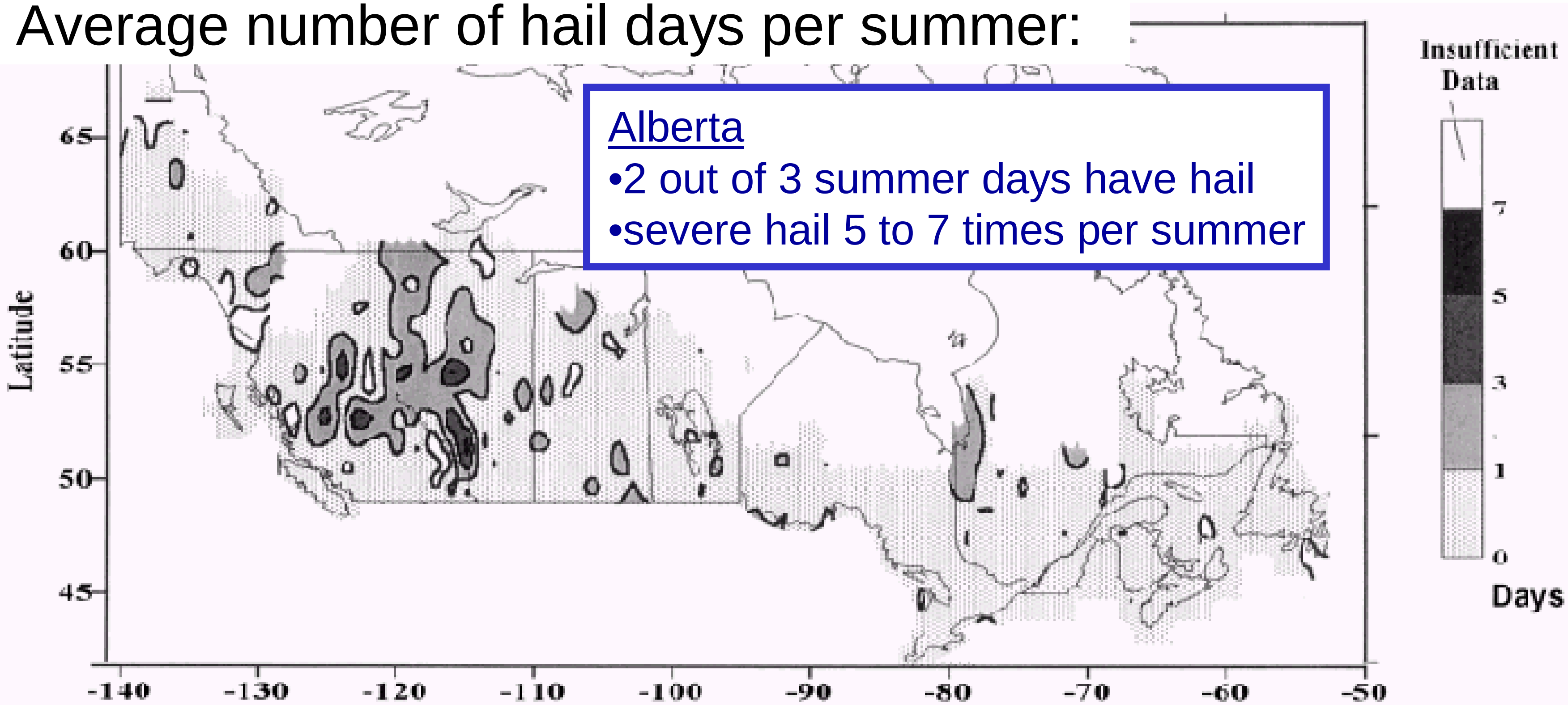
Resolution of NWP models is increasing

- There is in increasing likelihood of grid-scale saturation

The role of explicit microphysics schemes is becoming increasingly important

MOTIVATION

Average number of hail days per summer:



Alberta

•2 out of 3 summer days have hail

•severe hail 5 to 7 times per summer

Hail is one of Canada's most costly natural disasters

Source: Etkin and Brun, 1999 (Journal of Climatology)

MOTIVATION

Resolution of NWP models is increasing

- There is in increasing likelihood of grid-scale saturation

MOTIVATION

Resolution of NWP models is increasing

- There is in increasing likelihood of grid-scale saturation
- The ability to resolve convection is increasing

Can hail sizes be explicitly predicted by an operational NWP model?

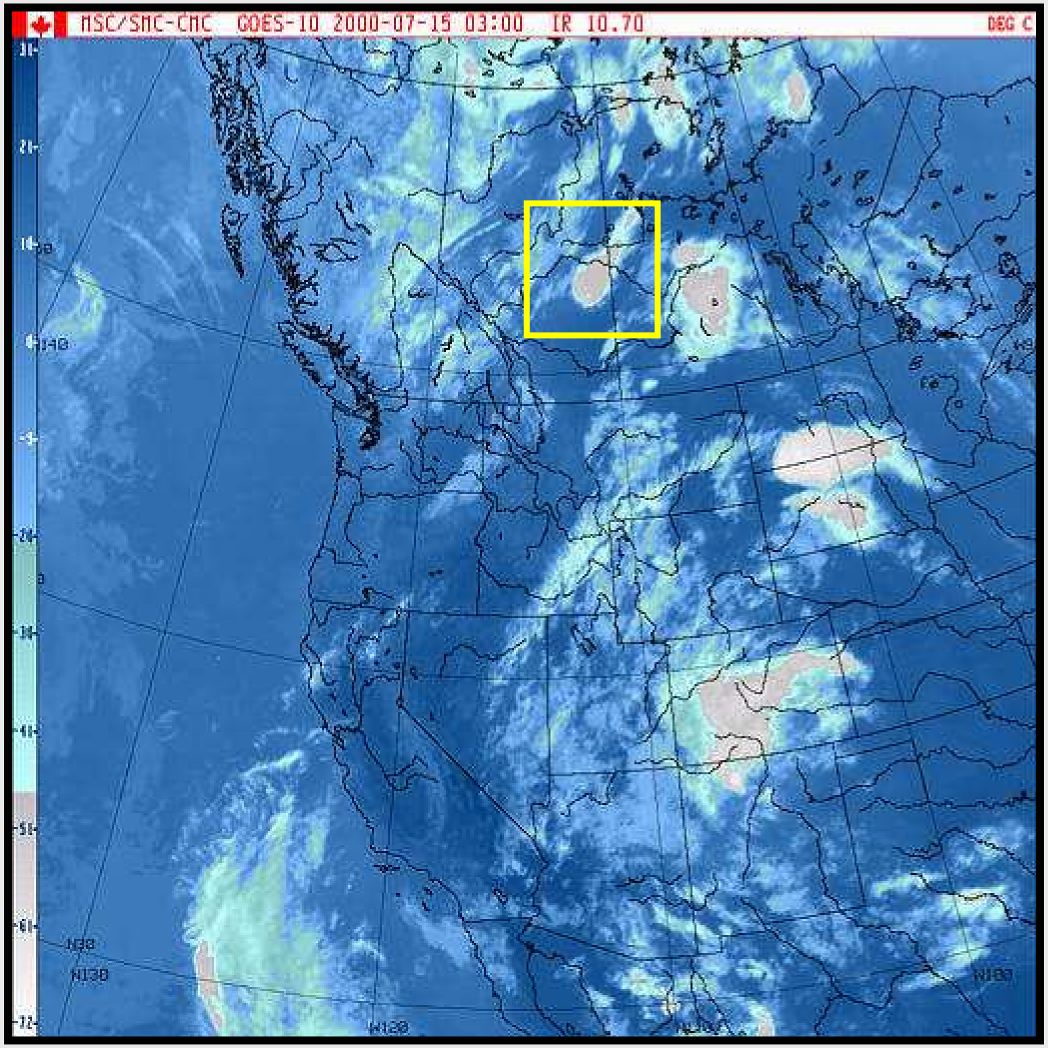
OUTLINE OF PRESENTATION

1. Background on microphysics parameterization
2. Comparison of methods to analytic solutions
3. Comparison in full 3D simulations

1. BACKGROUND ON MICROPHYSICS PARAMETERIZATION

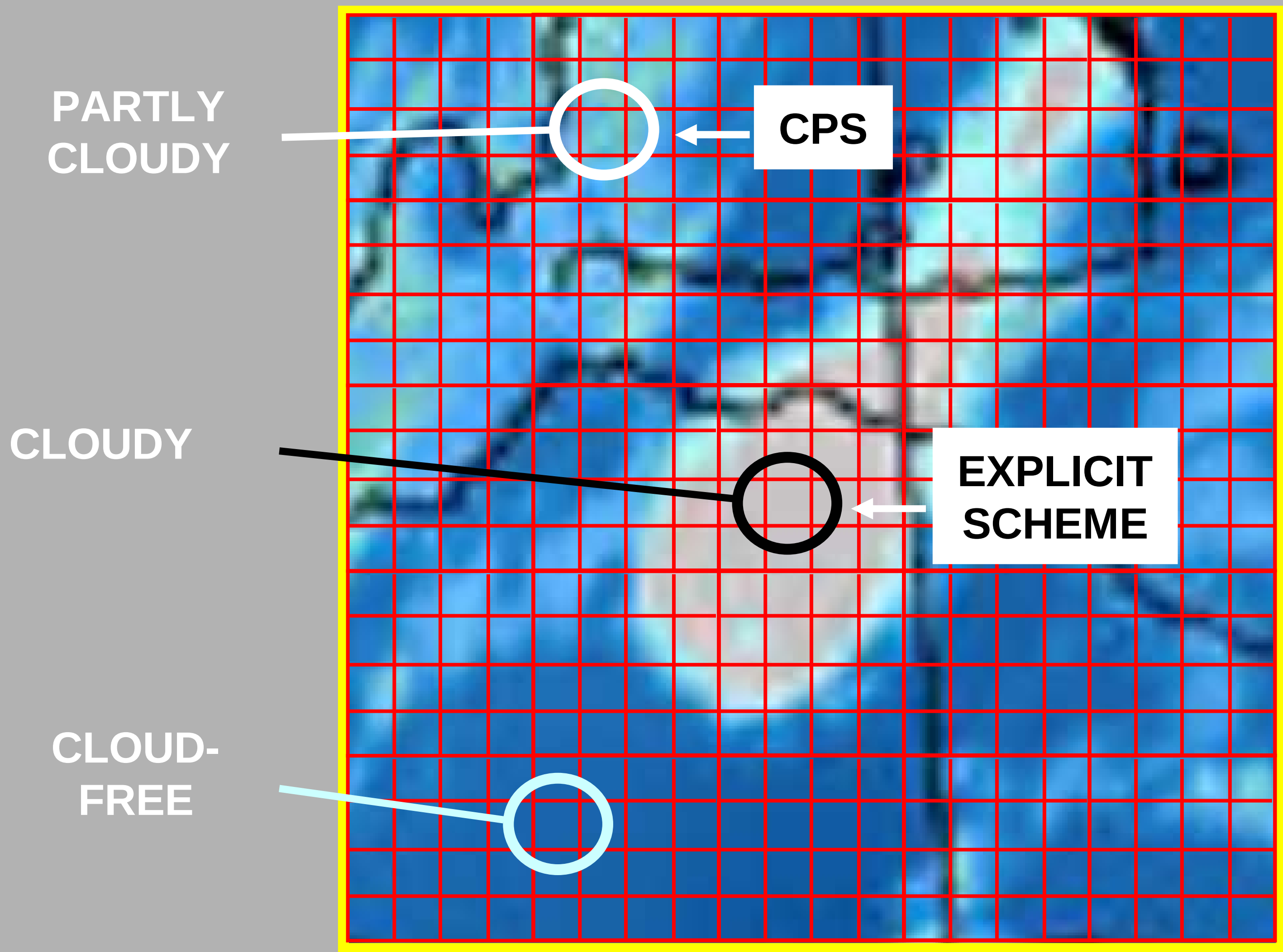
One of the goals of NWP model:

Predict the effects of the clouds

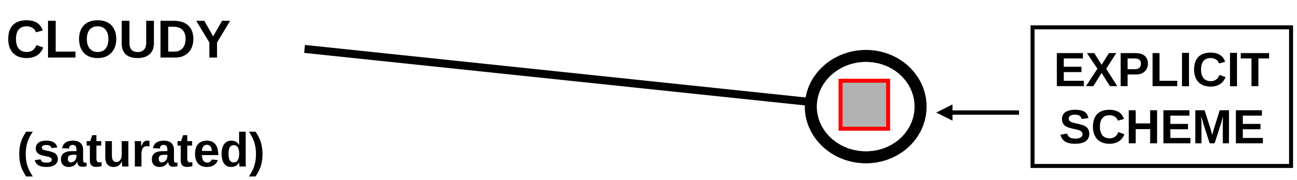


MODEL GRID:

(hypothetical NWP model)



Single cloudy grid element:

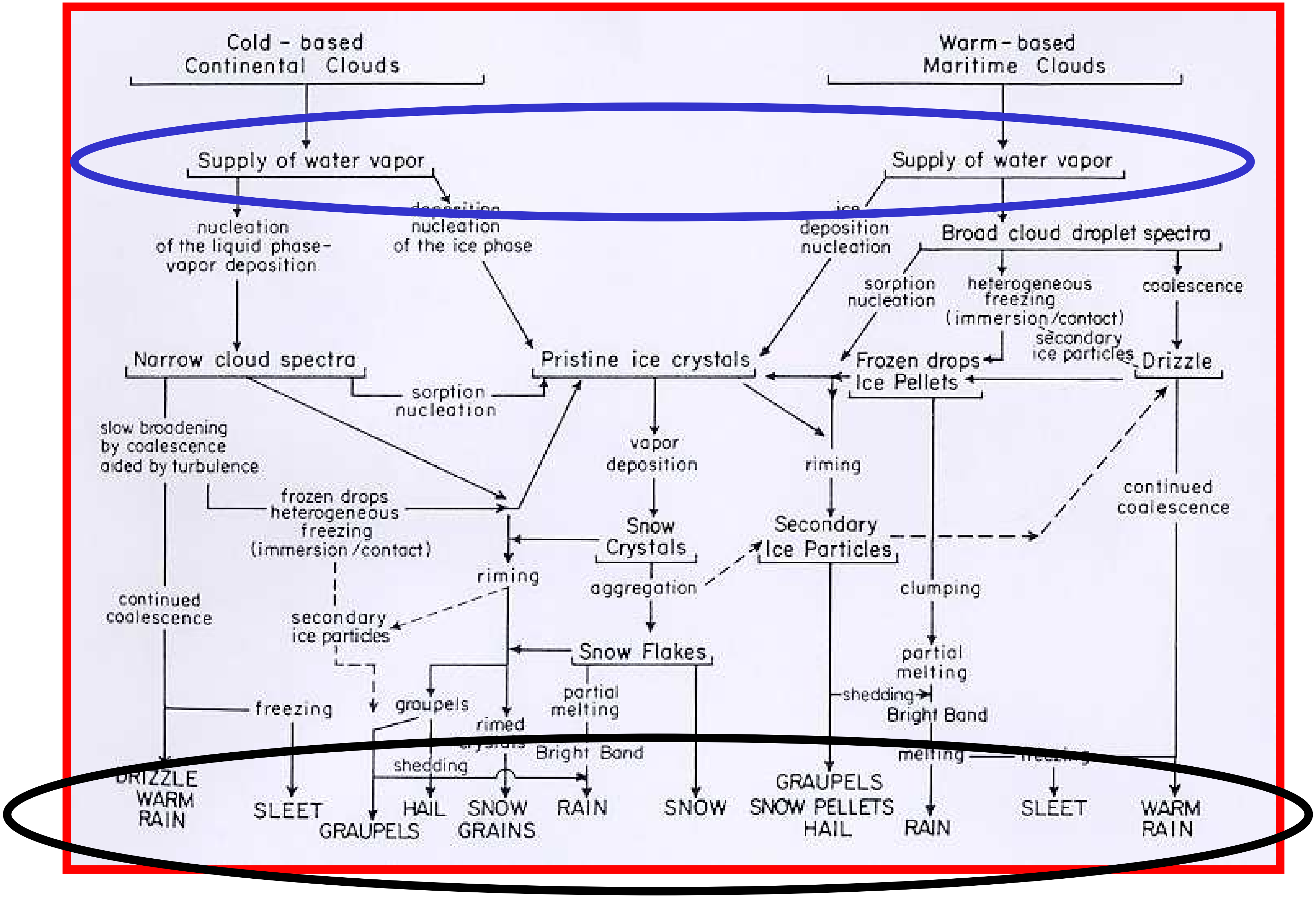


***Single cloudy grid element* – interaction with NWP model:**

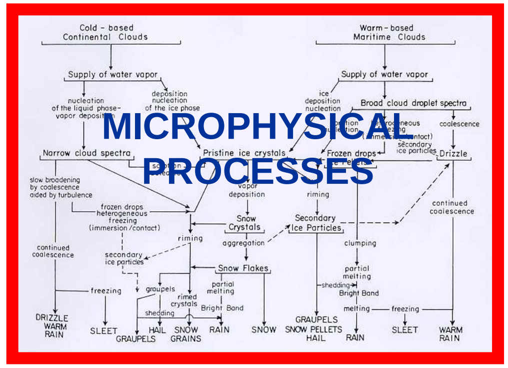
INPUT:

w, T, p, q_v

MICROPHYSICAL PROCESSES in the cloudy grid element



Single cloudy grid element – interaction with NWP model:



INPUT:

w, T, p, q_v

$q_c, q_r, q_i, \ldots$

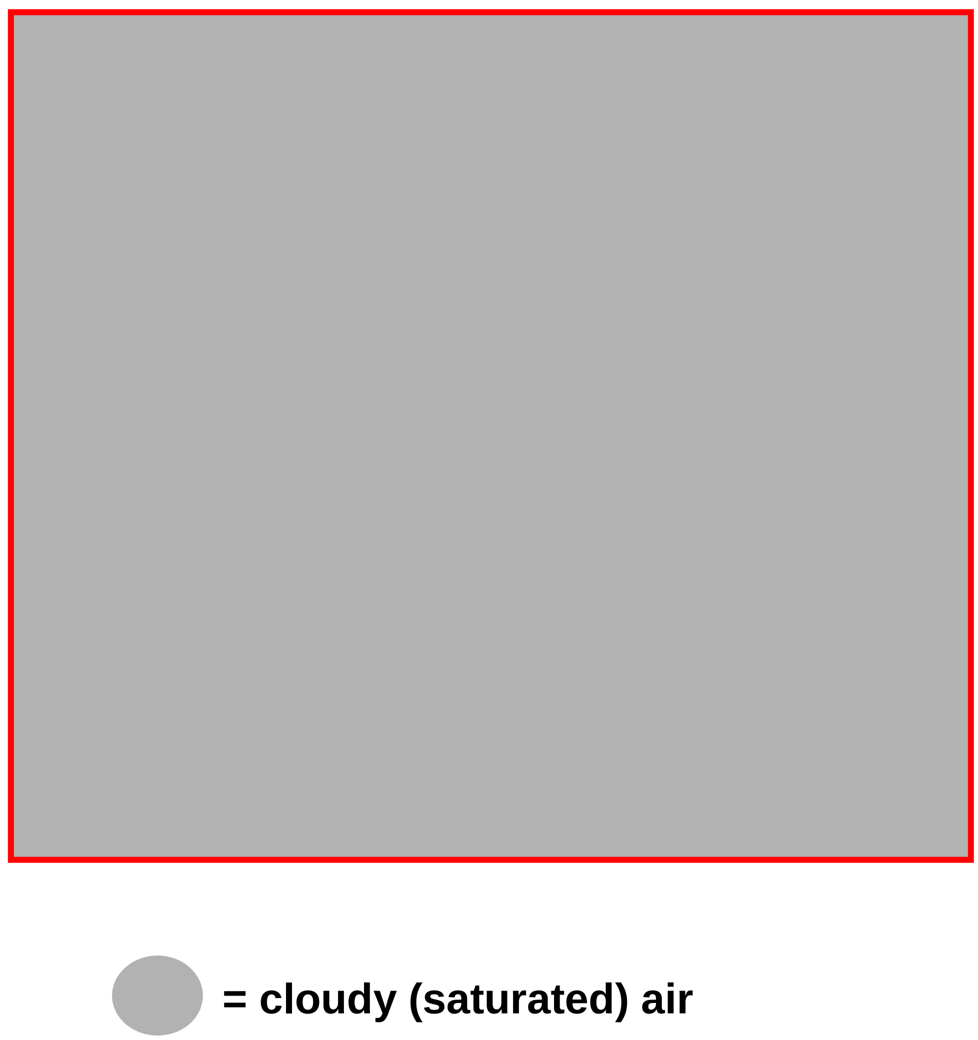
OUTPUT:

- Latent heating
- Hydrometeors (*cloud*, *rain*, *ice*,…) $\rightarrow q_c, q_r, q_i, \ldots$

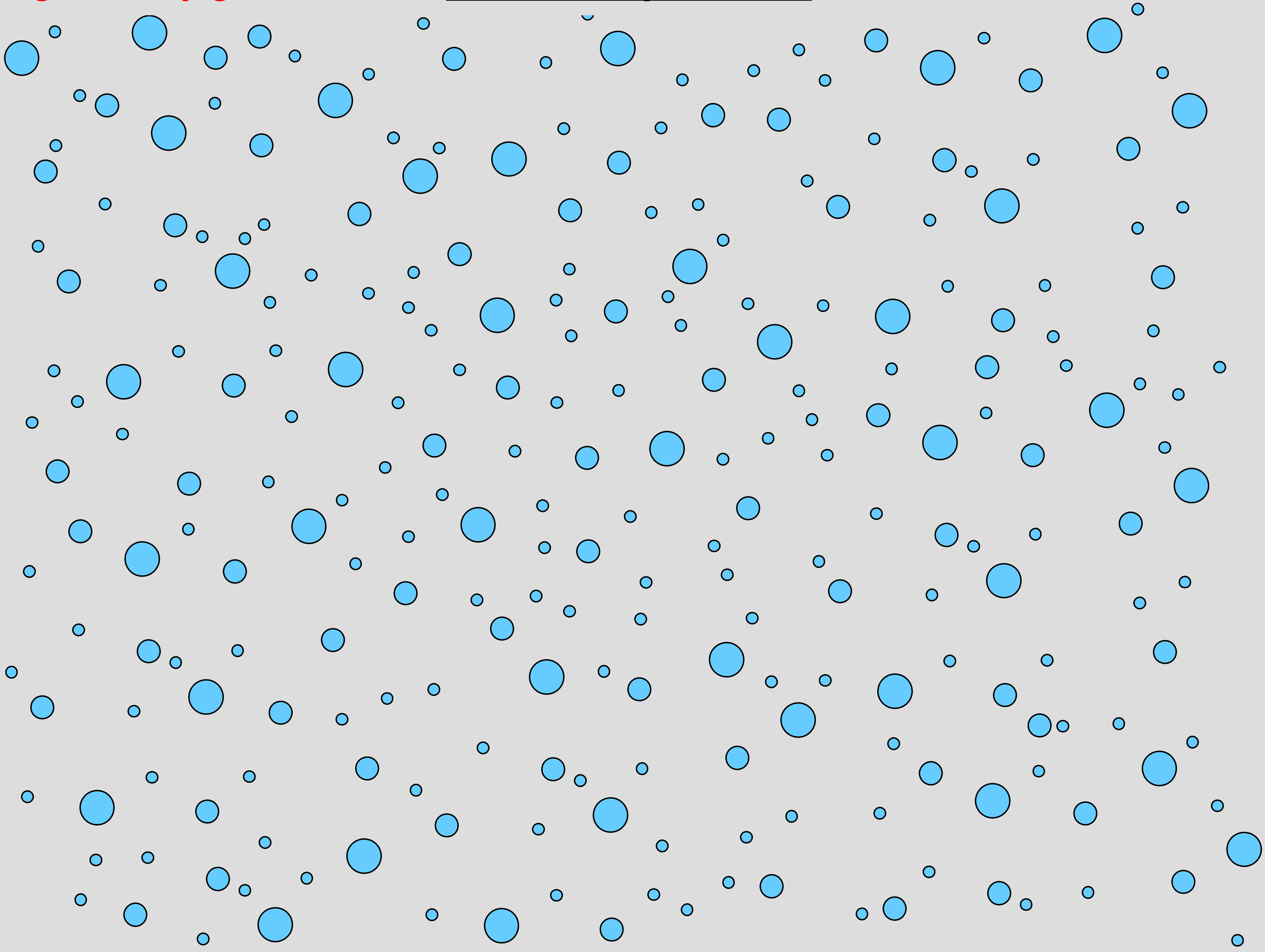
Changes to w, T, p, q_v and $q_c, q_r, q_i, \ldots$

Advection and Turbulent Mixing

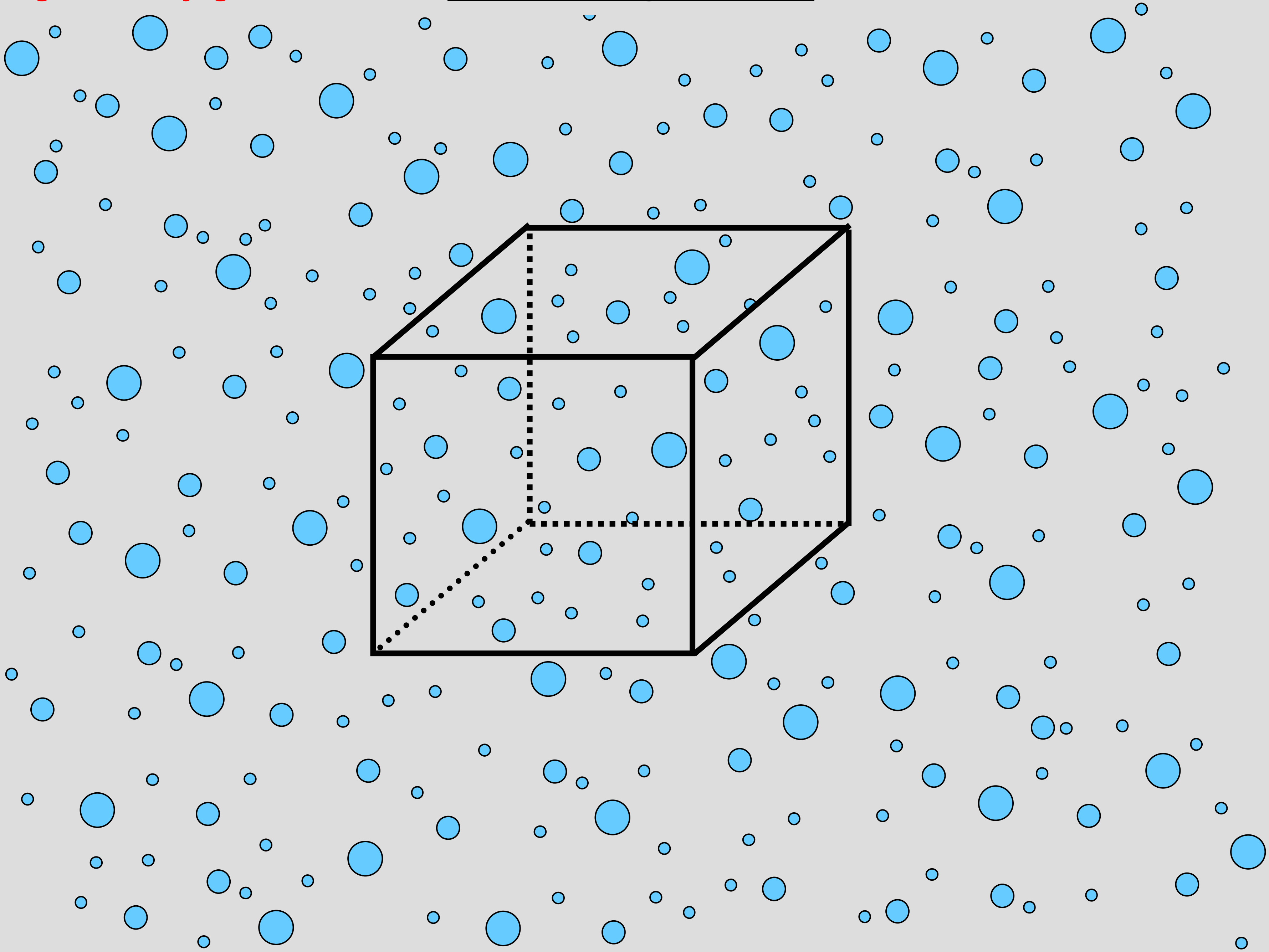
Single cloudy grid element: ***Slight magnification***

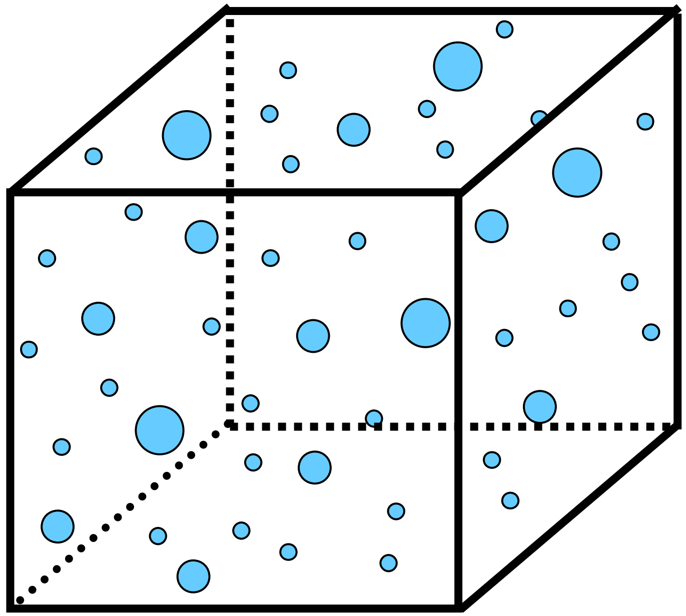


Single cloudy grid element: ***Extreme magnification***



Single cloudy grid element: ***Extreme magnification***

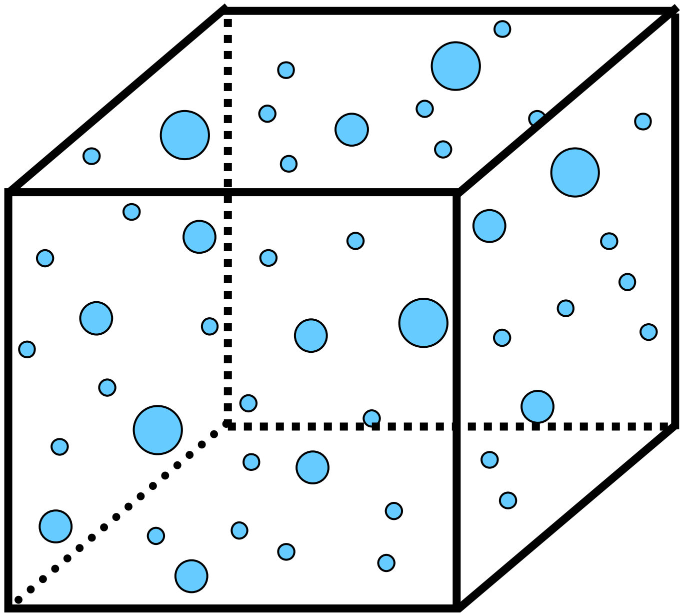




1 m³
(unit volume)

[e.g. Cloud droplets]

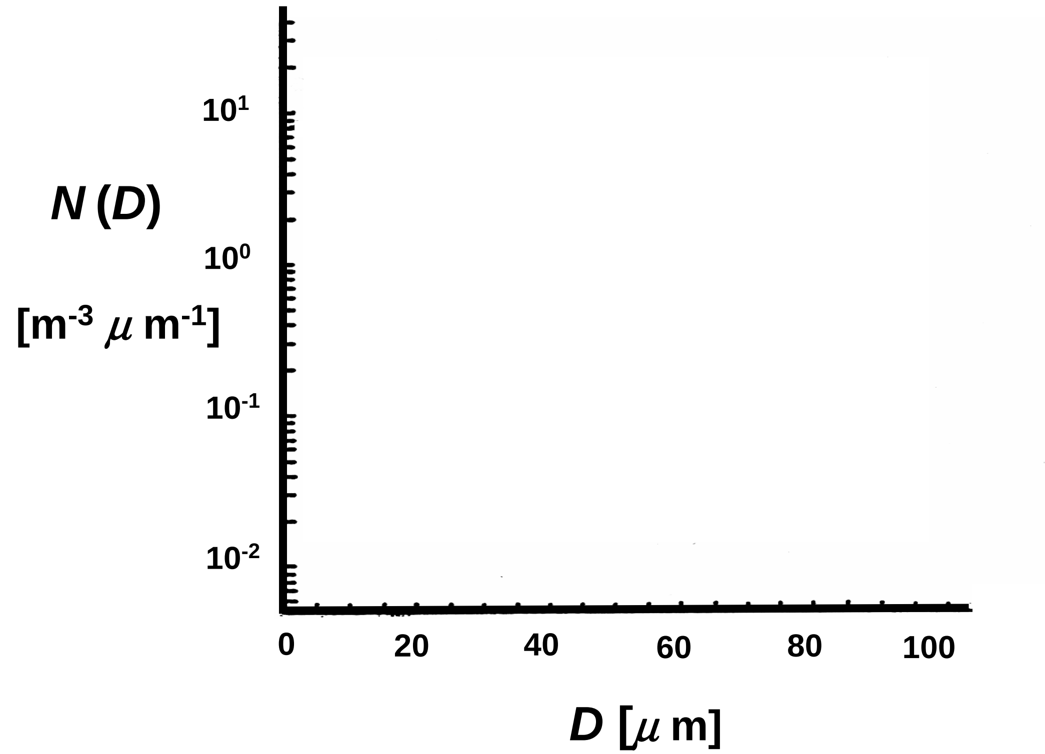
(*not to scale*)

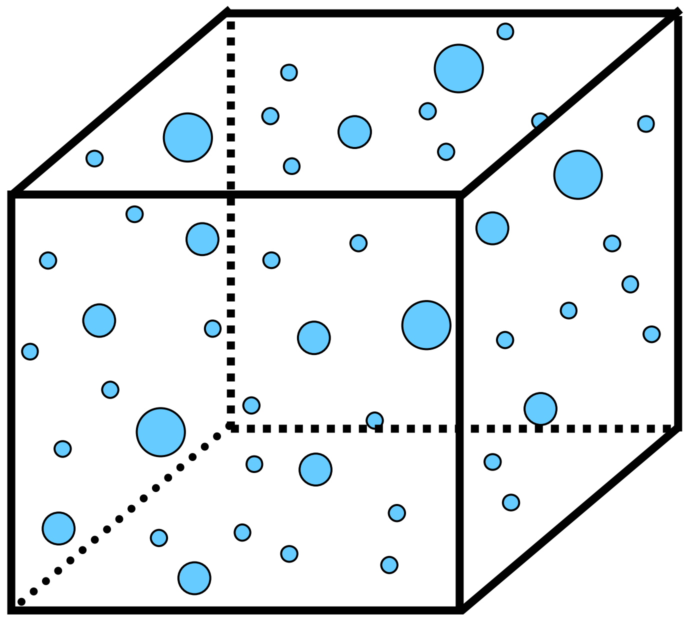


1 m³
(unit volume)

[e.g. Cloud droplets]

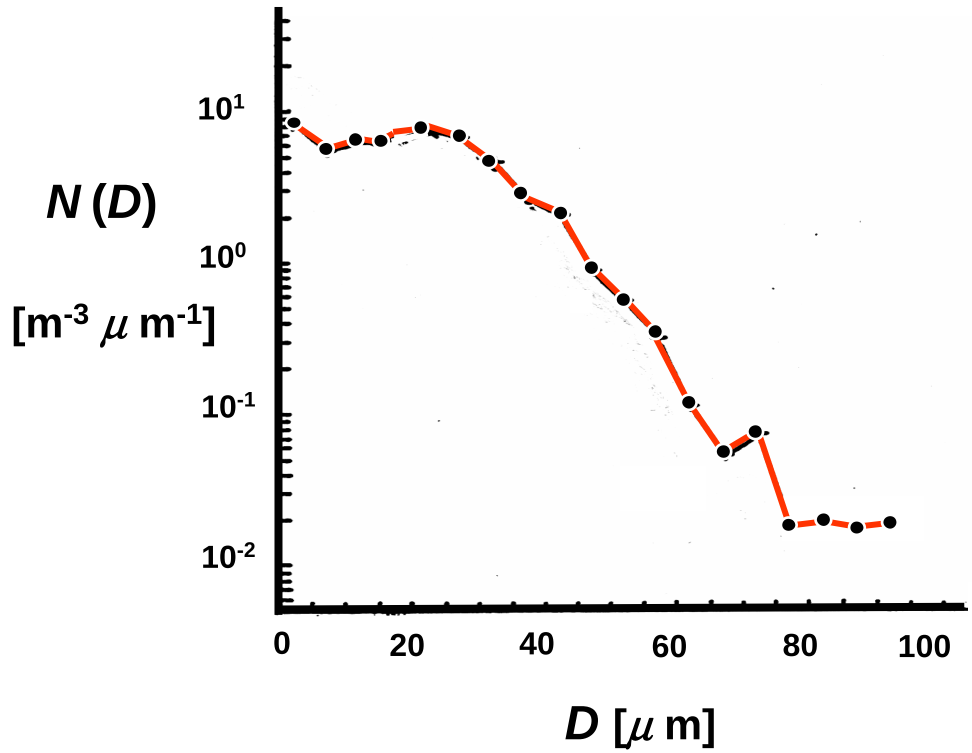
(*not to scale*)





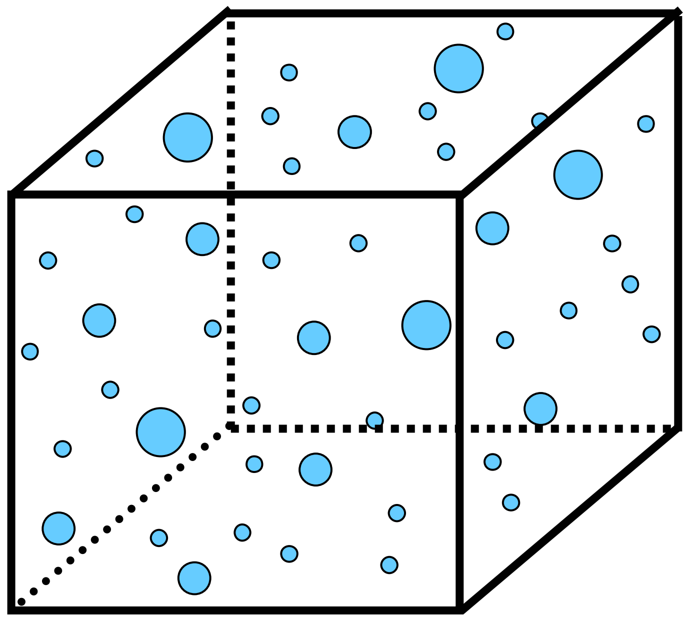
[e.g. Cloud droplets]

(*not to scale*)



(Example of observed cloud droplet spectrum)

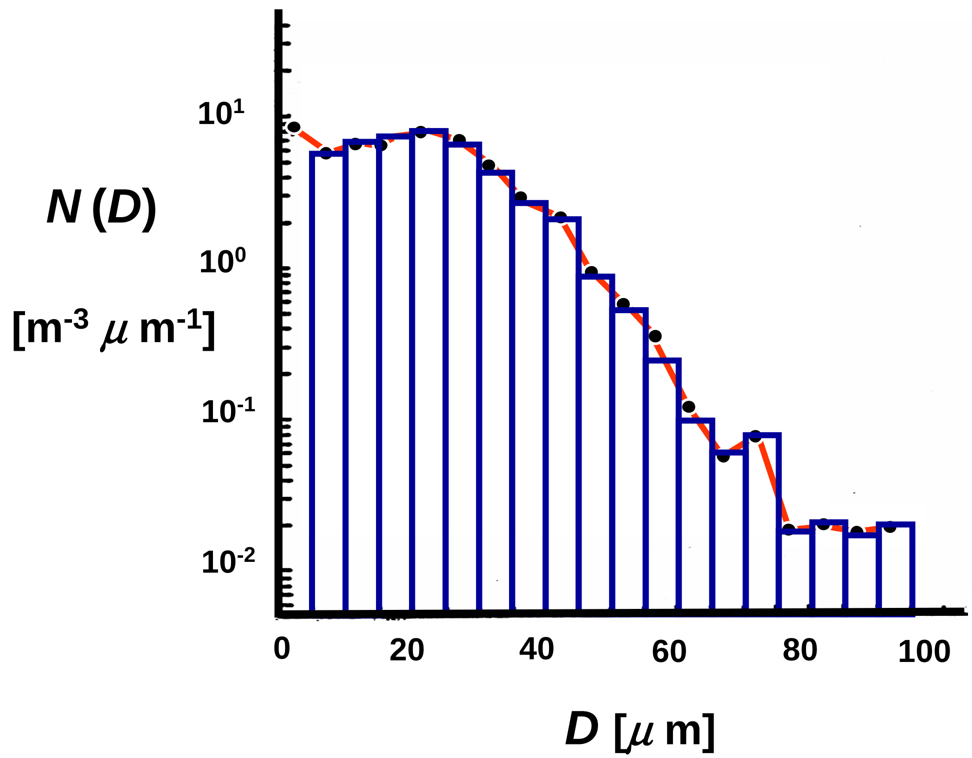
Representing the size spectrum



[e.g. Cloud droplets]

(*not to scale*)

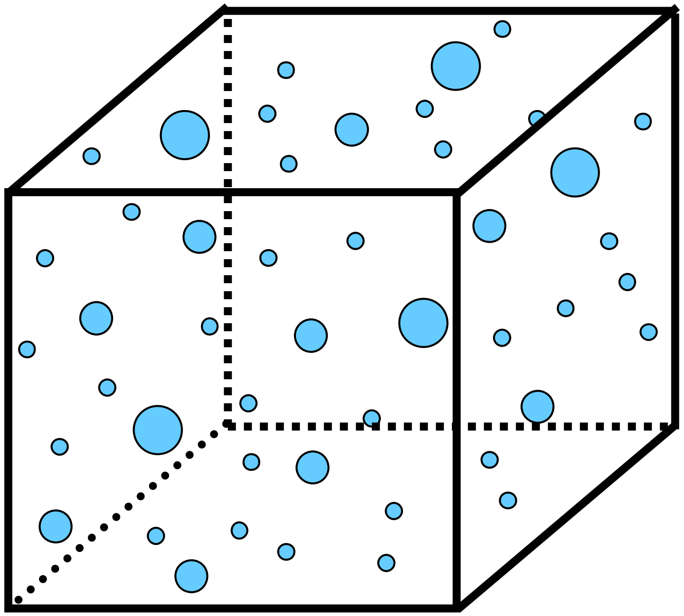
DISCRETE SIZE BINS



SPECTRAL METHOD

Representing the size spectrum

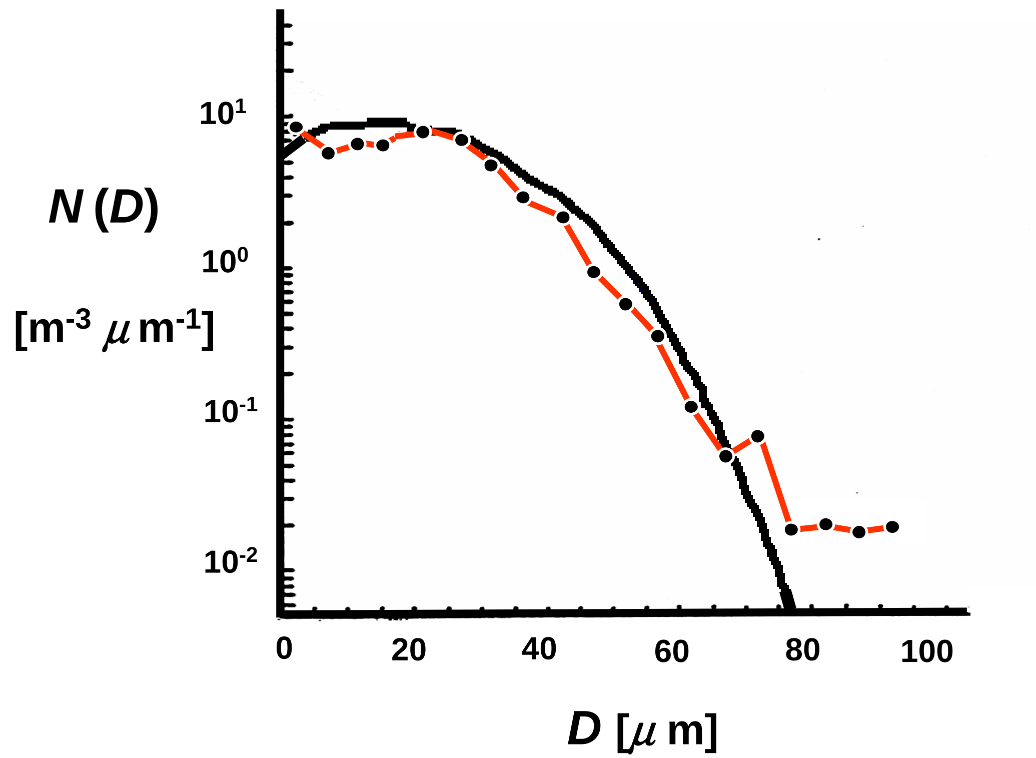
ANAYLTICAL FUNCTION



$1\ m^3$
(unit volume)

[e.g. Cloud droplets]

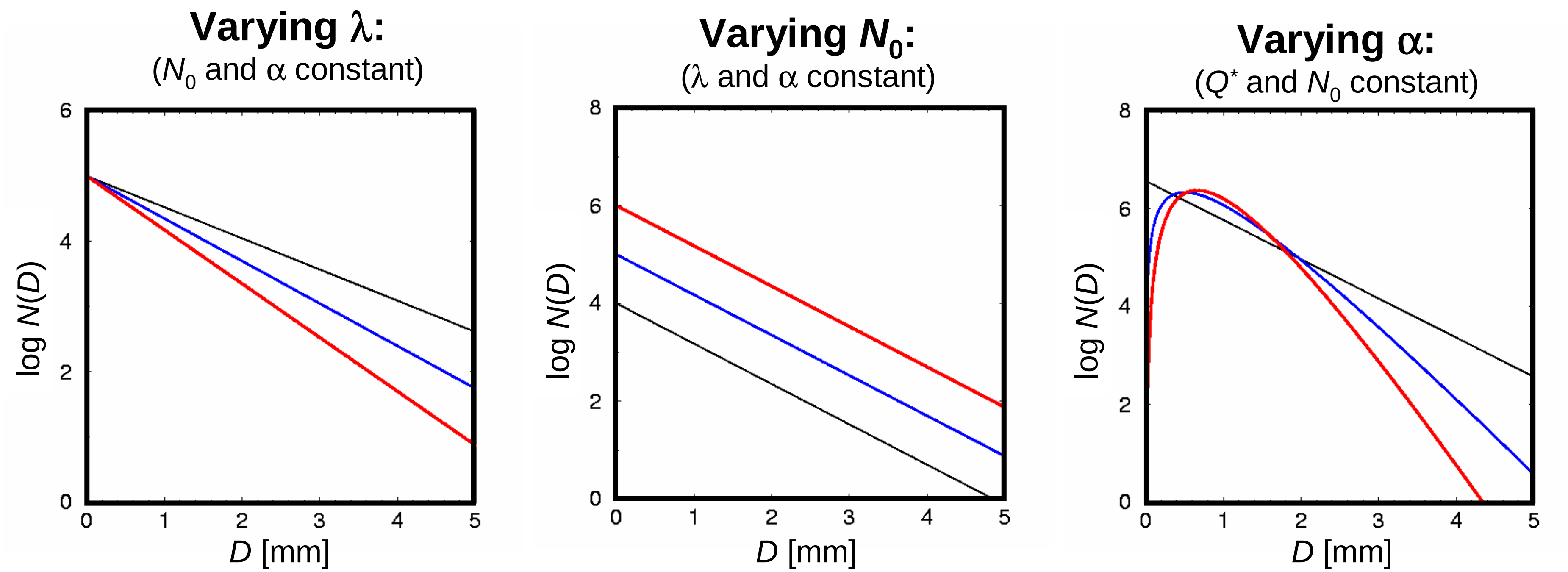
(*not to scale*)

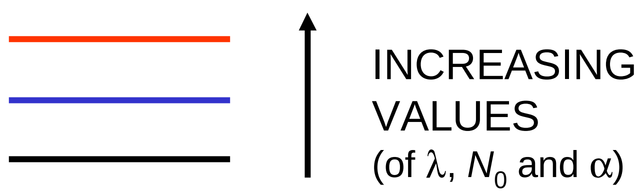


BULK METHOD

Gamma Distribution Function:

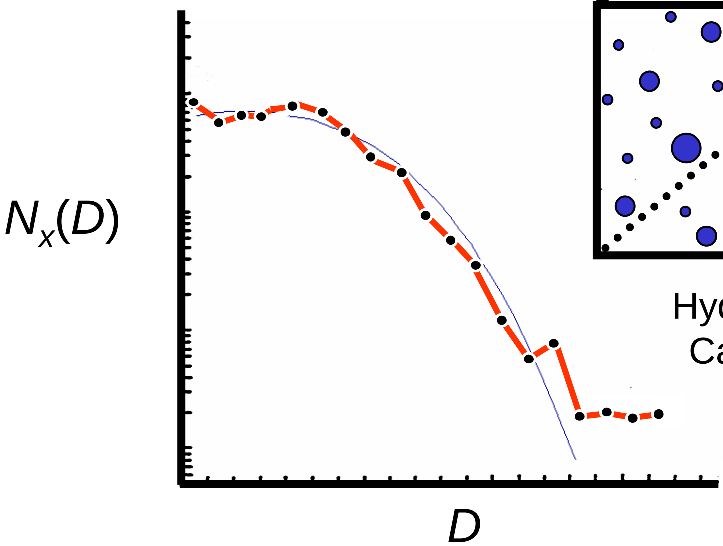
$$N(D) = N_0 D^{\alpha} e^{-\lambda D}$$

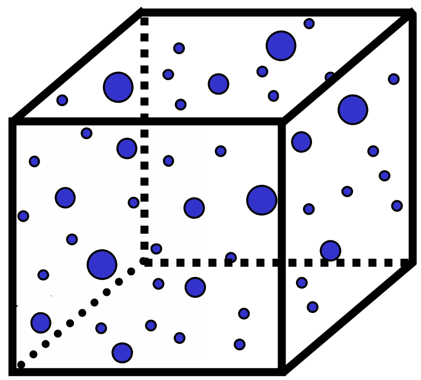




* $Q = \rho\, q$ (mass content)

BULK METHOD





Hydrometeor Category x

Total number concentration, N_{Tx}

$$N_{Tx} \equiv \int_0^{\infty} N_x(D)dD = M_x(0)$$

Mass mixing ratio, q_x

$$q_x \equiv \frac{\pi \rho_x}{6\rho} \int_0^{\infty} D^3 N_x(D)dD = \frac{\pi \rho_x}{6\rho} M_x(3)$$

Radar reflectivity factor, Z_x

$$Z_x \equiv \int_0^{\infty} D^6 N_x(D)dD = M_x(6)$$

***Size Distribution Function*:**

$$N_x(D) = N_{0x} D^{\alpha_x} e^{-\lambda_x D}$$

p^{th} moment:

$$M_x(p) \equiv \int_0^{\infty} D^p N_x(D)dD = N_{0x} \frac{\Gamma(1+\alpha_x + p)}{\lambda_x^{p+1+\alpha_x}}$$

BULK METHOD

Predict changes to specific moment(s)

e.g. q_x, N_{Tx}, ...

↓

Implies changes to values of parameters

i.e. N_{0x}, λ_x, ...

Total number concentration, N_{Tx}

$$N_{Tx} \equiv \int_0^\infty N_x(D)dD = M_x(0)$$

Mass mixing ratio, q_x

$$q_x \equiv \frac{\pi\rho_x}{6\rho}\int_0^\infty D^3 N_x(D)dD = \frac{\pi\rho_x}{6\rho}M_x(3)$$

Radar reflectivity factor, Z_x

$$Z_x \equiv \int_0^\infty D^6 N_x(D)dD = M_x(6)$$

***Size Distribution Function*:**

$$N_x(D) = N_{0x}D^{\alpha_x}e^{-\lambda_x D}$$

p^{th} moment:

$$M_x(p) \equiv \int_0^\infty D^p N_x(D)dD = N_{0x}\frac{\Gamma(1+\alpha_x+p)}{\lambda_x^{p+1+\alpha_x}}$$

BULK METHOD

Predict changes to specific moment(s)

e.g. q_x, N_{Tx}, ...

↓

Implies changes to values of parameters

i.e. N_{0x}, λ_x, ...

For every predicted moment, there is one prognostic parameter.

All other parameter are prescribed or diagnosed.

e.g.

One-moment scheme:

q_x is predicted;
$\rightarrow \lambda_x$ is prognosed
(N_{0x} and α_x are specified)

Two-moment scheme:

q_x and N_{Tx} are predicted;
$\rightarrow \lambda_x$ and N_{0x} are prognosed;
(α_x is specified)

Three-moment scheme:

q_x, N_{Tx} and Z_x are predicted;
$\rightarrow \lambda_x$, N_{0x} and α_x is prognosed

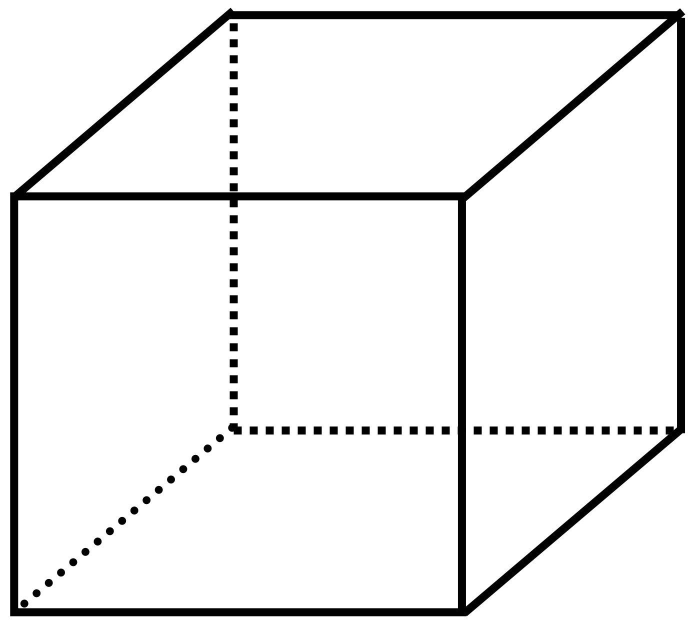
***Size Distribution Function*:**

$$N_x(D) = N_{0x} D^{\alpha_x} e^{-\lambda_x D}$$

p^{th} moment:

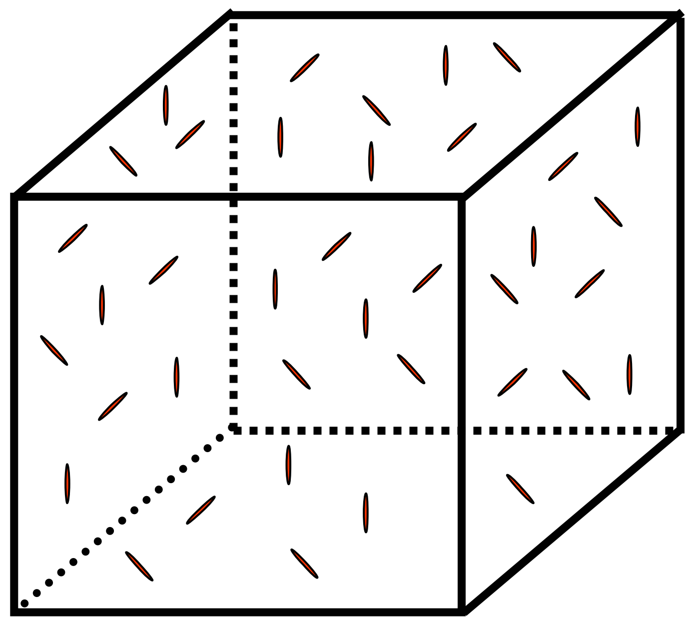
$$M_x(p) \equiv \int_0^\infty D^p N_x(D) dD = N_{0x} \frac{\Gamma(1+\alpha_x+p)}{\lambda_x^{p+1+\alpha_x}}$$

$T < 0°C$ *



* (May contain traces of supercooled water)

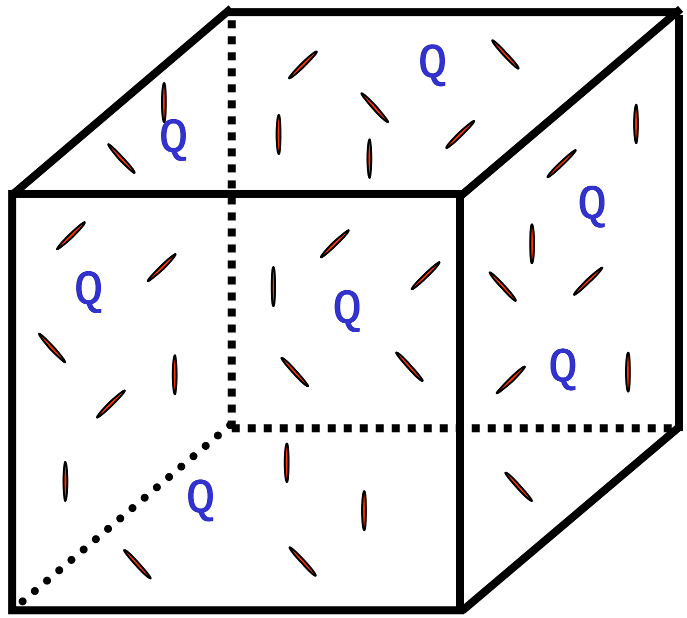
$T < 0°C$



= ICE CRYSTAL

(May contain traces of supercooled water)

T < 0°C

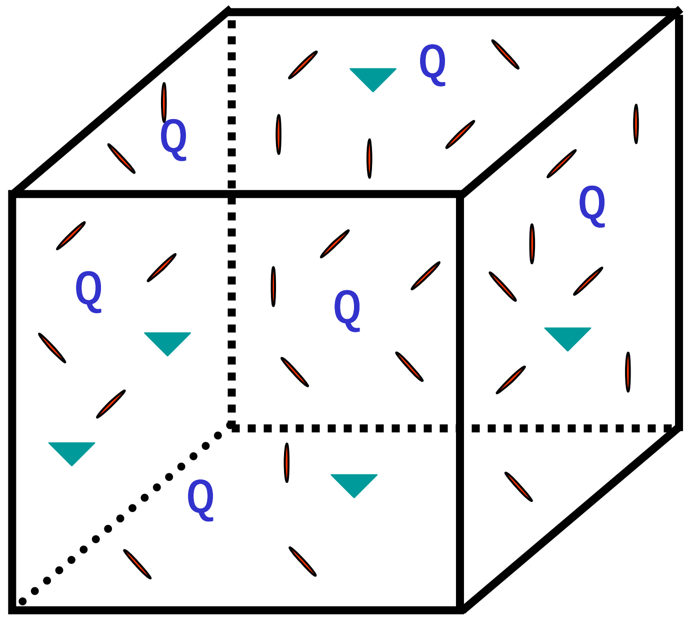


= ICE CRYSTAL

Q = SNOW CRYSTAL / AGGRETATE

(May contain traces of supercooled water)

$T < 0°C$



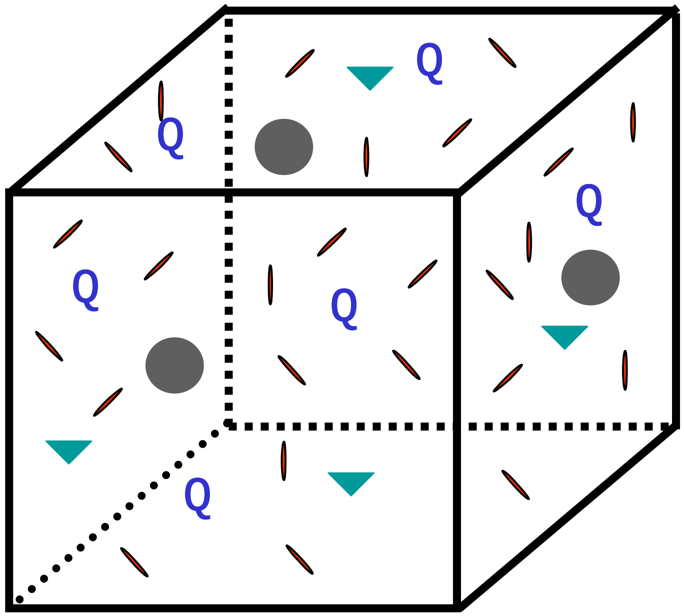
= ICE CRYSTAL

Q = SNOW CRYSTAL / AGGREGATE

= GRAUPEL

(May contain traces of supercooled water)

T < 0°C



= ICE CRYSTAL

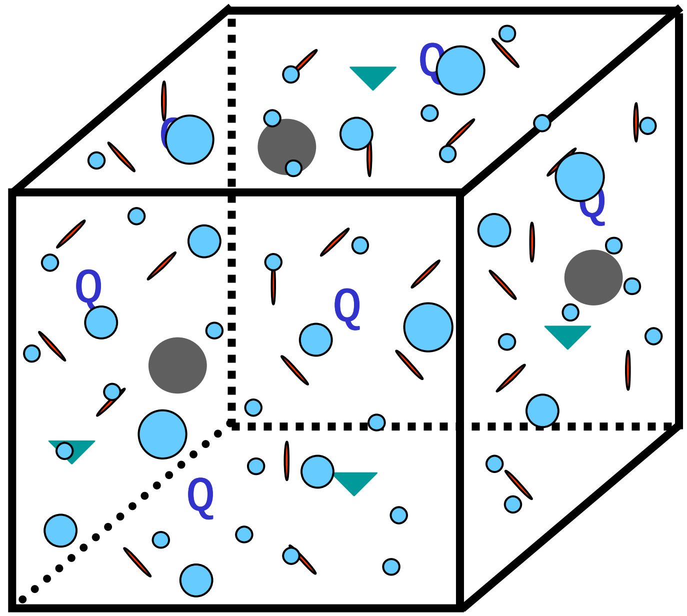
Q = SNOW CRYSTAL / AGGREGATE

= GRAUPEL

= HAIL

(May contain traces of supercooled water)

T < 0°C



= ICE CRYSTAL

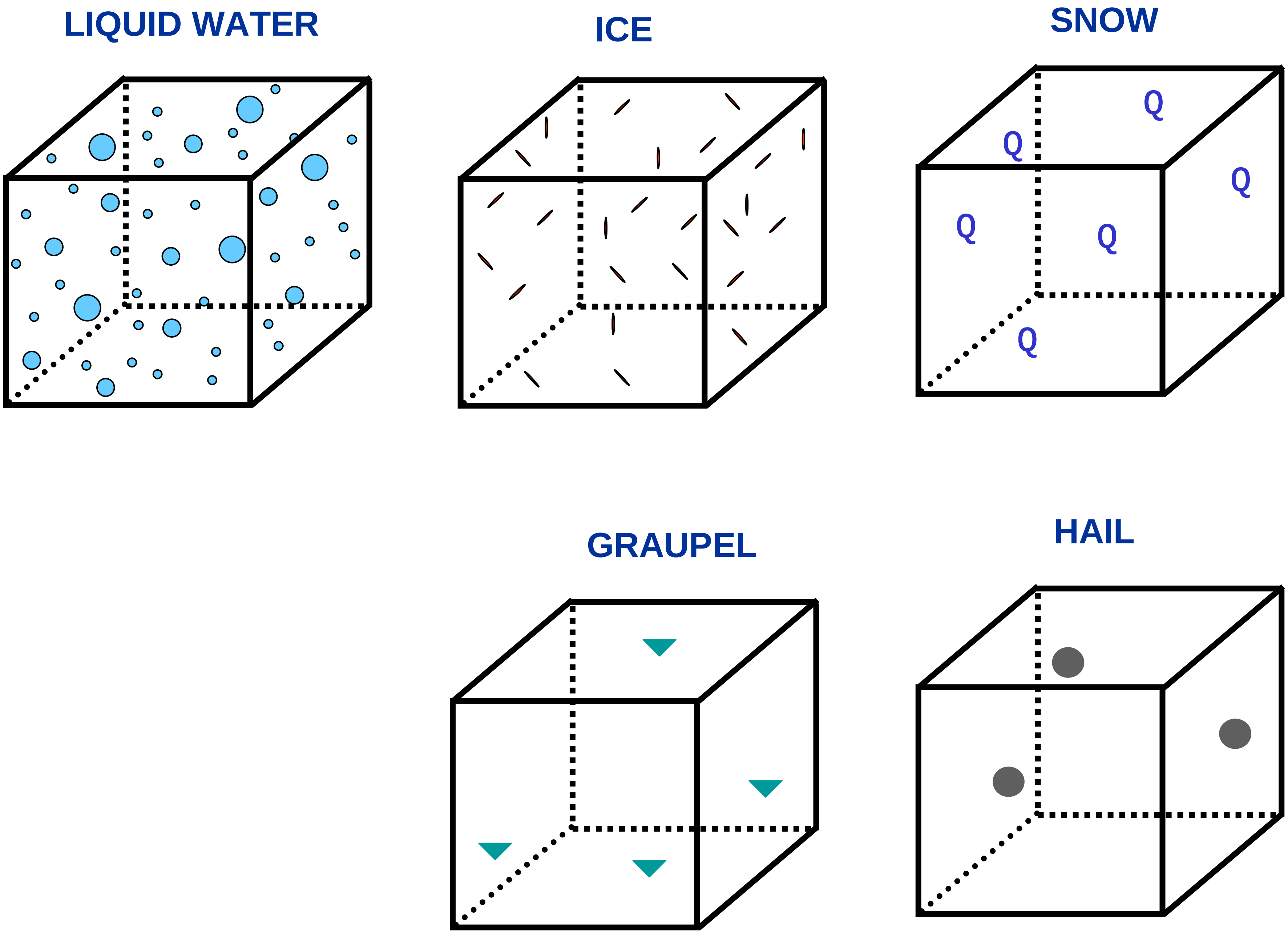
Q = SNOW CRYSTAL / AGGREGATE

= GRAUPEL

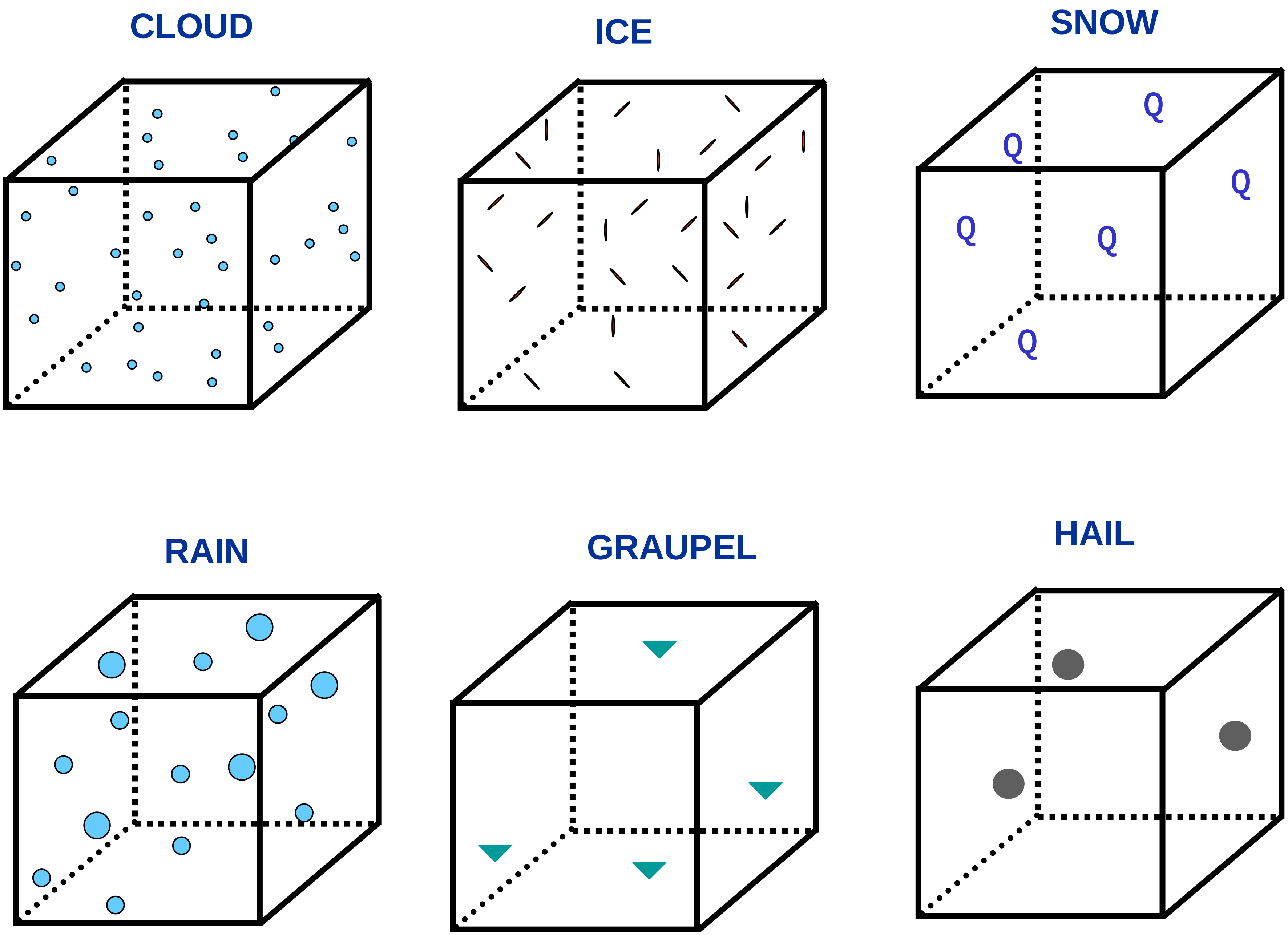
= HAIL

= LIQUID WATER

PARTITIONING THE HYDROMETEOR SPECTRUM



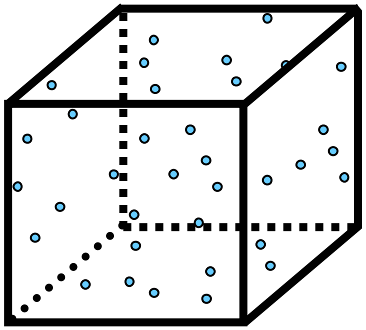
PARTITIONING THE HYDROMETEOR SPECTRUM



BULK METHOD

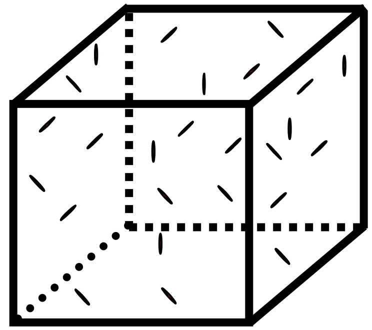
PARTITIONING THE HYDROMETEOR SPECTRUM

CLOUD



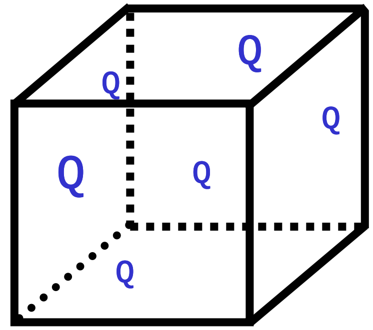
$$N_c(D) = N_{0c} D^{\nu_c(1+\alpha_c)-1} \exp\left[-(\lambda_c D)^{\nu_c}\right]$$

ICE



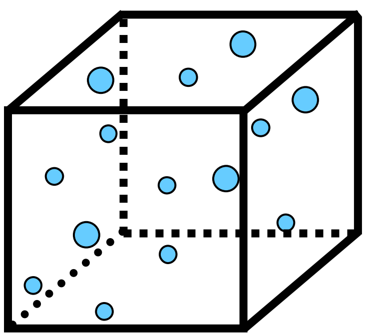
$$N_i(D) = N_{0i} D^{\alpha_i} e^{-\lambda_i D}$$

SNOW



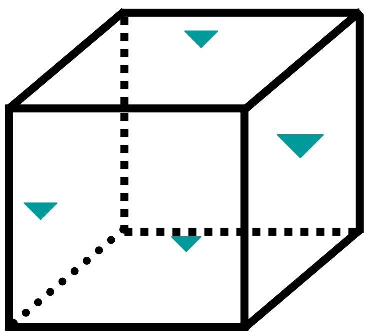
$$N_s(D) = N_{0s} D^{\alpha_s} e^{-\lambda_s D}$$

RAIN



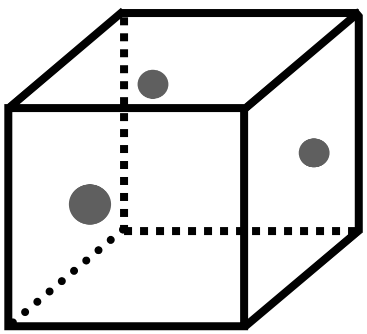
$$N_r(D) = N_{0r} D^{\alpha_r} e^{-\lambda_r D}$$

GRAUPEL



$$N_g(D) = N_{0g} D^{\alpha_g} e^{-\lambda_g D}$$

HAIL



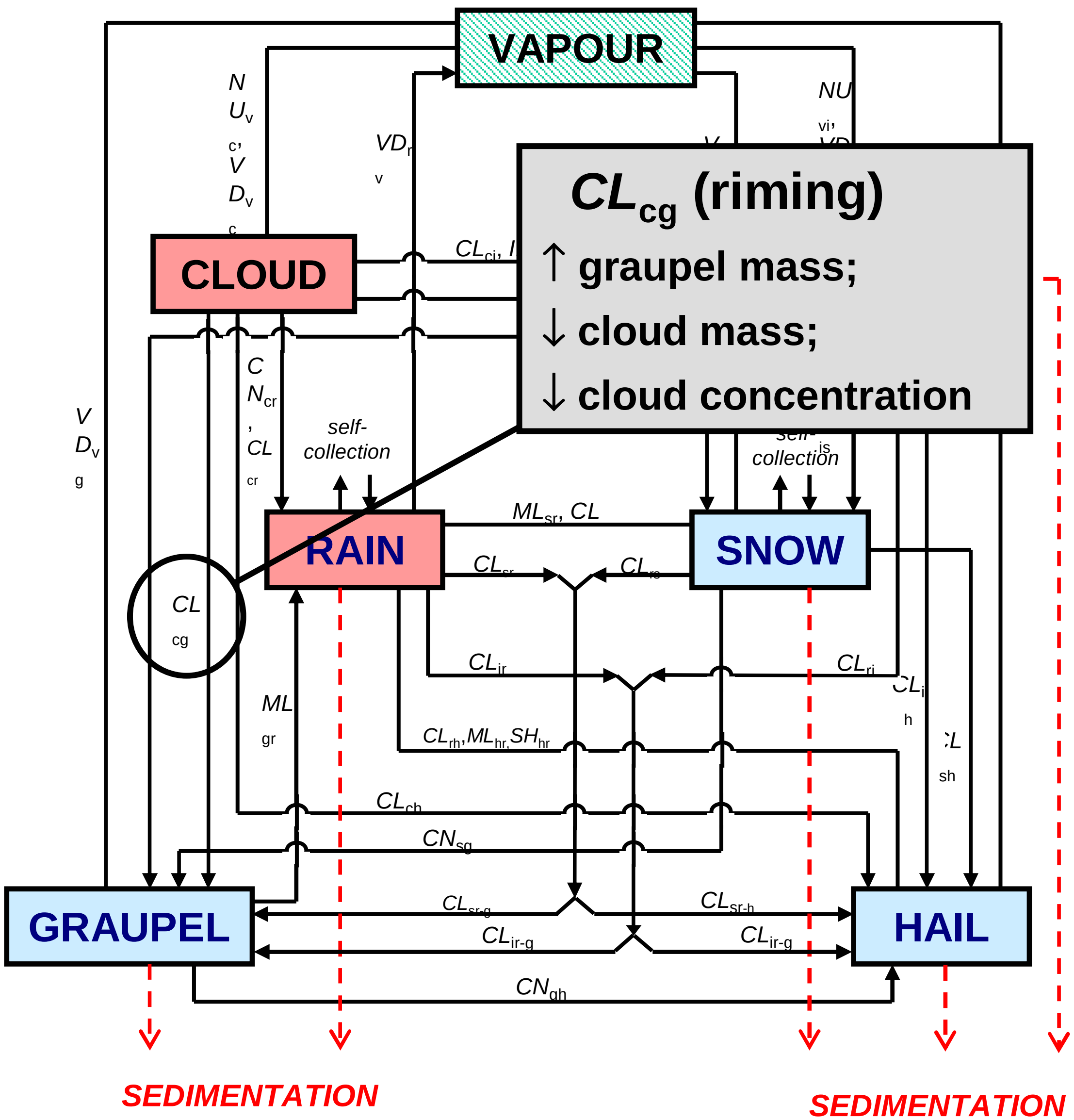
$$N_h(D) = N_{0h} D^{\alpha_h} e^{-\lambda_h D}$$

The Multi-Moment Microphysics Scheme *

- Six hydrometeor categories:
  - 2 liquid: **cloud** and **rain**
  - 4 frozen: **ice**, **snow**, **graupel** and **hail**
- ~50 distinct microphysical processes
- warm-rain scheme based on Cohard and Pinty (2000a)
- ice-phase based on Murakami (1990), Ferrier (1994), Meyers et al. (1997), Reisner et al. (1998), etc.
- **diagnostic-α_x** relations added for double-moment version*
- **predictive equations for Z_x** added for triple-moment version*

* **Milbrandt and Yau, 2004 [*J. Atmos. Sci.* (accepted)]**

The Multi-Moment Microphysics Scheme



Hydrometeor Categories:

LIQUID	**Density**	**Fallspeed***
Cloud	**1000 kg m^{-3}**	**---**
Rain	**1000 kg m^{-3}**	**6.0 m s^{-1}**
SOLID		
Ice	**500 kg m^{-3}**	**0.5 m s^{-1}**
Snow	**100 kg m^{-3}**	**1.5 m s^{-1}**
Graupel	**400 kg m^{-3}**	**3.0 m s^{-1}**
Hail	**900 kg m^{-3}**	**10.0 m s^{-1}**

*** e.g. for a mass content of 1.0 g m^{-3} and a prescribed concentration**

$$N_{\mathbf{x}}(D) = N_{0\mathbf{x}} D^{\alpha_{\mathbf{x}}} e^{-\lambda_{\mathbf{x}} D}$$

(for **x** = c, r, i, s, g, h)

2. EVALUATION OF BULK APPROACHES – COMPARISON TO ANALYTIC SOLUTIONS

Evaluation of the various bulk methods

How many moments should be used?

PREMISE:

The most important function of a microphysics scheme (in NWP) is to <u>predict hydrometeor mass.</u>

DRIVING QUESTION:

How much better are higher-moment schemes at predicting hydrometeor mass?

DRIVING QUESTION: How much better are higher-moment schemes at predicting hydrometeor mass?

Mass continuity equation:

$$\frac{\partial q_x}{\partial t} = -\frac{1}{\rho}\vec{\nabla}\cdot\left(\rho q_x \vec{U}\right) + TURB(q_x) + \frac{1}{\rho}\frac{\partial}{\partial z}\left(\rho q_x \overline{V}_{xq}\right) + \left.\frac{dq_x}{dt}\right|_S$$

ADVECTION / COMPRESSION — TURBULENT MIXING — SEDIMENTATION — SOURCES / SINKS

MODIFIED DRIVING QUESTION:

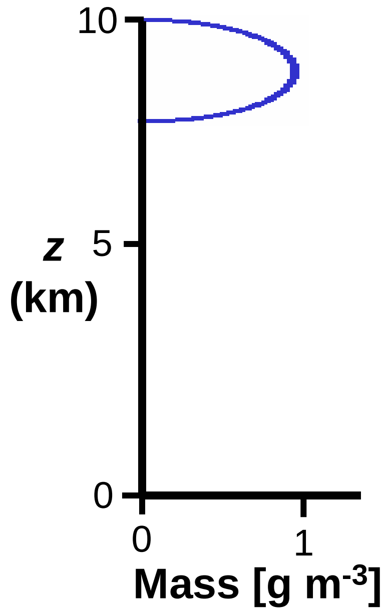
How much better are higher-moment schemes at predicting SEDIMENTATION and SOURCES/SINKS ?

SEDIMENTATION:

Analytic bin model calculation:

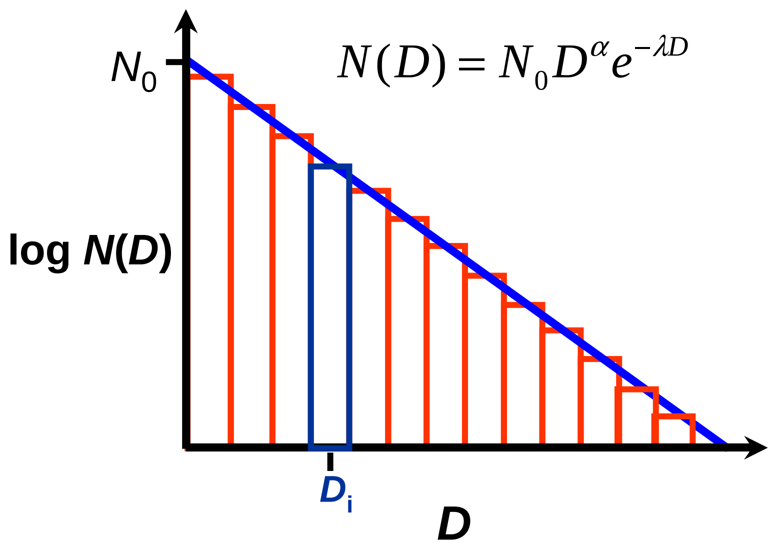
(1D column)

1. Prescribe $Q(z)$:



2. Compute $N(D_i, z)$:

[from a prescribed distribution]



3. Compute locations of each particle after sedimentation for time t:

For every size bin i:

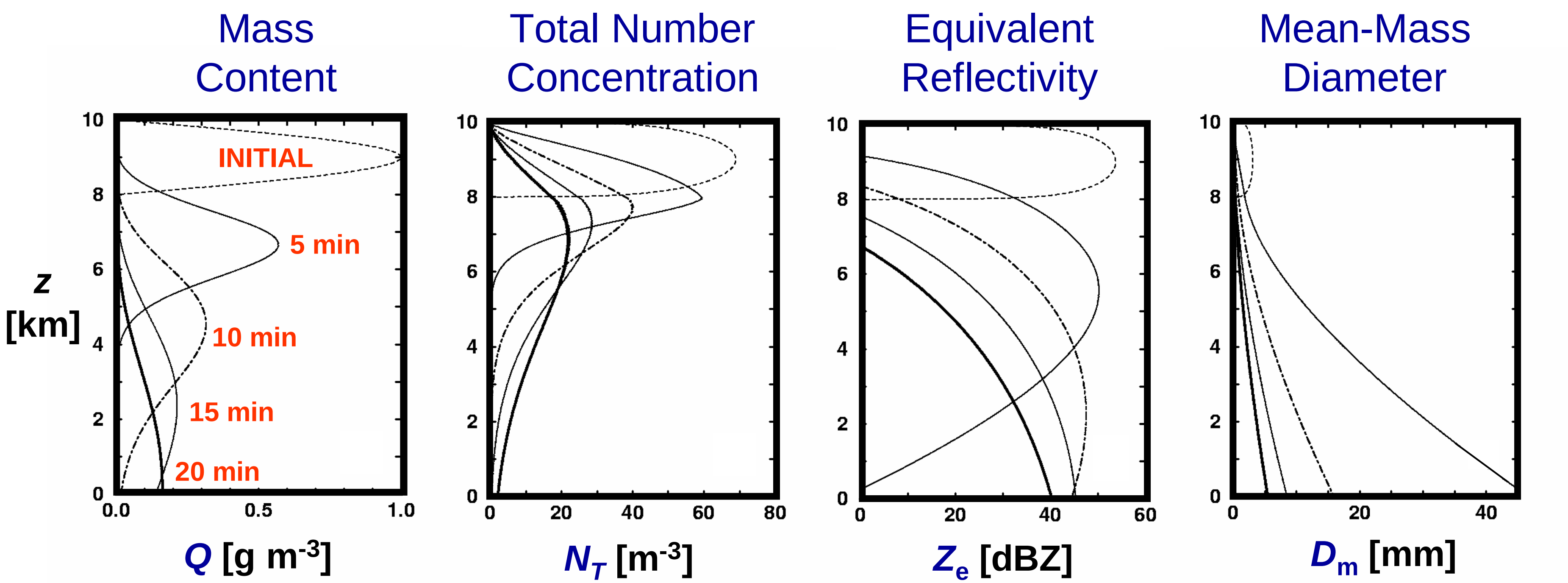
$$V_i(D_i) = aD^b$$

$$z_i(t) = z_i(0) - V_i(D_i) \cdot t$$

SEDIMENTATION:

Analytic bin model calculation:

(1D column)



Contours every 5 min

SEDIMENTATION: Bulk scheme

$$\left.\frac{\partial q_x}{\partial t}\right|_{SEDI} = \frac{\partial\left(\rho q_x \overline{V}_{xq}\right)}{\partial z}$$

$\overline{V}_{xq}$ = mass-weighted fall velocity

$$\left.\frac{\partial N_x}{\partial t}\right|_{SEDI} = \frac{\partial\left(N_x \overline{V}_{xN}\right)}{\partial z}$$

$\overline{V}_{xN}$ = number-weighted fall velocity

$$\left.\frac{\partial Z_x}{\partial t}\right|_{SEDI} = \frac{\partial\left(Z_x \overline{V}_{xZ}\right)}{\partial z}$$

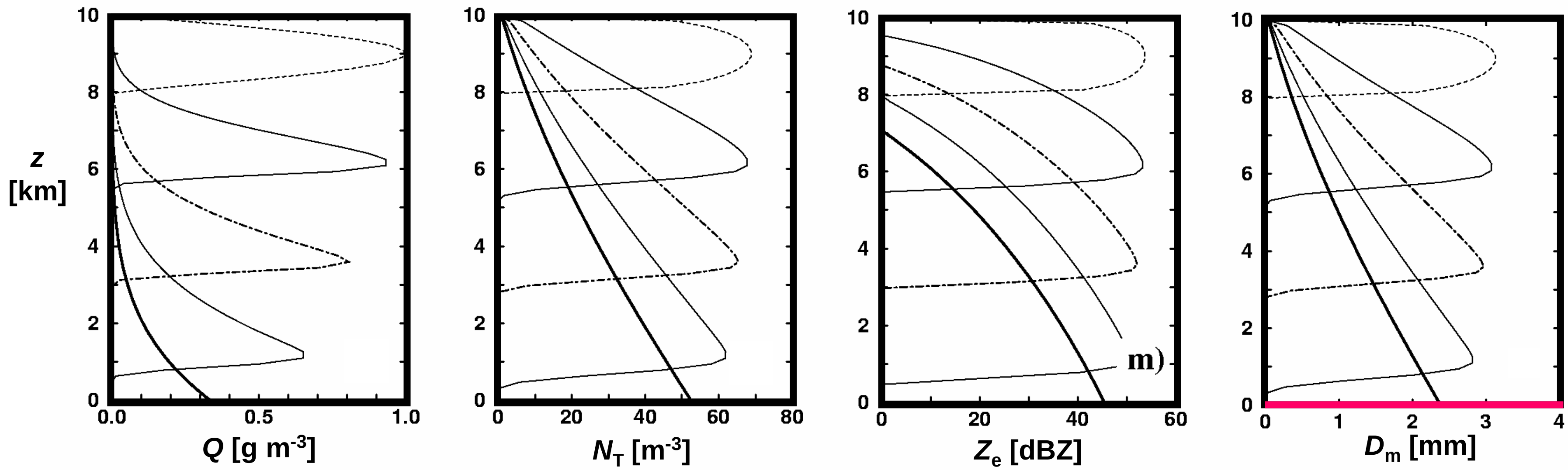
$\overline{V}_{xZ}$ = reflectivity-weighted fall velocity

SM (mass equation)

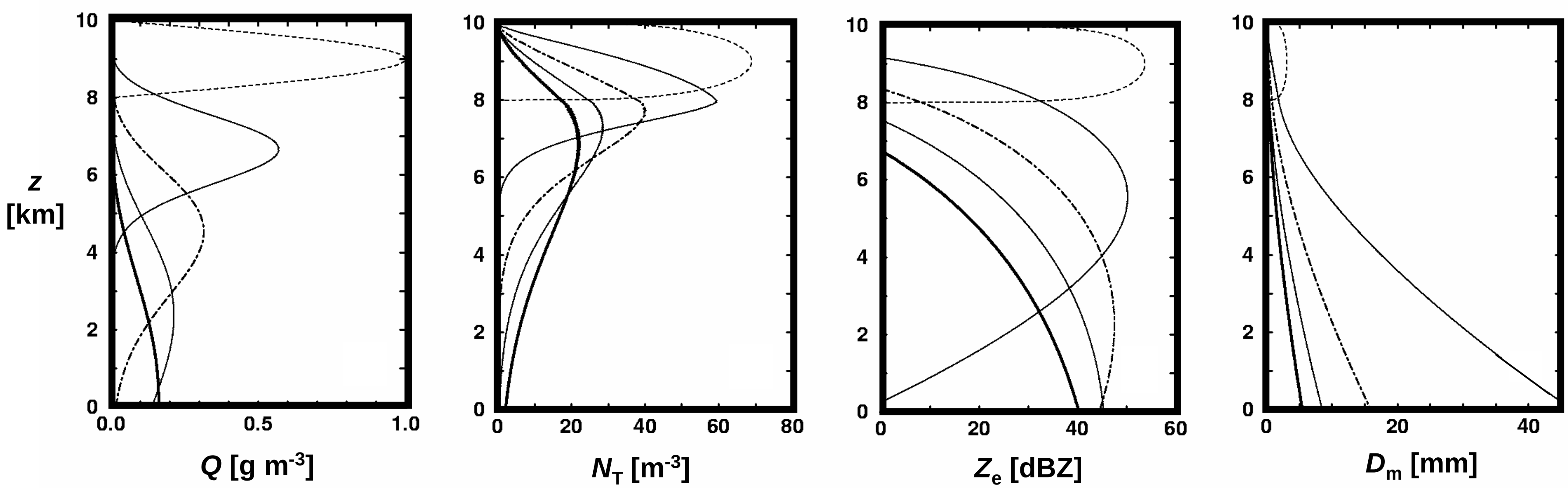
DM (mass and number equations)

TM (mass, number and reflectivity equations)

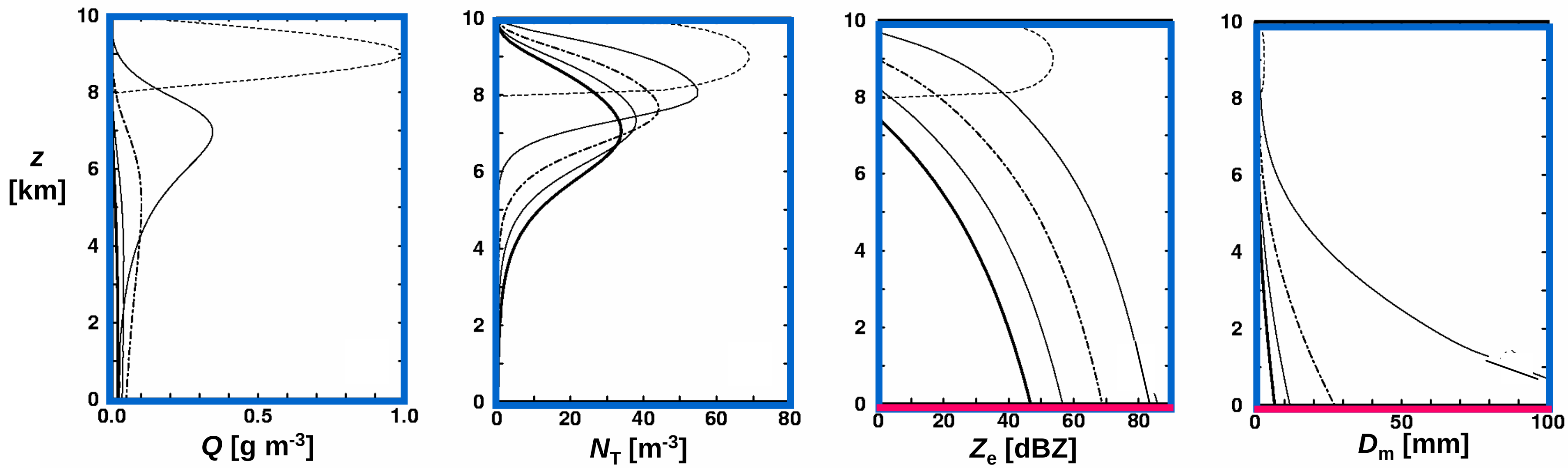
SINGLE-moment scheme (SM):



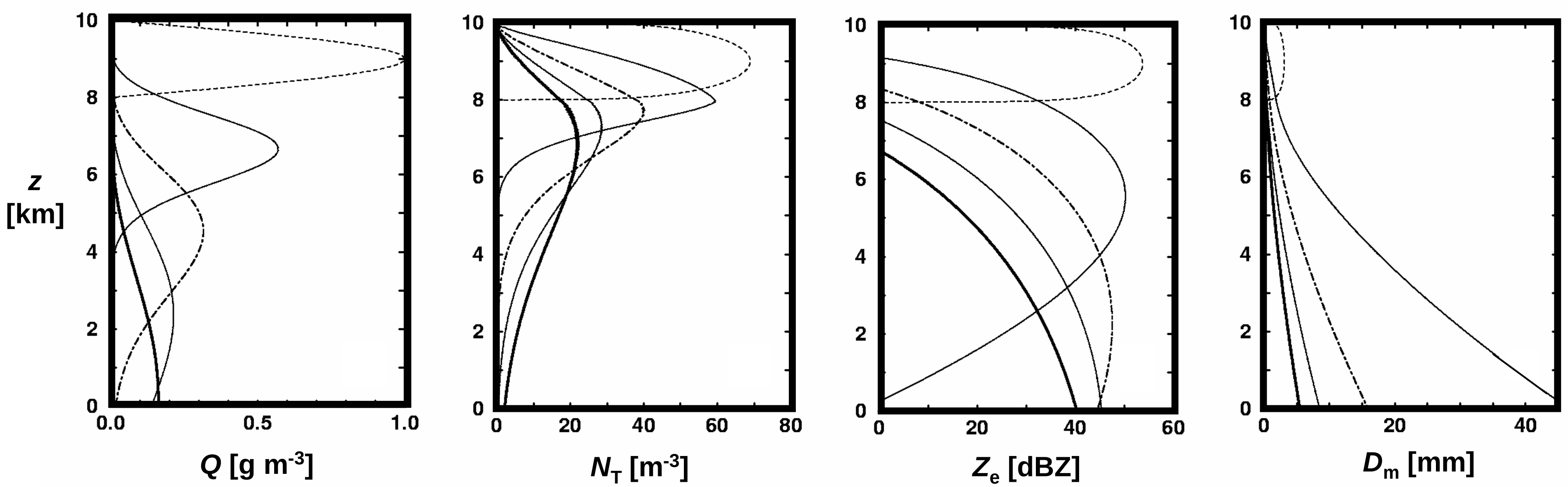
ANALYTIC BIN model (ANA):



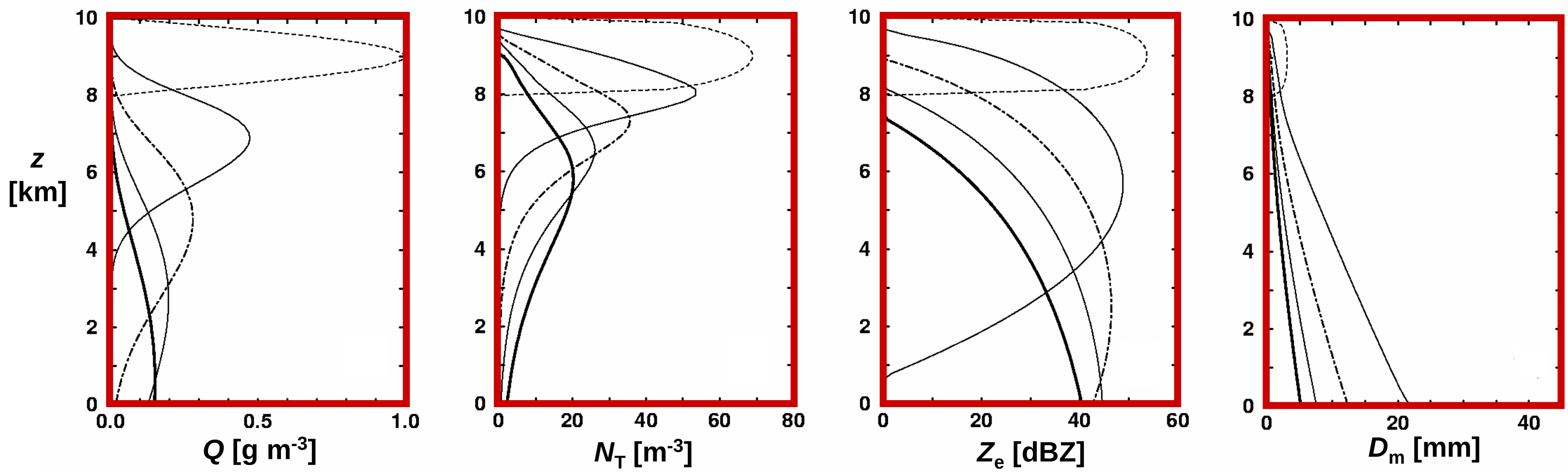
DOUBLE-moment scheme, fixed α = 0 (FIX0):



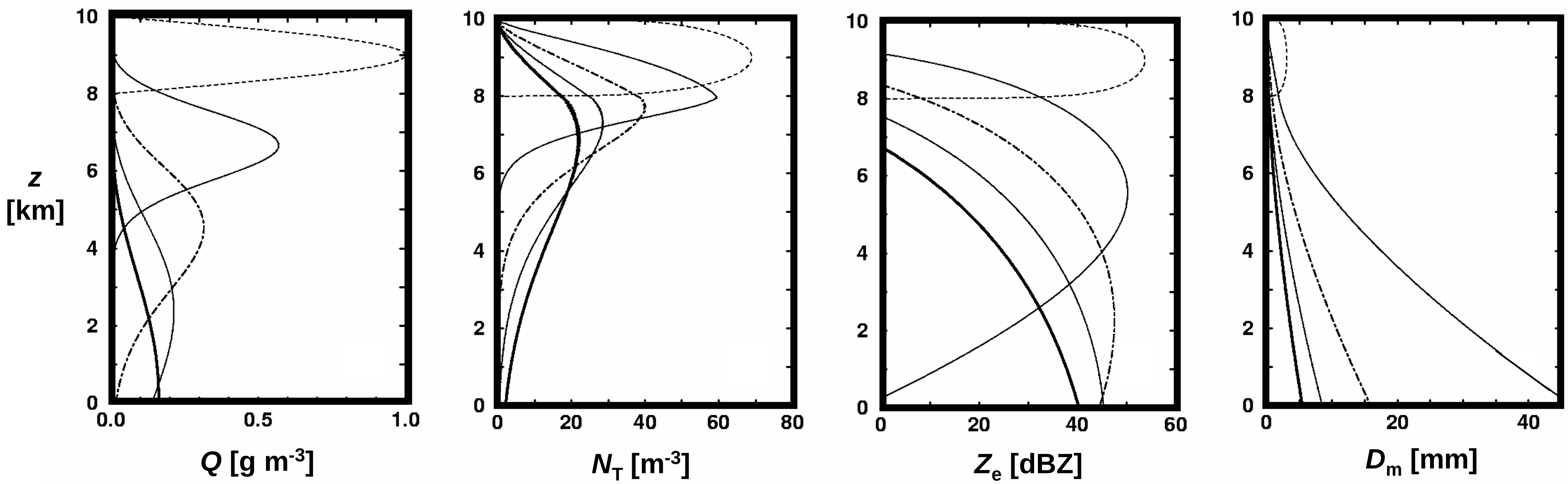
ANALYTIC BIN model (ANA):



TRIPLE-moment bulk scheme (TM):

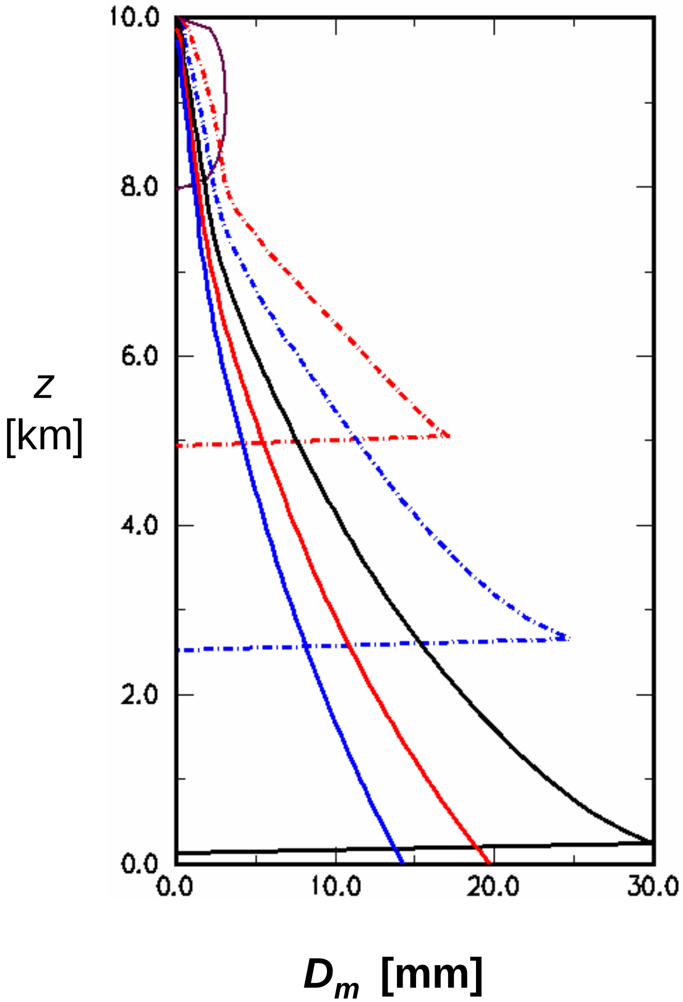


ANALYTIC BIN model (ANA):

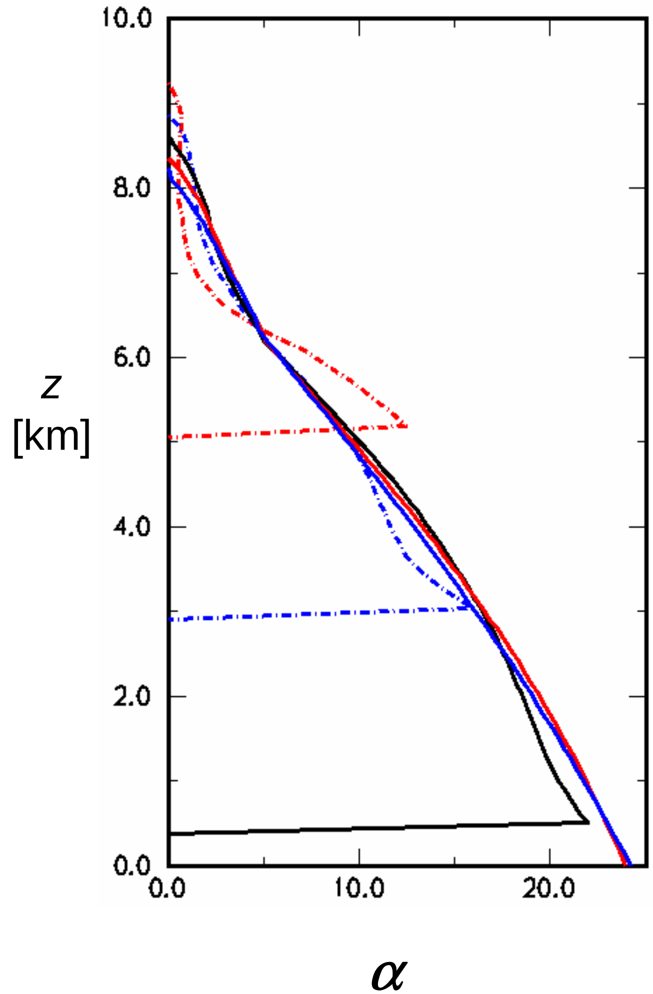


From TRIPLE- MOMENT sedimentation profiles:

Mean-mass Diameter

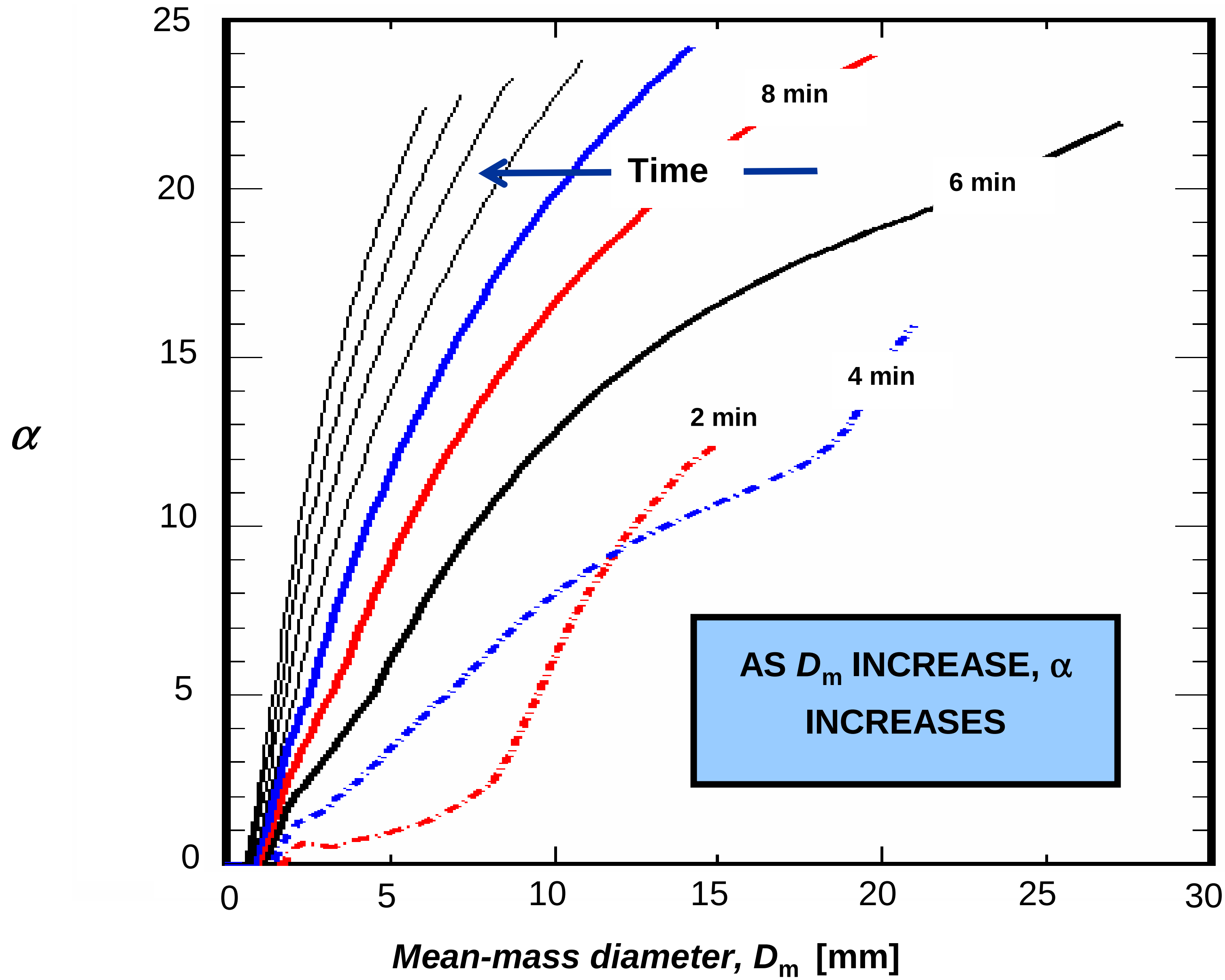


Shape Parameter

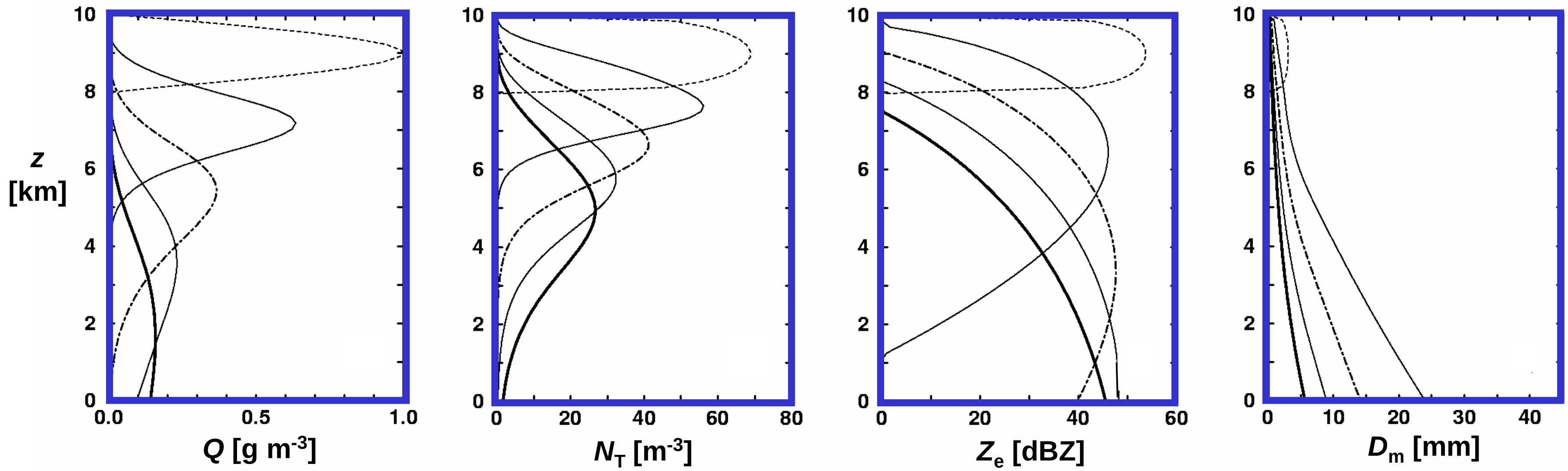


Contours every 2 min

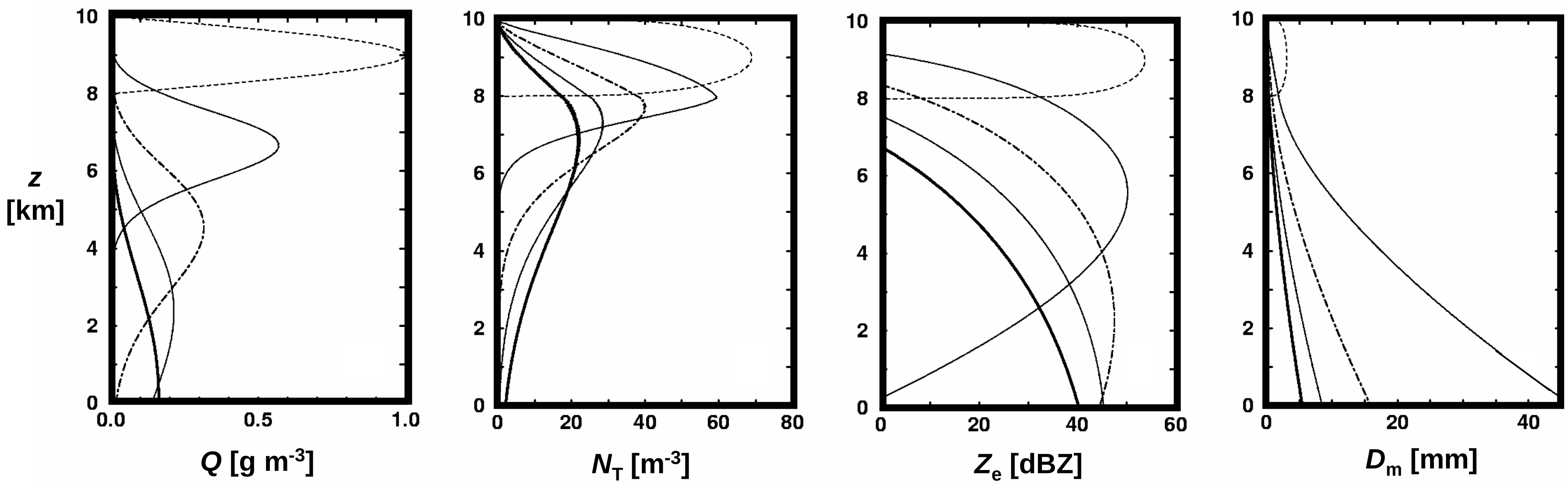
From TRIPLE- MOMENT sedimentation profiles:



DOUBLE-moment scheme, $\alpha = f(D_m)$ (DIAG):

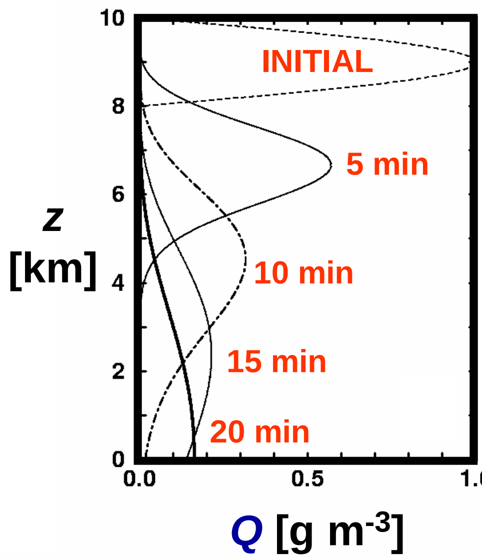


ANALYTIC BIN model (ANA):



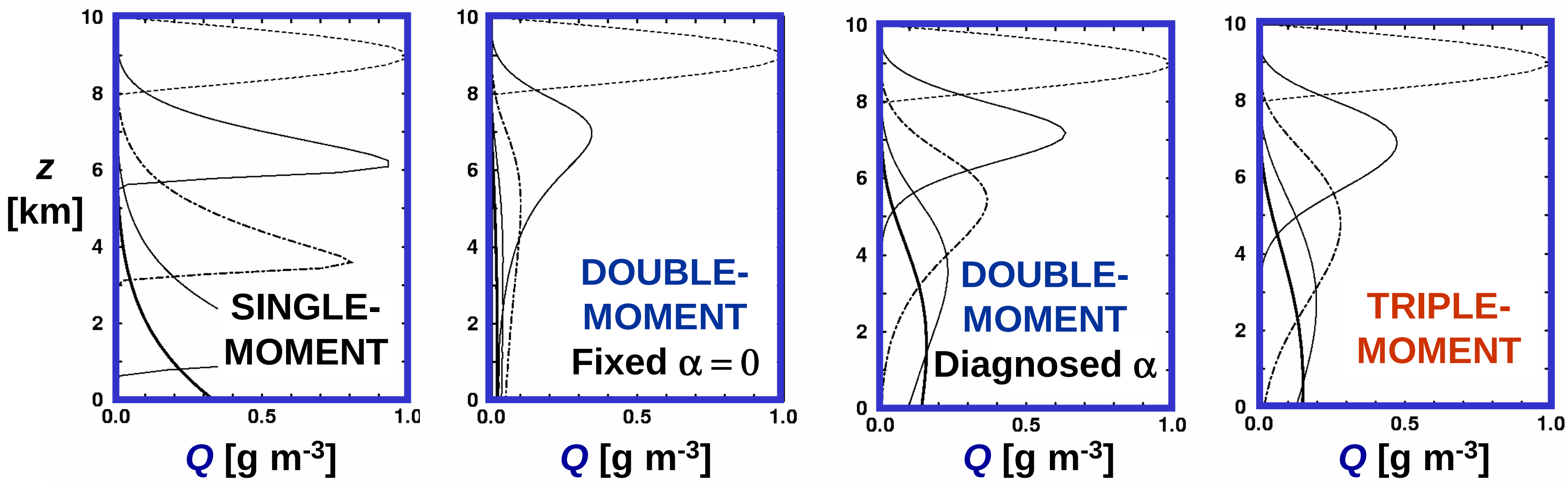
SEDIMENTATION ***BULK* vs. *ANALYTIC***

Analytic model:



Mass Content

Bulk schemes:



GROWTH RATES

MICROPHYSICS SOURCES/SINKS

CONTINUOUS COLLECTION OF CLOUD WATER (CL_{cx}):

$$\left.\frac{dq_x}{dt}\right|_{CL} = \int_0^\infty \left.\frac{dm(D)}{dt}\right|_{CL} N(D)dD$$

$$\left.\frac{dm(D)}{dt}\right|_{CL} = \frac{\pi D^2}{4} V(D) E_{xc} \rho q_c = \left(\frac{\pi}{4} E_{xc} \rho q_c\right) D^{2+b_x}$$

$$\left.\frac{dq_x}{dt}\right|_{CL} = \left(\frac{\pi}{4} E_{xc} \rho q_c\right) \int_0^\infty D^{2+b_x} N(D)dD$$

$$\left.\frac{dq_x}{dt}\right|_{CL} \propto M_{2+b_x}$$

$$\left[M_x(p) \equiv \int_0^\infty D^p N_x(D)dD \right]$$

The p^{th} moment of $N_x(D)$

GROWTH RATES

A scheme's ability to predict the **growth rates** depends on its ability to compute the value of **certain moments** [ranging from $M_x(b_x)$ to $M_x(2+b_x)$]

e.g. The **accretion rate** for hail (***CL*****ch**) is proportional to $\mathbf{M_h(2.6)}$

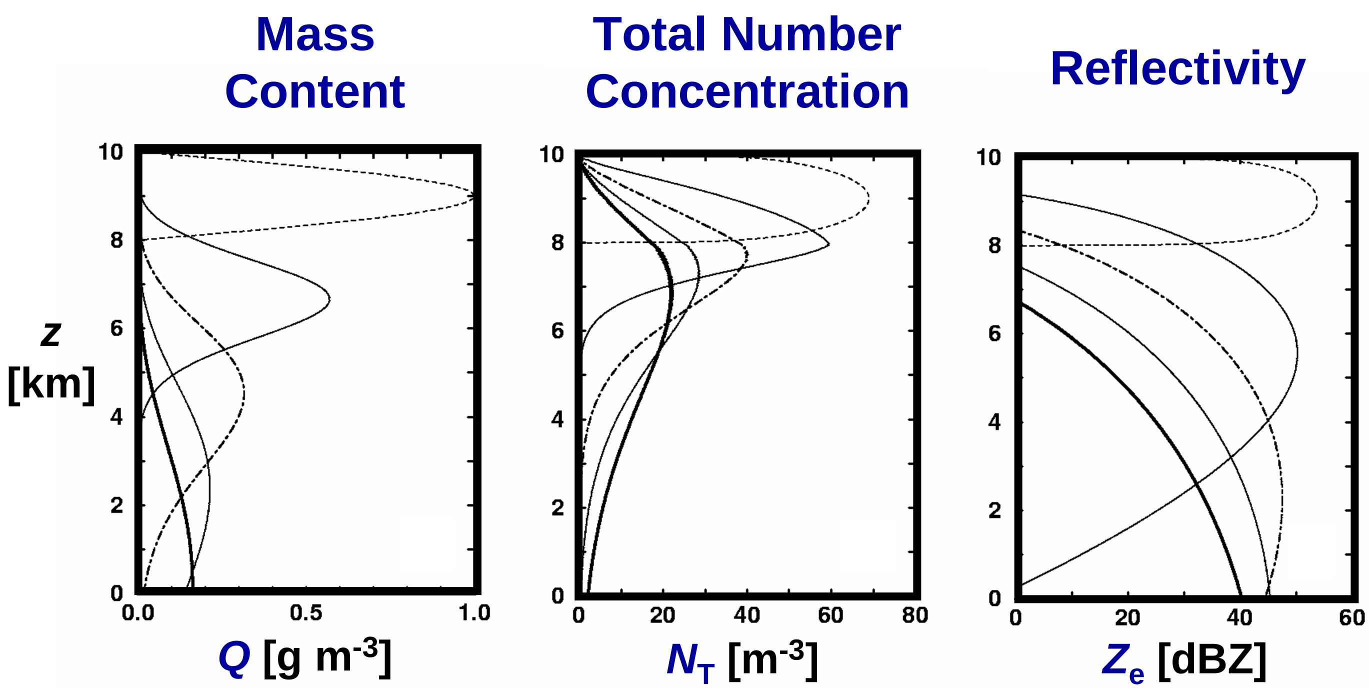
$$V_x(D) = \gamma a_x D^{b_x}$$

HAIL
$b_x \approx 0.6$

GROWTH RATES

RECALL:

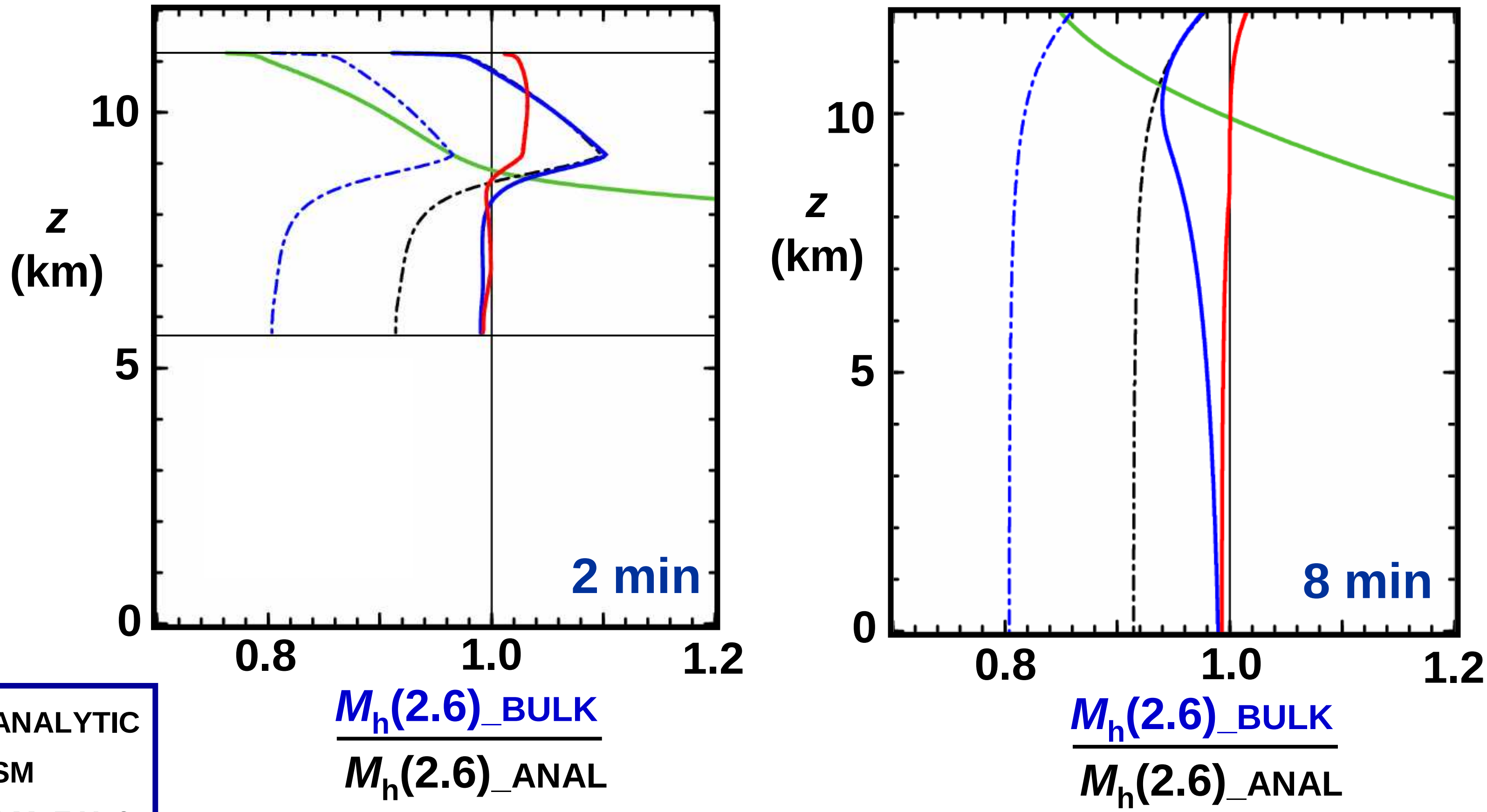
Analytic bin model calculation for sedimentation:

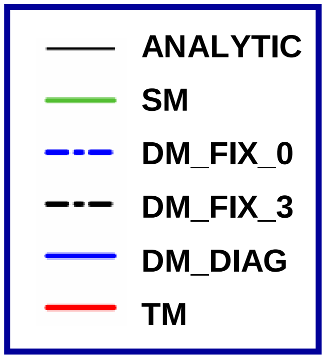


At each level: $\left. \begin{array}{c} Q \\ N_T \\ Z \end{array} \right\} \Rightarrow \left\{ \begin{array}{c} N_0 \\ \lambda \\ \alpha \end{array} \right. \Rightarrow$ *M* (2.6)_BULK

GROWTH RATES (e.g. CL_{ch})

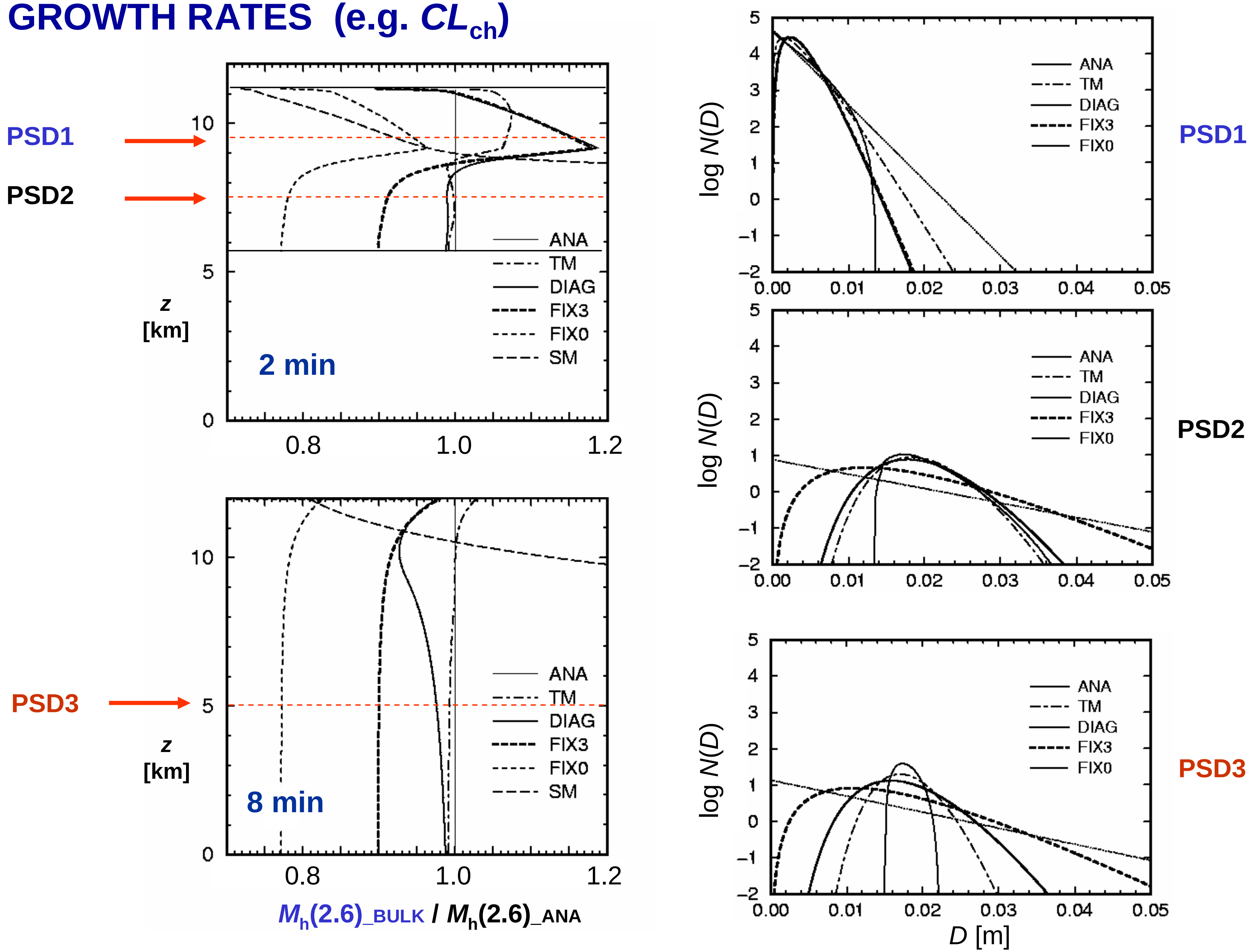
Ratio of Growth Rates: $\frac{CL_{ch\_}BULK}{CL_{ch\_}ANAL} = \frac{M_h(2.6)\_BULK}{M_h(2.6)\_ANAL}$





(e.g. A **ratio of 0.95** ⇒ growth rate **underestimated by 5%**)

GROWTH RATES (e.g. CL_{ch})



3. EVALUATION OF BULK APPROACHES – COMPARISON OF 3D SIMULATIONS

PART I: TRIPLE-MOMENT CONTROL RUN

CONTROL SIMULATION

MODEL:

Canadian MC2 mesoscale model (v4.9.5)

- non-hydrostatic, fully compressible
- initialized with GEM-24 km regional analysis
- triply-nested to 1-km grid
- interfaced with new microphysics scheme
  (triple-moment version for CONTROL SIMULATION)

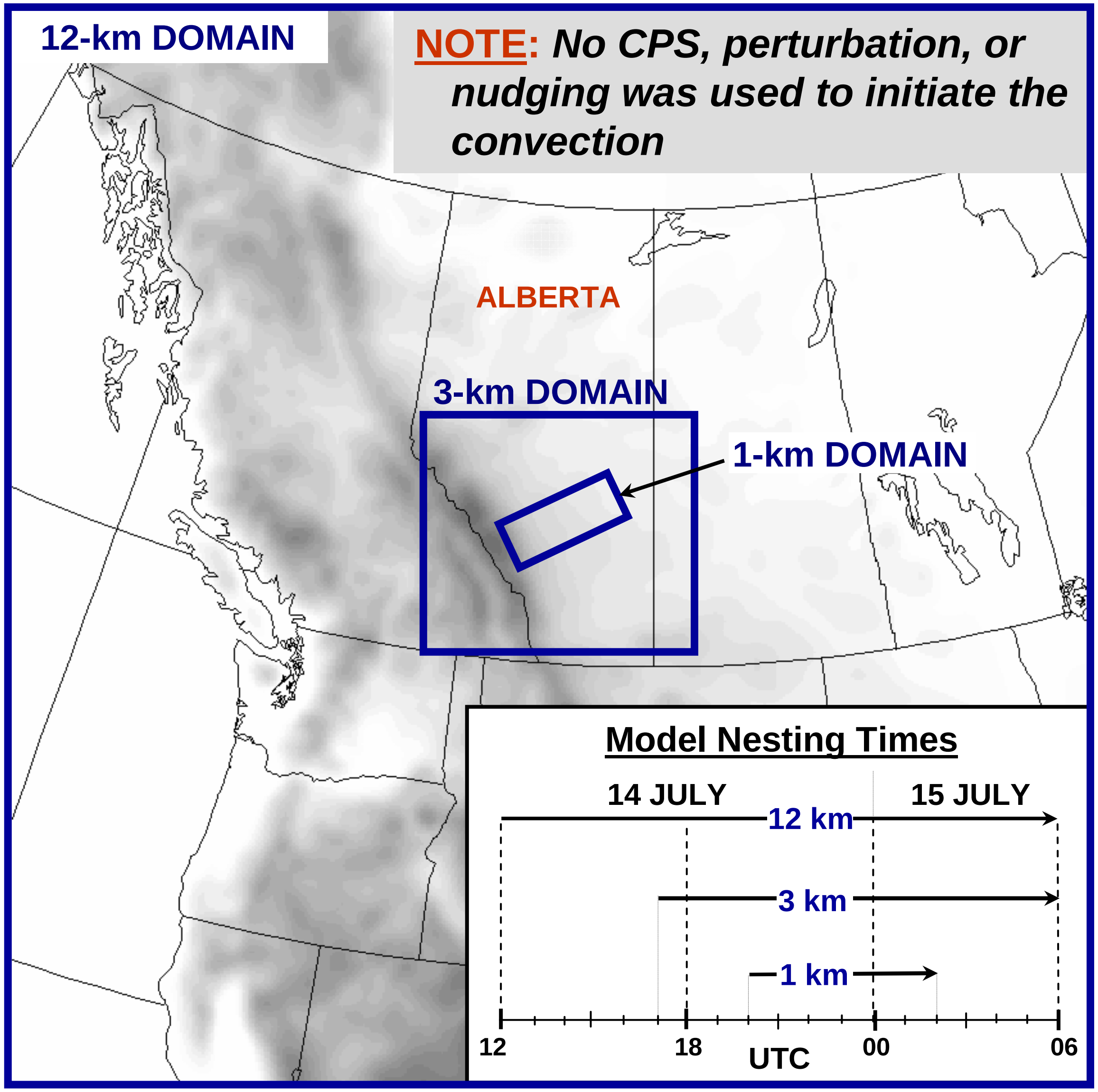
CASE:

14 July 2000 “Pine Lake storm”, Alberta, Canada

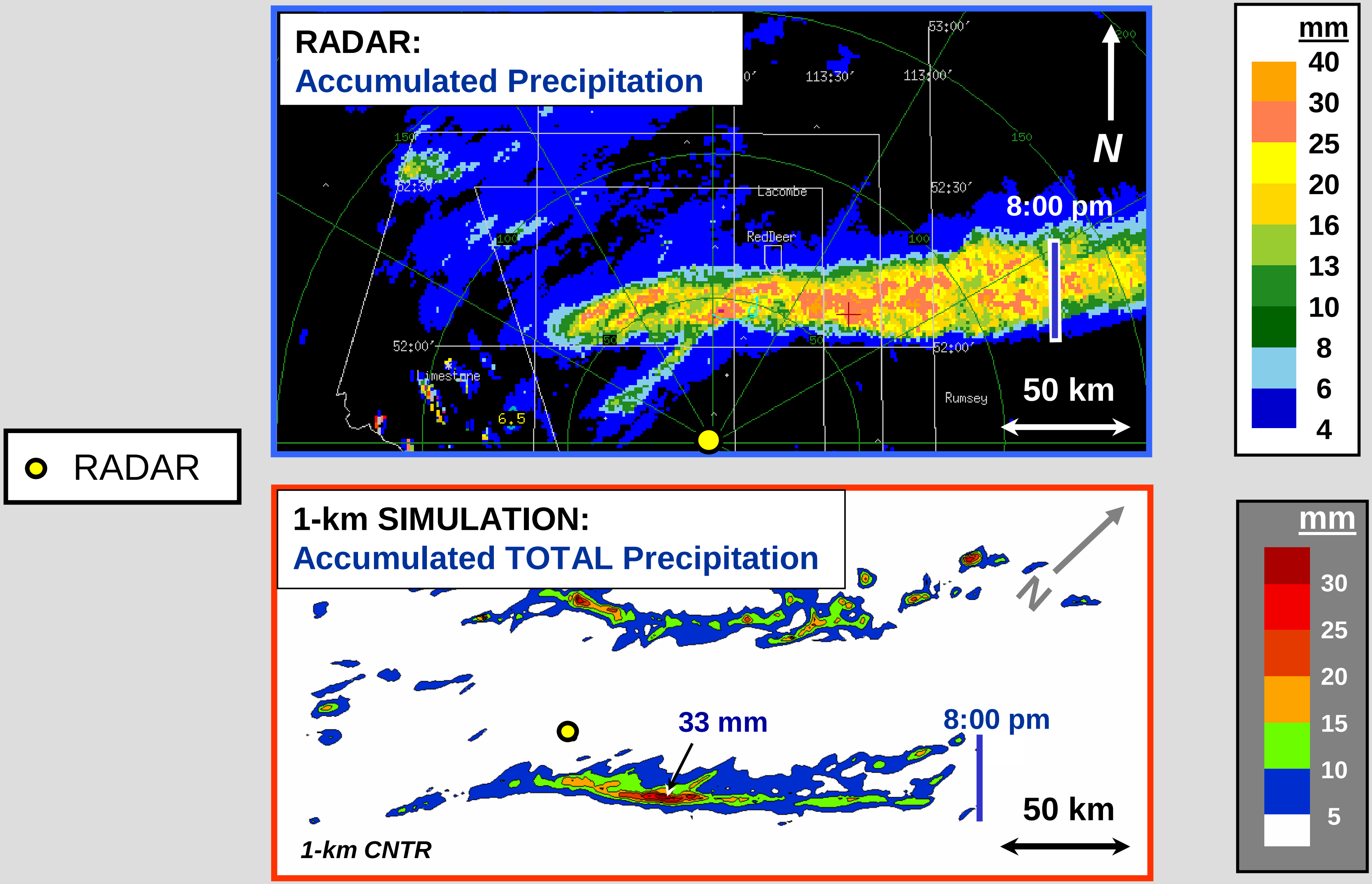
- long-lasting supercell
- well-observed by nearby radar
- large hail observed

CONTROL SIMULATION

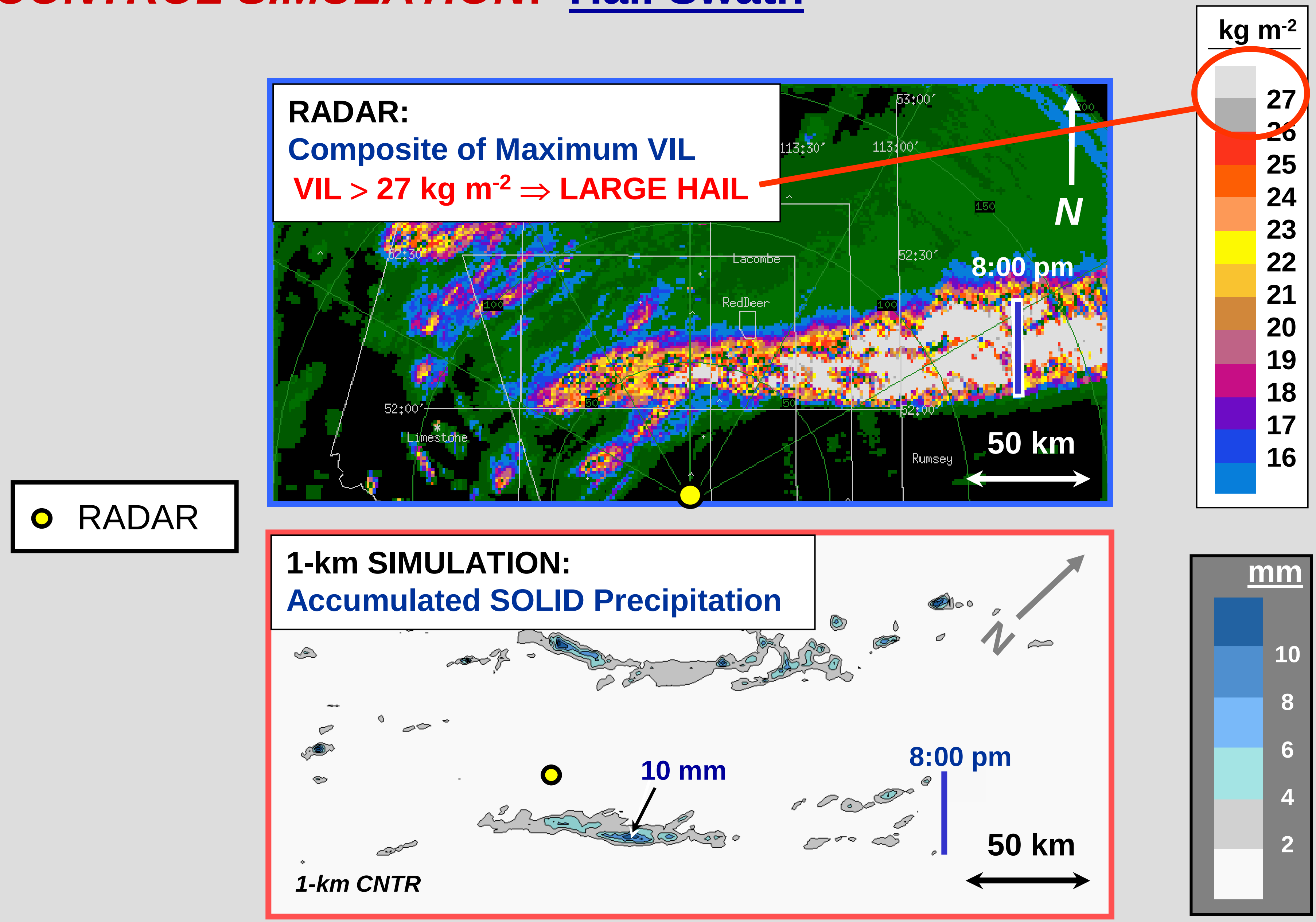
MC2 Grid Configuration:



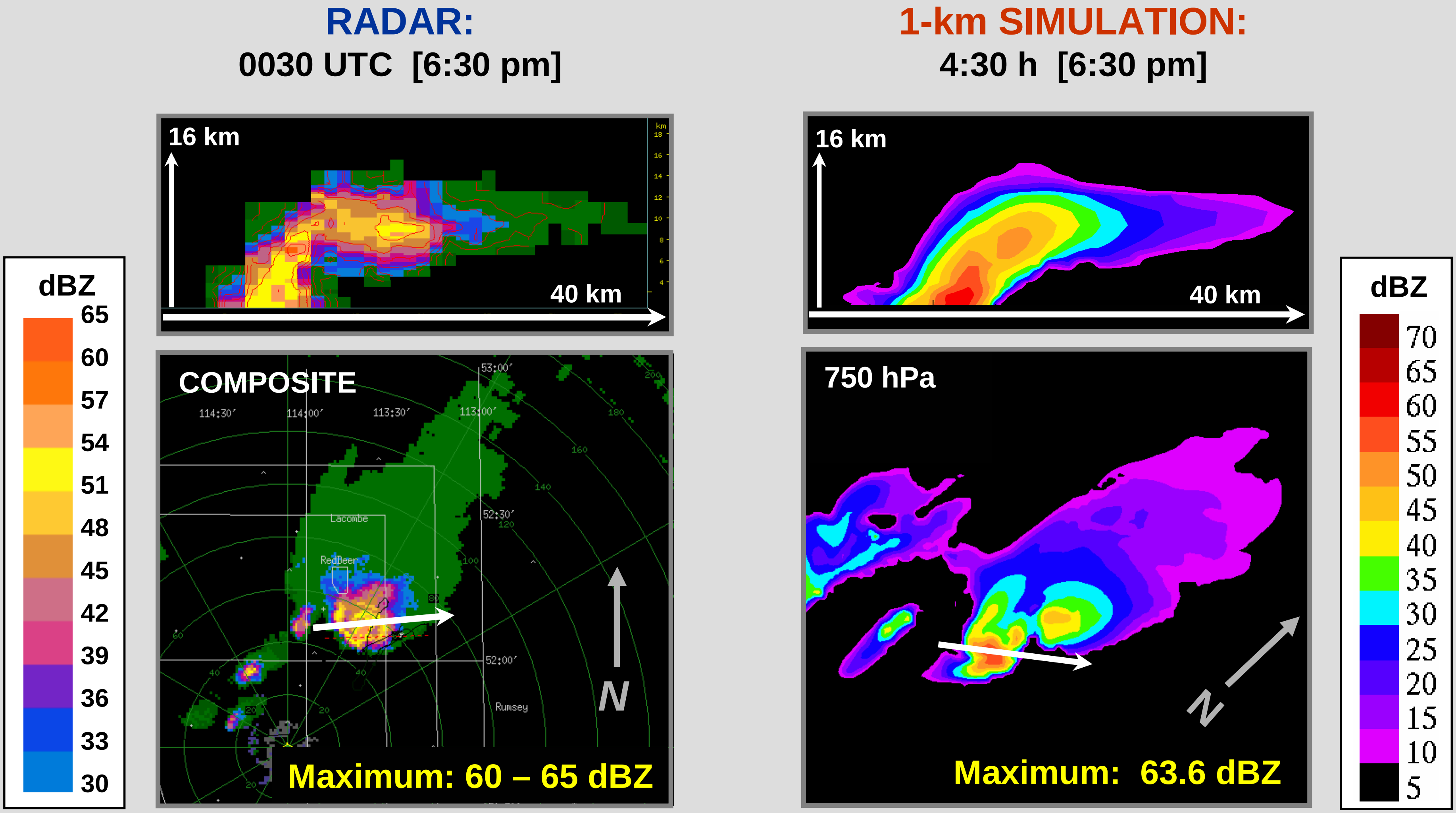
CONTROL SIMULATION: Accumulated Total Precipitation



CONTROL SIMULATION: Hail Swath

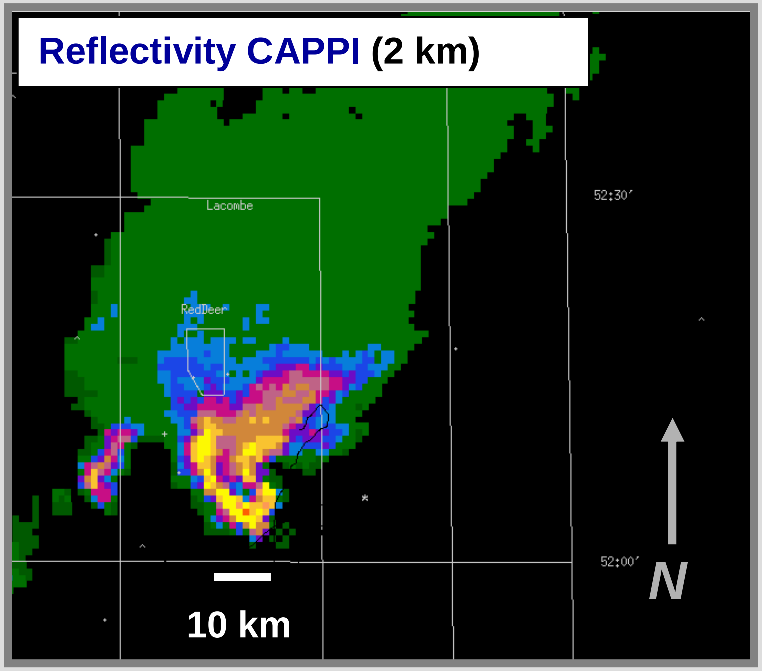


CONTROL SIMULATION: Storm Structure: REFLECTIVITY

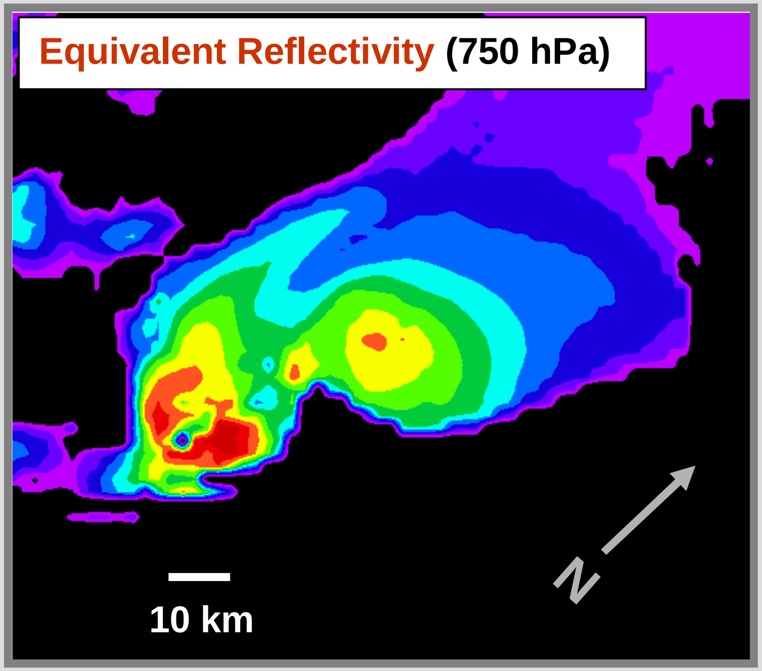


CONTROL SIMULATION: Storm Structure: HOOK ECHO

RADAR:
0030 UTC [6:30 pm]

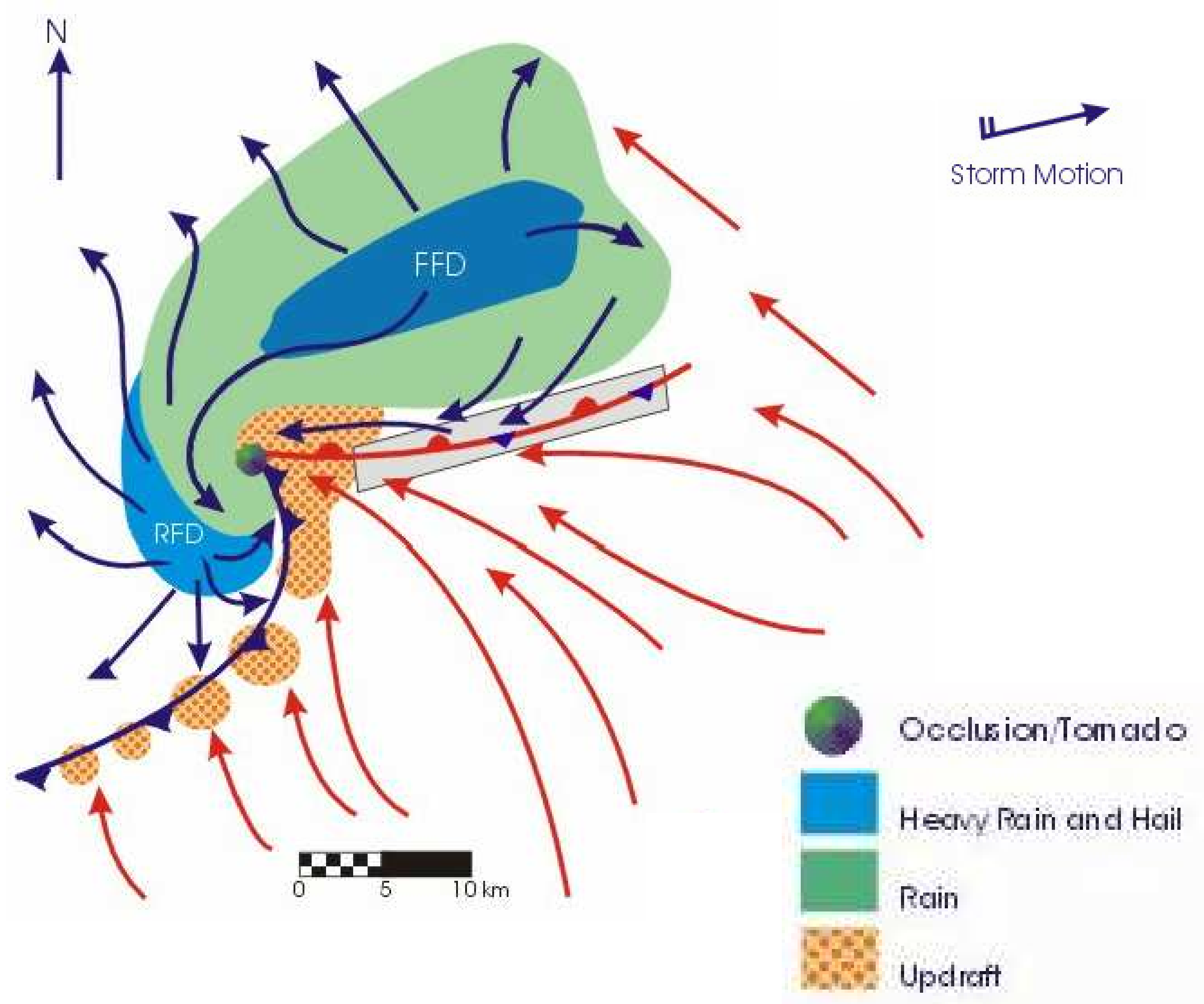


1-km SIMULATION:
4:15 h [6:15 pm]



Schematic of a Classic Supercell:

(Modified from Lemon and Doswell, 1979 MWR)



http://mrd3.nssl.ucar.edu/~pietrych/www/mocise/mociseclassic.jpg

CONTROL SIMULATION: Validation

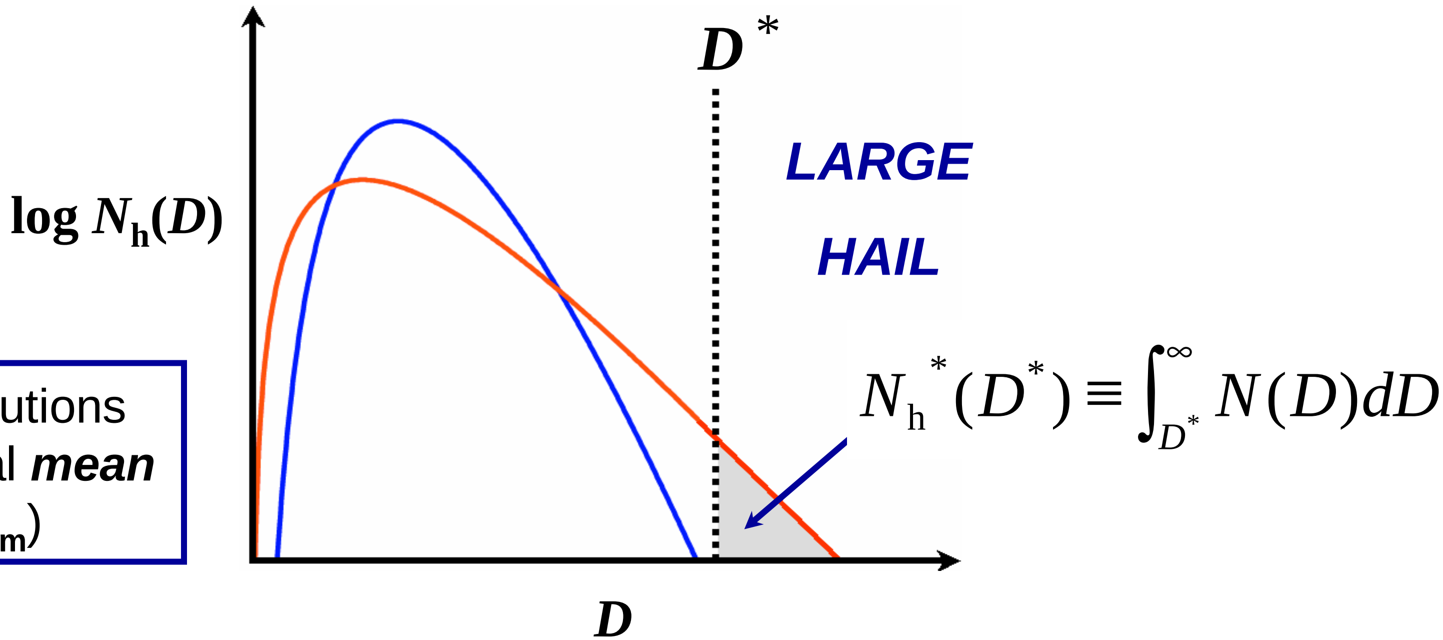
The following radar observations were correctly simulated:

- time of appearance of the first-echo
- propagation speed and direction (relative to steering-flow)
- reflectivity structure of the supercell
  - spatial dimensions
  - low-level mesocyclone
  - maximum reflectivity values
- precipitation track and accumulated quantities
- hail at the surface (before 7:00 pm)

CONTROL SIMULATION: Hail Sizes

How can the maximum hail sizes at the ground be inferred?

These distributions have identical ***mean*** diameters (D_m)



Flux of large of hail ($D > D^*$):

$$R_h^*(D^*) \equiv N_h^*(D^*) \cdot V_T(D^*)$$

CONTROL SIMULATION: Simulated Hail Sizes

At 5:45 pm (simulation time 4:45 h):

D* = 2 cm	***D* = 3 cm***
R_h*(2 cm) = 5.0×10^{-2} m^{-2} s^{-1}	R_h*(3 cm) = 2.3×10^{-4} m^{-2} s^{-1}
or,	or,
1 hailstone ***D*** > **2 cm** per 20 m^2 every 20 seconds	1.4 hailstones ***D*** > **3 cm** per 100 m^2 every 1 minute
OBSERVABLE	***NEGLIGIBLE***

MAXIMUM:

Walnut-sized (2 – 3 cm) hail was *simulated*

Golf ball-sized (3 – 4 cm) hail was *observed*

3. EVALUATION OF BULK APPROACHES – COMPARISON OF 3D SIMULATIONS

PART I: TRIPLE-MOMENT CONTROL RUN

PART II: SENSITIVITY EXPERIMENTS

***SENSITIVITY EXPERIMENTS*:**

<u>List of Runs</u>

1. **TRIPLE-MOMENT** (*control simulation*)
2. DOUBLE-MOMENT with **DIAGNOSED-** α_x
3. DOUBLE-MOMENT with **FIXED-** $\alpha_x = 0$
4. **SINGLE-MOMENT**

ALL RUNS USE DIFFERENT

VERSIONS OF THE <u>SAME SCHEME</u>

SENSITIVITY EXPERIMENTS: Maximum hail sizes (at surface)

3 – 4 cm (Golf ball-sized) hail was observed

[at 5:45 pm, time of maximum hail rate in CONTROL RUN]

TRIPLE-MOMENT

2 – 3 cm (Walnut-sized)

DOUBLE-MOMENT
Diagnosed α

1 – 2 cm (Grape-sized)

SINGLE-MOMENT

4 – 5 cm (Baseball-sized)

DOUBLE-MOMENT
Fixed $\alpha = 0$

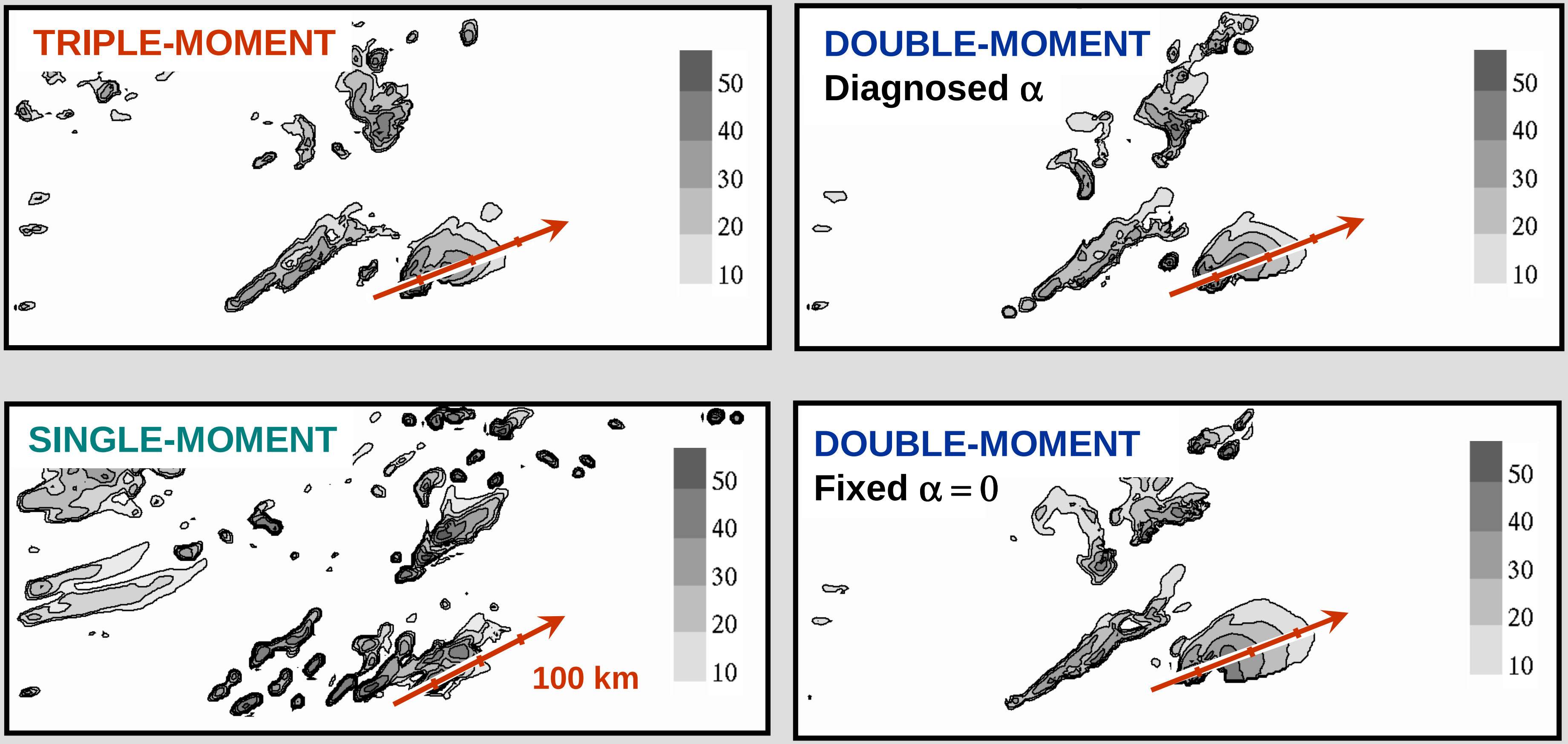
8 – 9 cm (Grapefruit-sized)

SENSITIVITY EXPERIMENTS: Equivalent Hail Reflectivity

700 hPa:

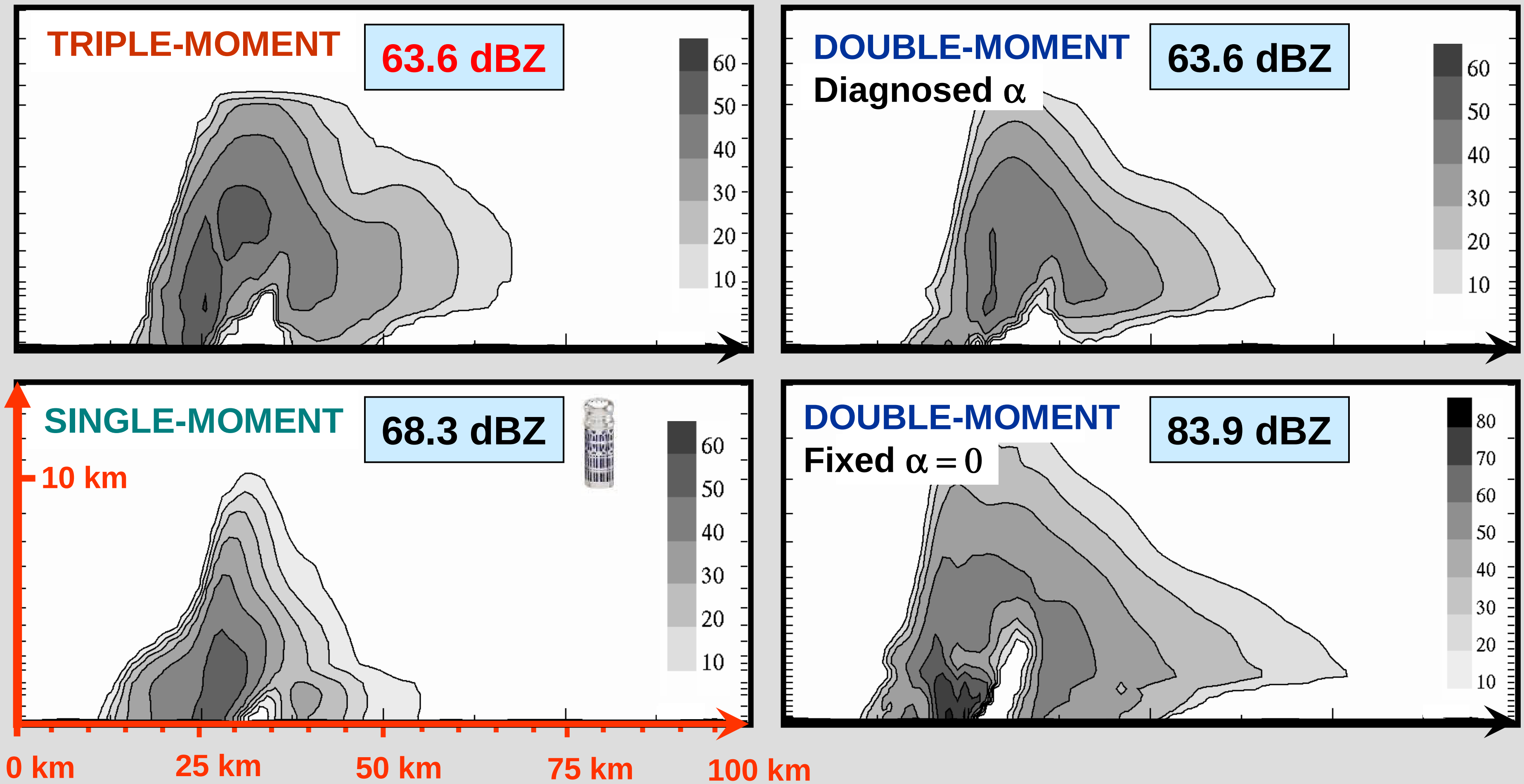
Local time: 6:30 pm
(Simulation time: 4:30 h)

Z_{eh} [dBZ]



SENSITIVITY EXPERIMENTS: Equivalent Hail Reflectivity, Z_{eh} [dBZ]

Local time: 6:30 pm
(Simulation time: 4:30 h)

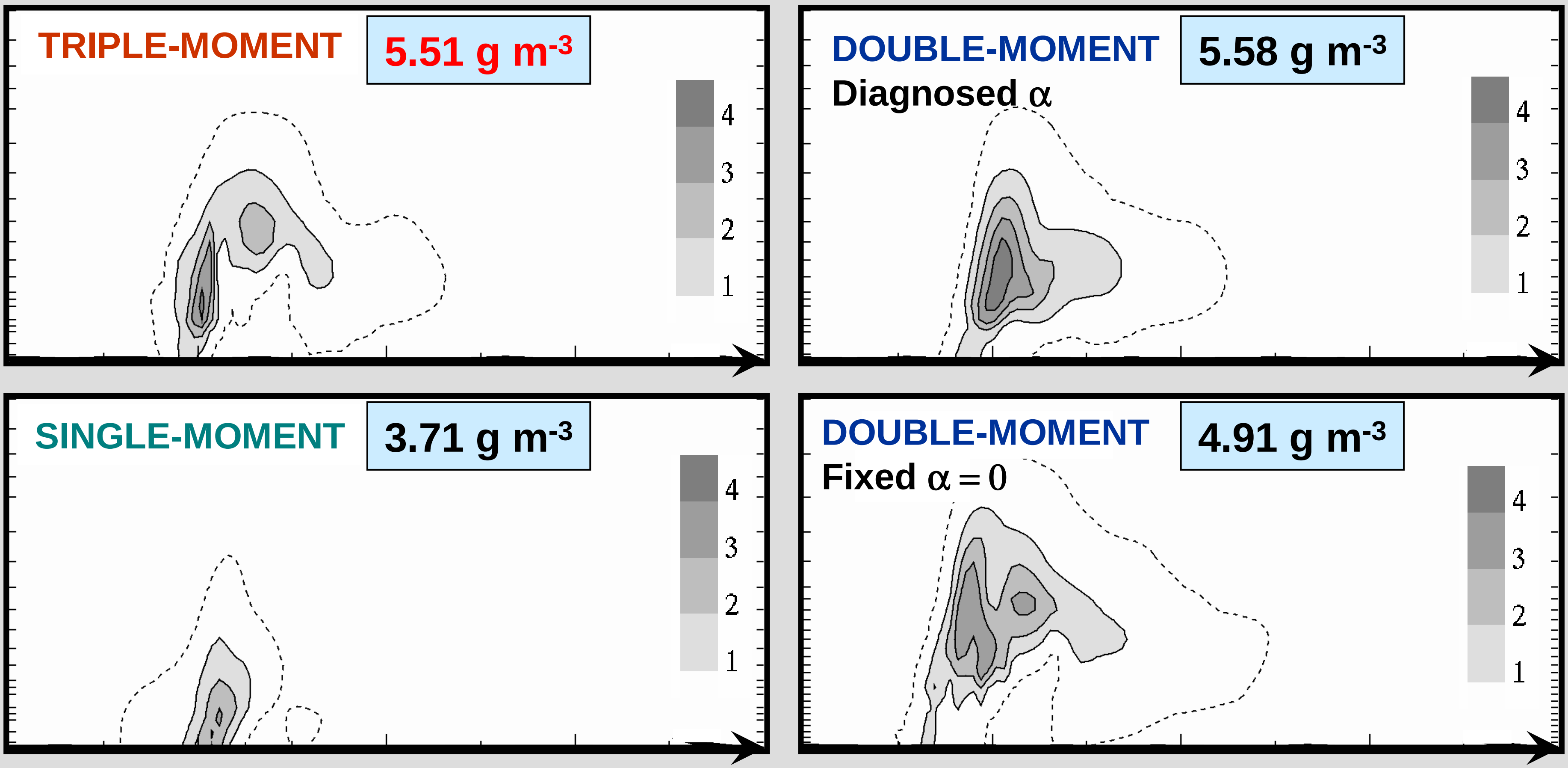


MAXIMUM VALUE

SENSITIVITY EXPERIMENTS: Hail Mass Content,

Q_h [g m^{-3}]

Local time: 6:30 pm
(Simulation time: 4:30 h)



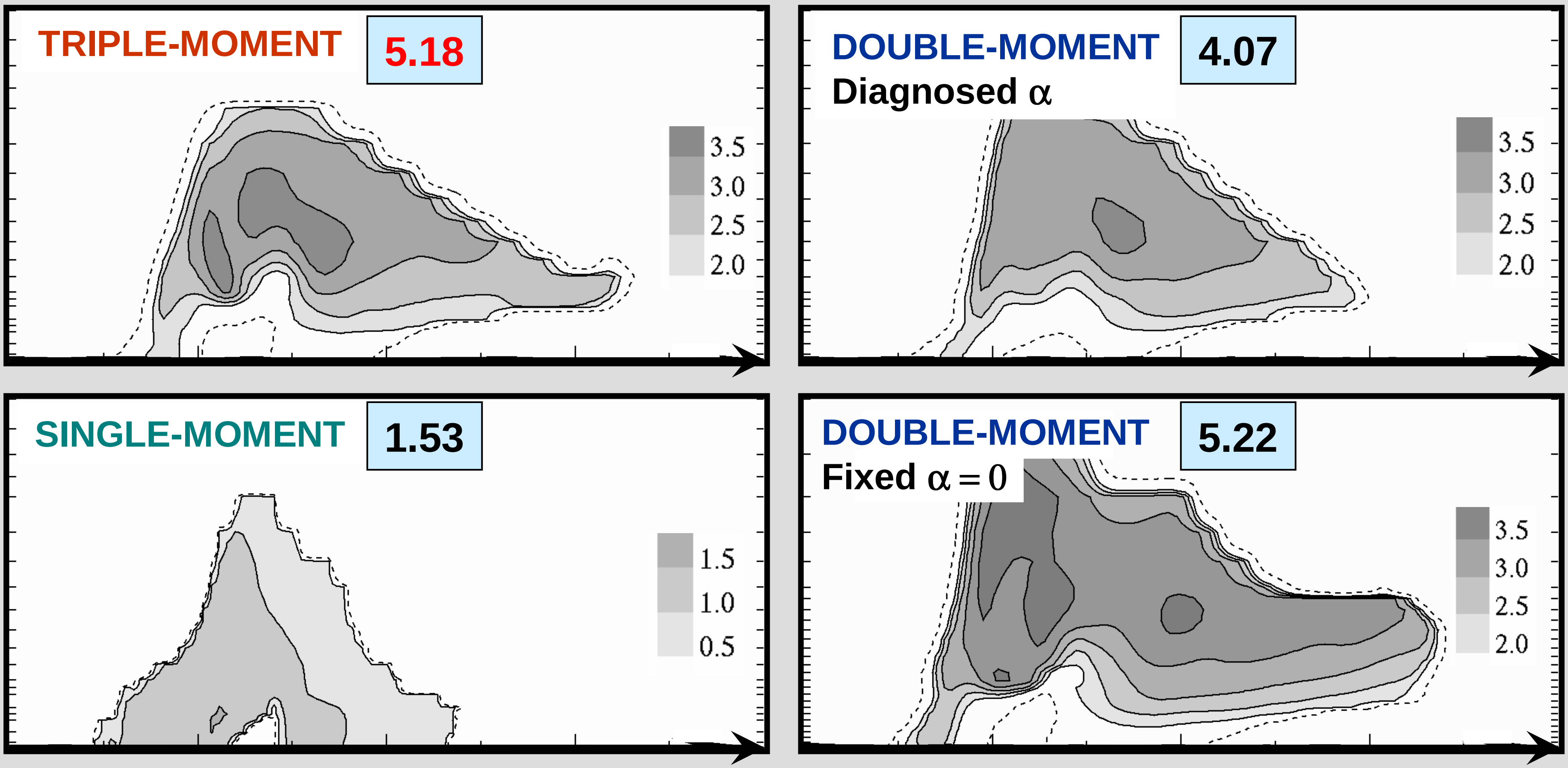
Dashed contour: 0.1 g m^{-3}

MAXIMUM VALUE

SENSITIVITY EXPERIMENTS: Hail Number Concentration

log N_{Th} [m^{-3}]

Local time: 6:30 pm
(Simulation time: 4:30 h)



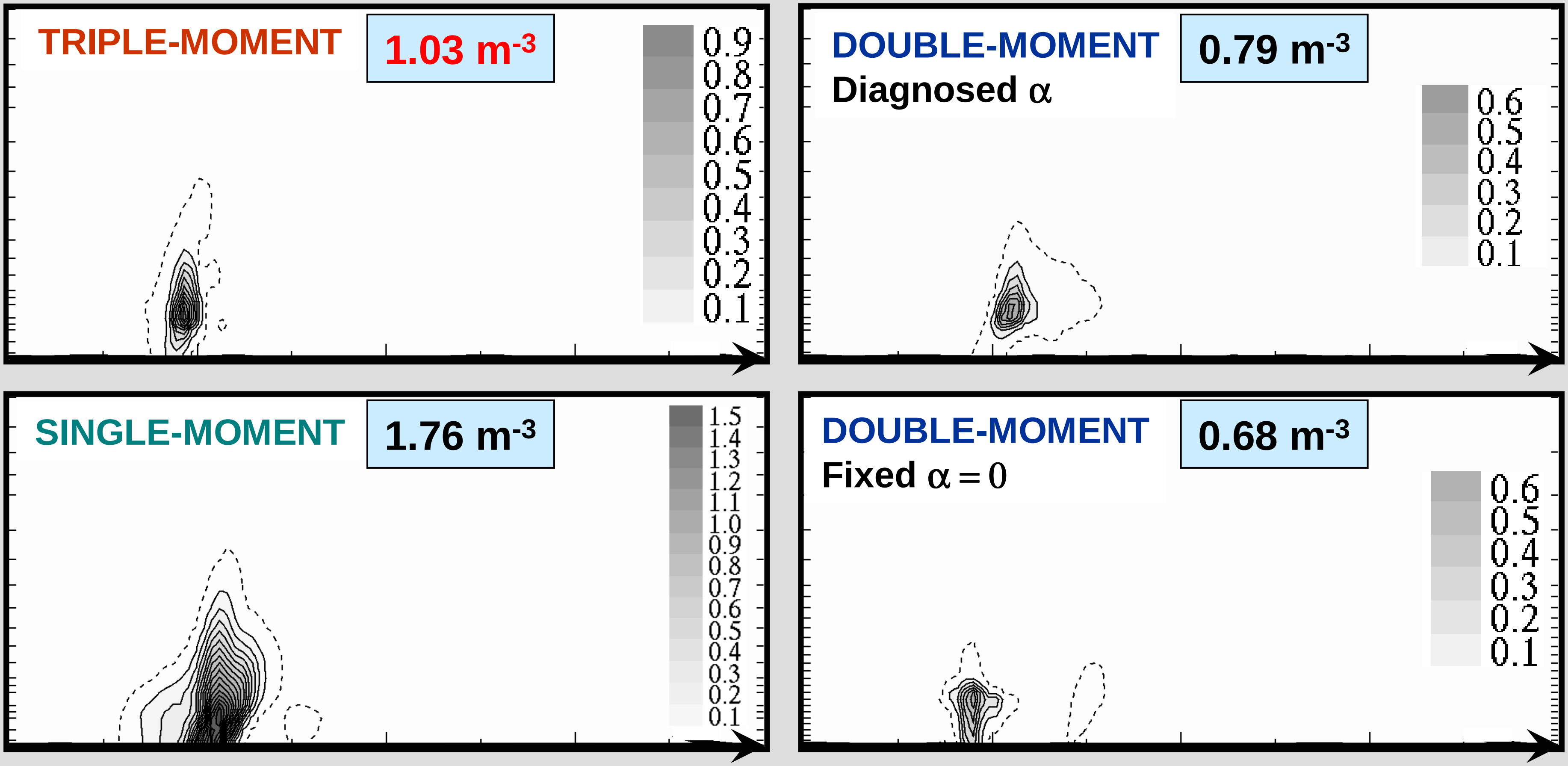
Dashed contour: 1.0 m^{-3}

MAXIMUM VALUE

SENSITIVITY EXPERIMENTS: Large Hail Concentration,

Local time: 6:30 pm
(Simulation time: 4:30 h)

N_h*{1 cm} [m^{-3}]
(grape-sized or larger)



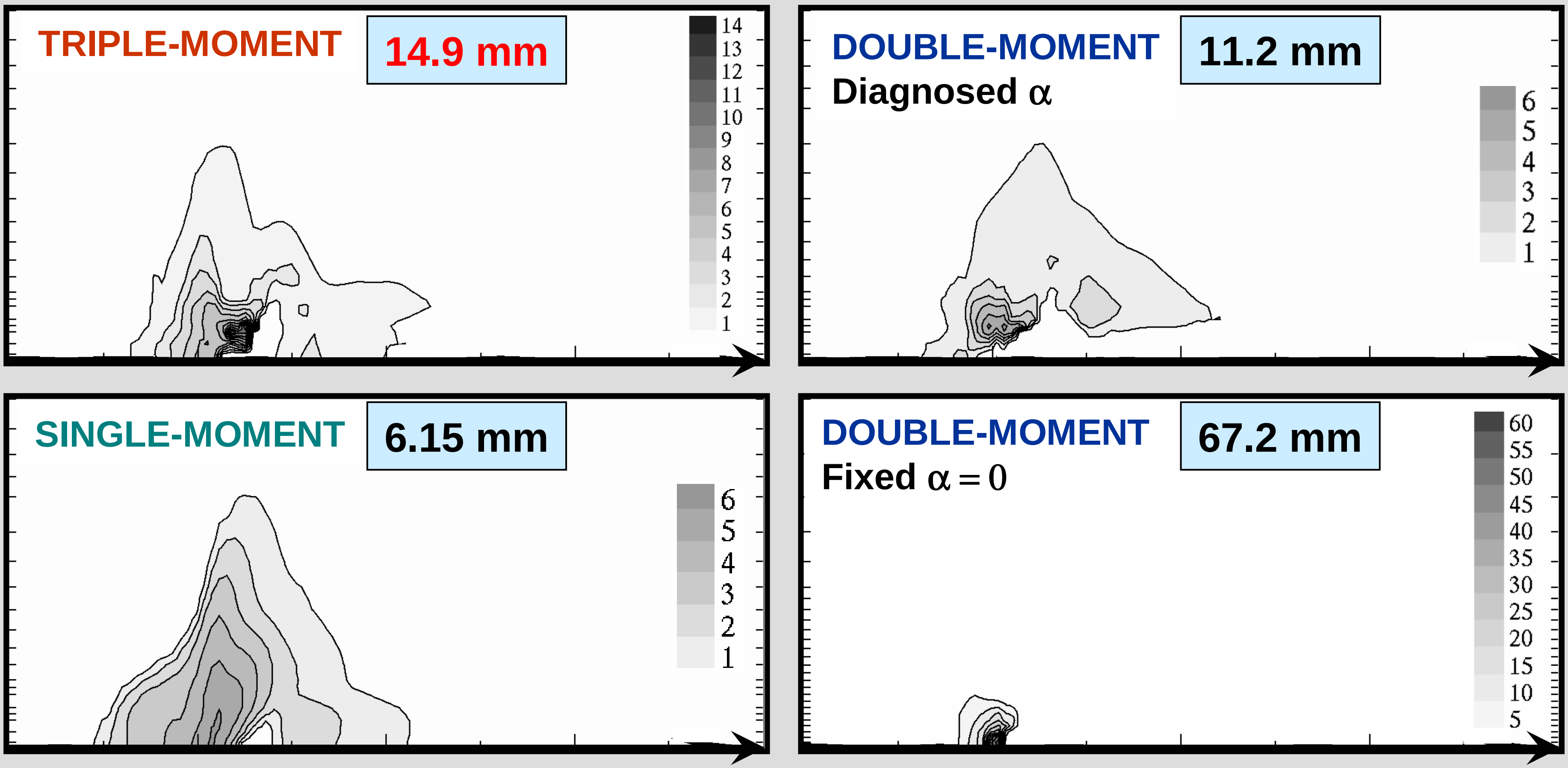
Dashed contour: 0.01 m^{-3}

MAXIMUM VALUE

SENSITIVITY EXPERIMENTS: Mean Hail Diameters,

***D**$_{mh}$* **[mm]**

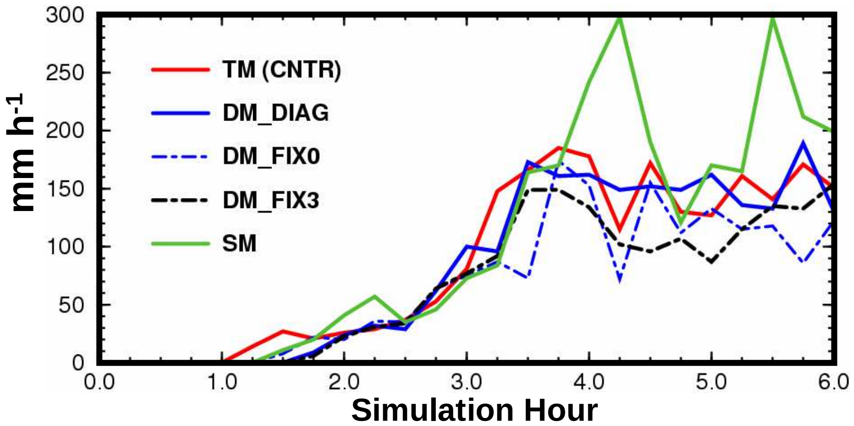
Local time: 6:30 pm
(Simulation time: 4:30 h)



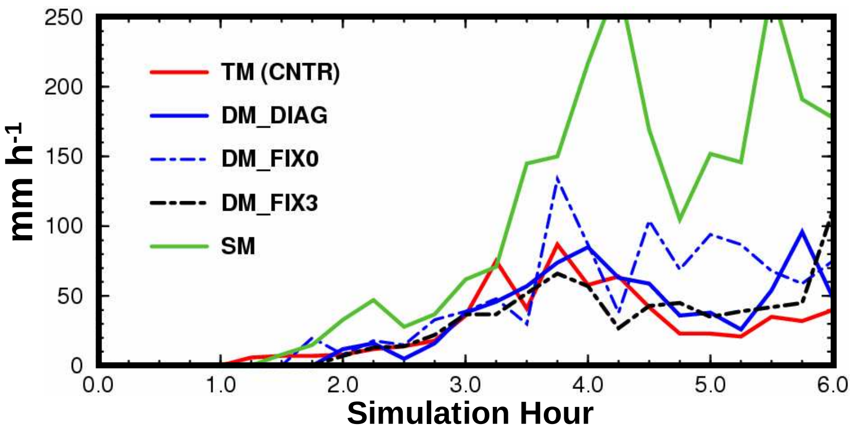
MAXIMUM VALUE

SENSITIVITY EXPERIMENTS Maximum precipitation rates

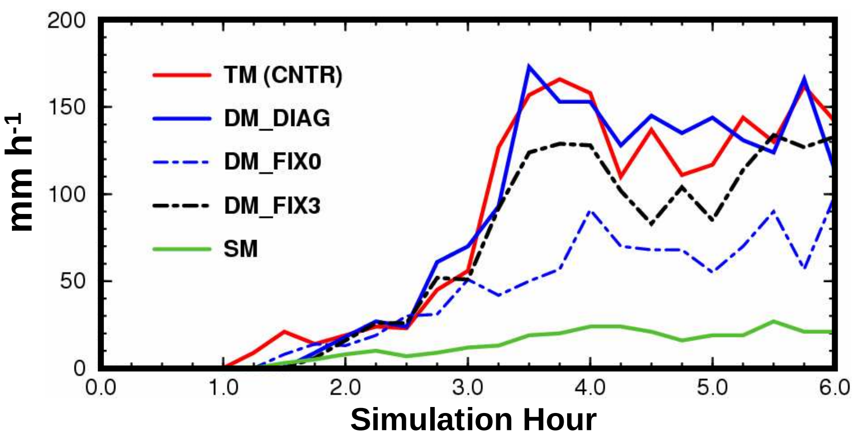
TOTAL



SOLID
(hail)



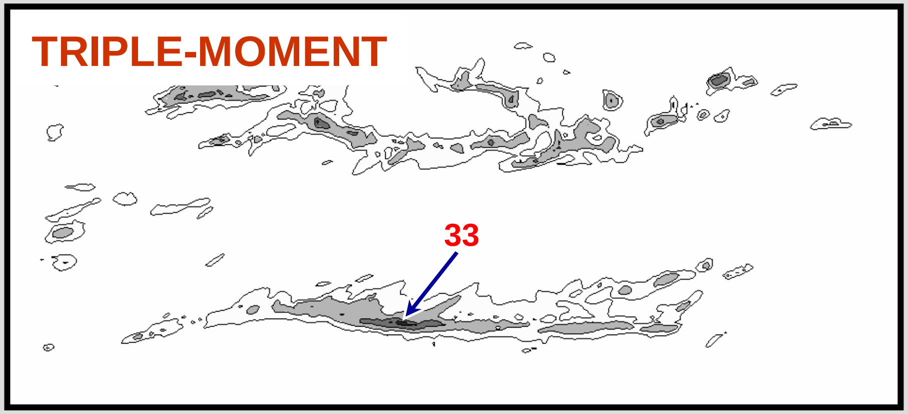
LIQUID
(rain)

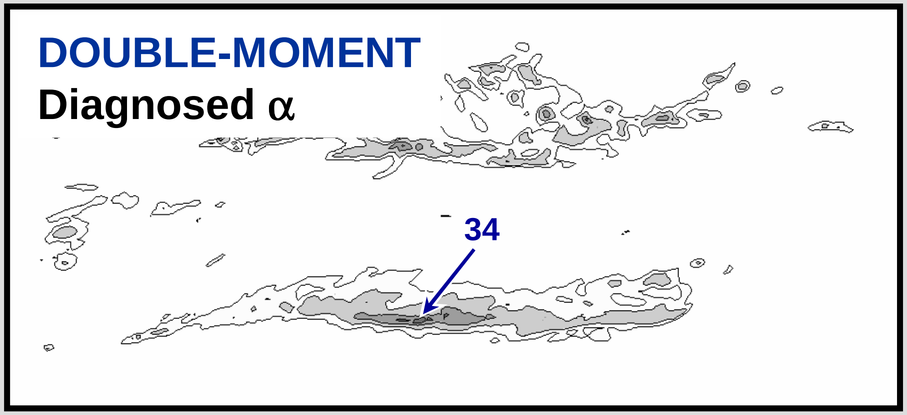


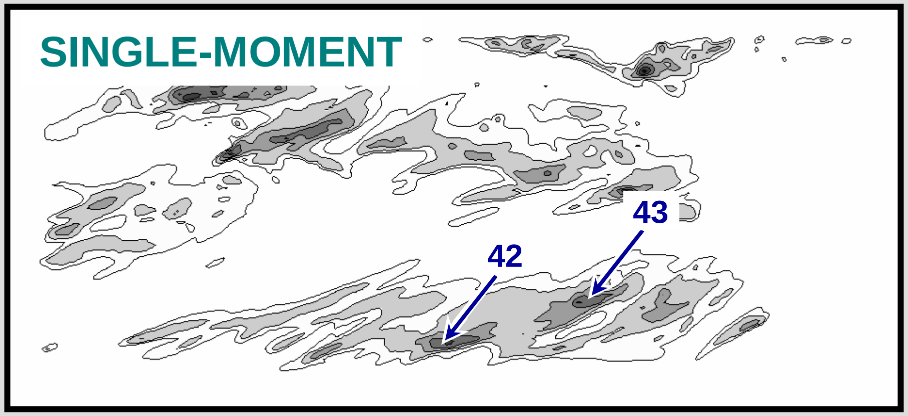
SENSITIVITY EXPERIMENTS:

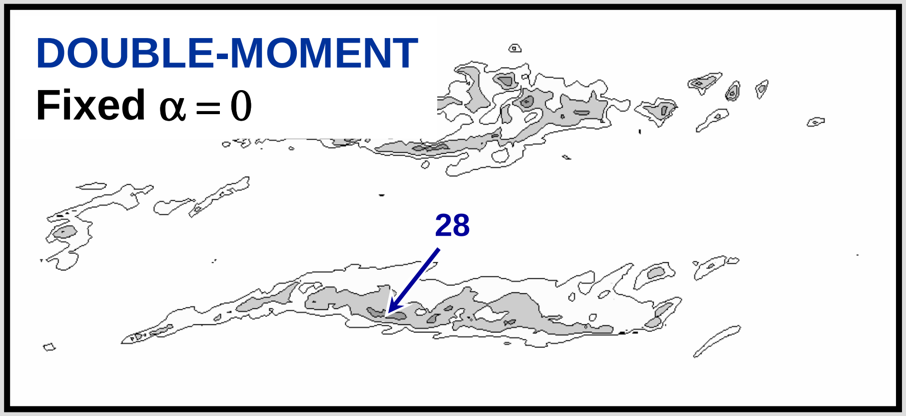
6-h ACCUMLATED TOTAL PRECIPITATION

[mm]







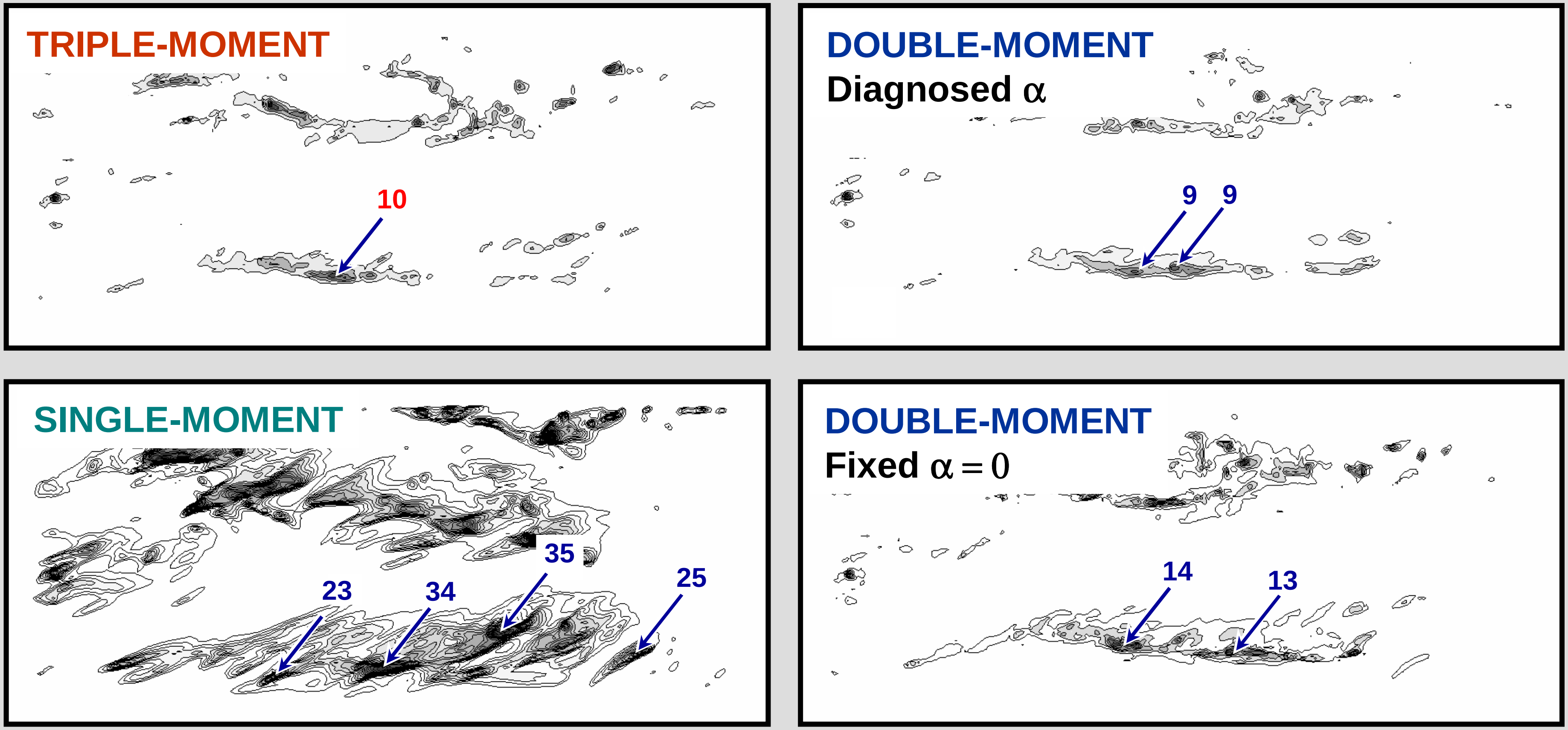


CONTOURS: 5, 10, 20, 30, 40 mm

***SENSITIVITY EXPERIMENTS*:**

6-h ACCUMLATED SOLID PRECIPITATION

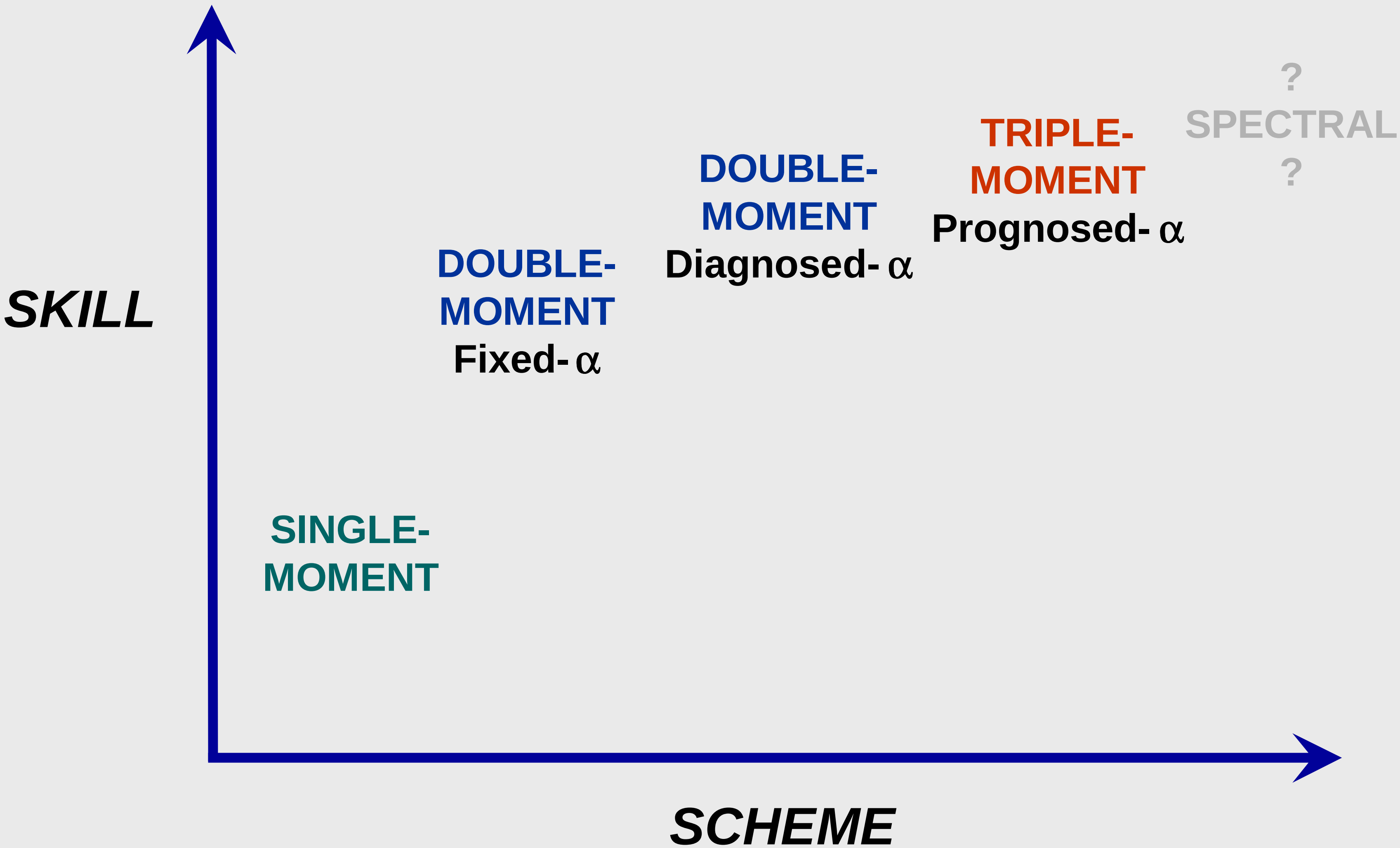
[mm]



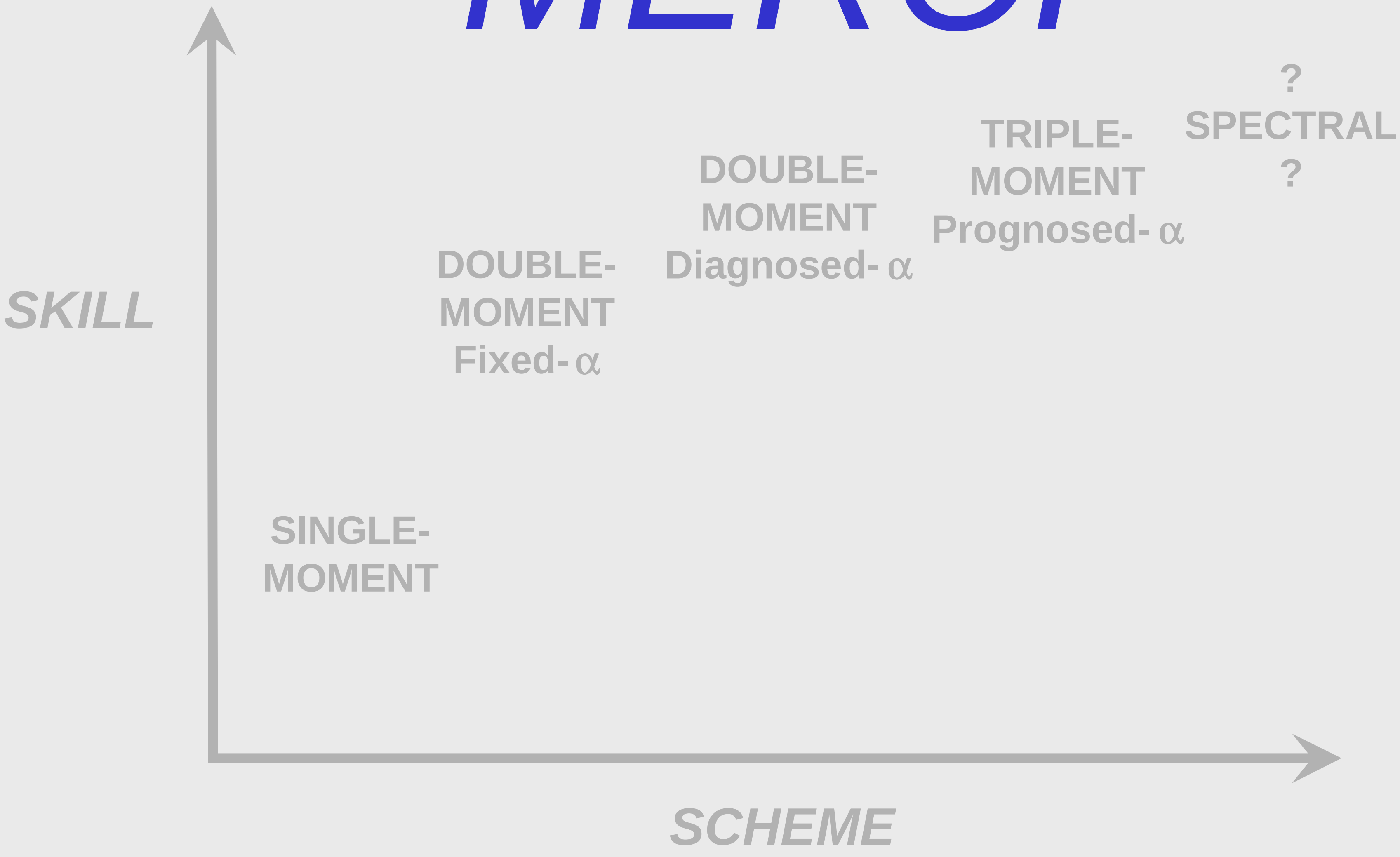
CONTOUR INTERVAL: 2 mm

CONCLUSION

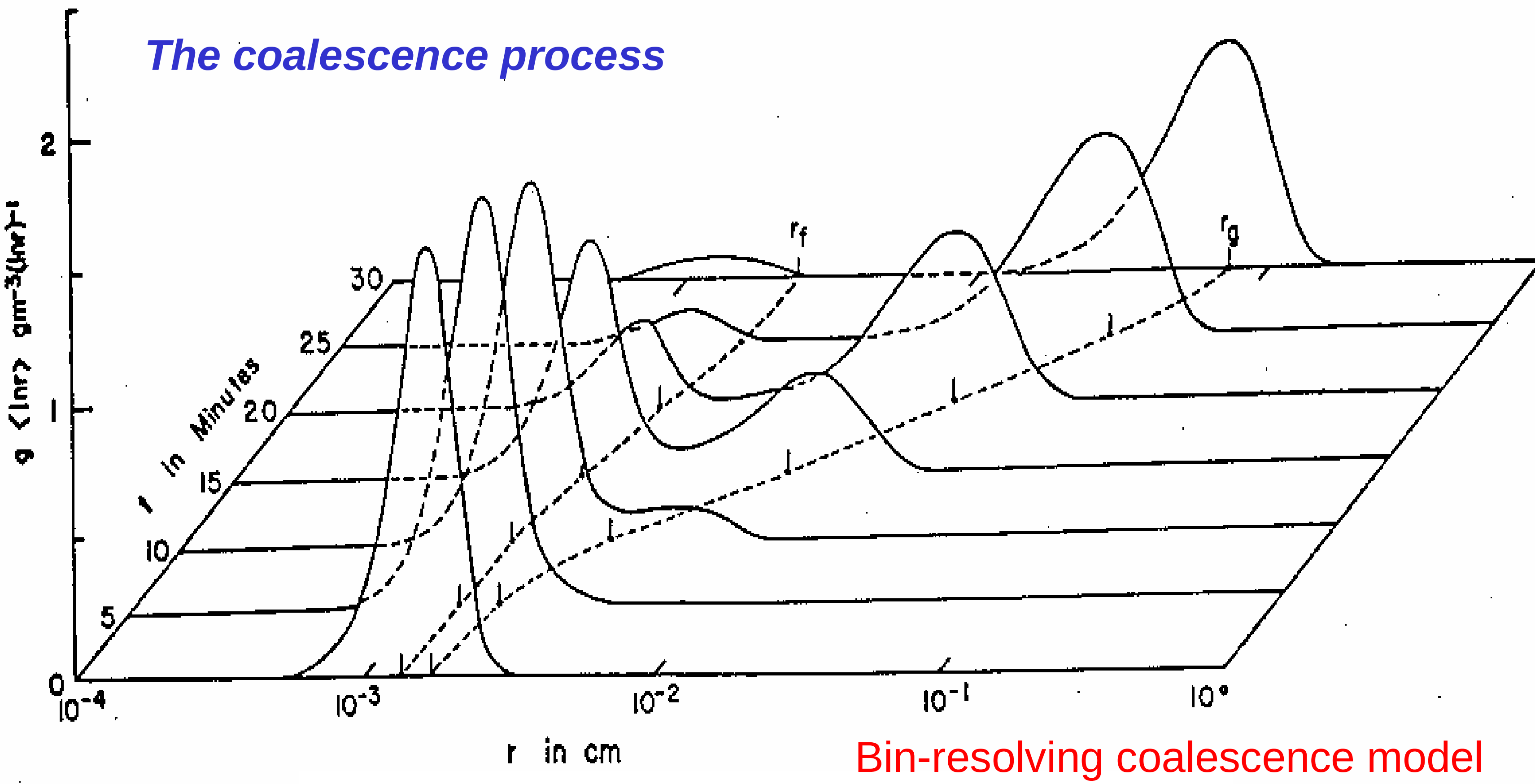
Overall Ranking of Bulk Schemes



MERCI


SKILL
SCHEME
SINGLE-MOMENT
DOUBLE-MOMENT Fixed-α
DOUBLE-MOMENT Diagnosed-α
TRIPLE-MOMENT Prognosed-α
? SPECTRAL ?

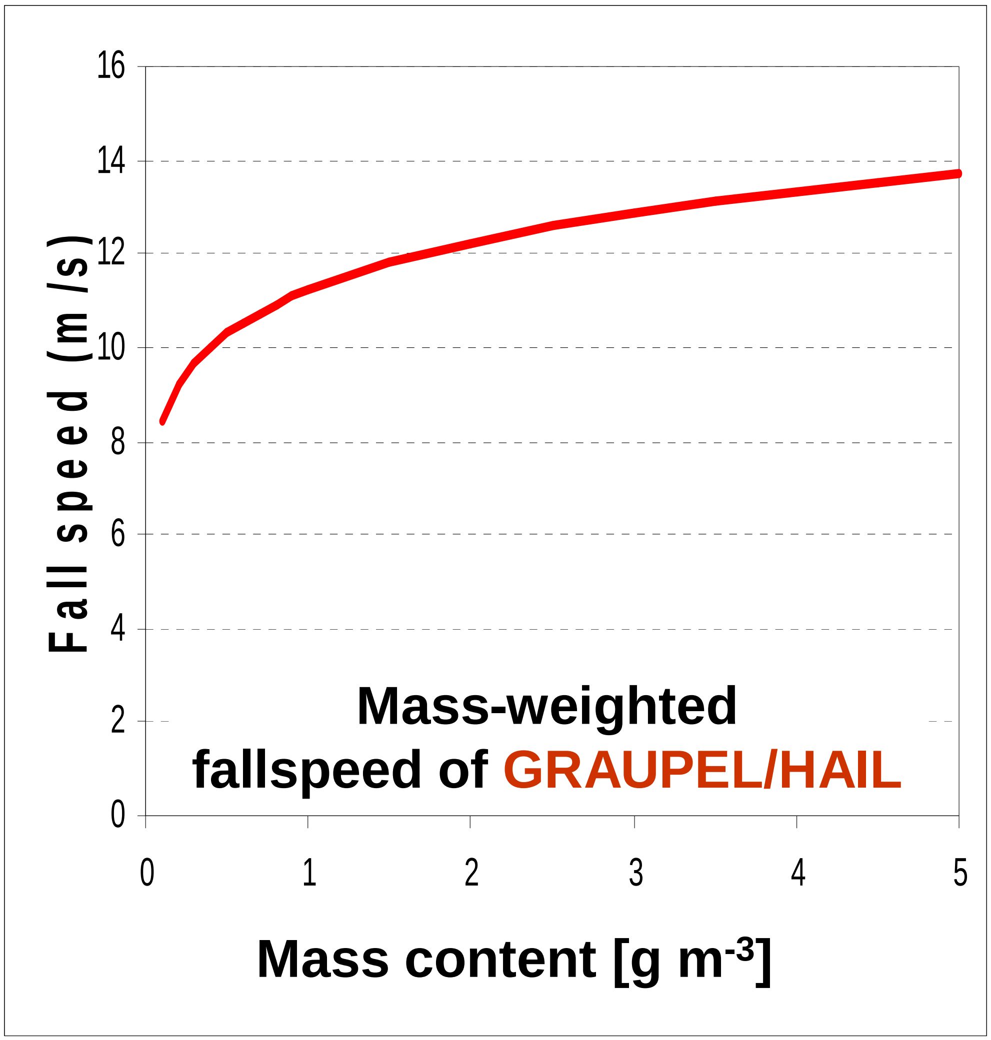
The coalescence process



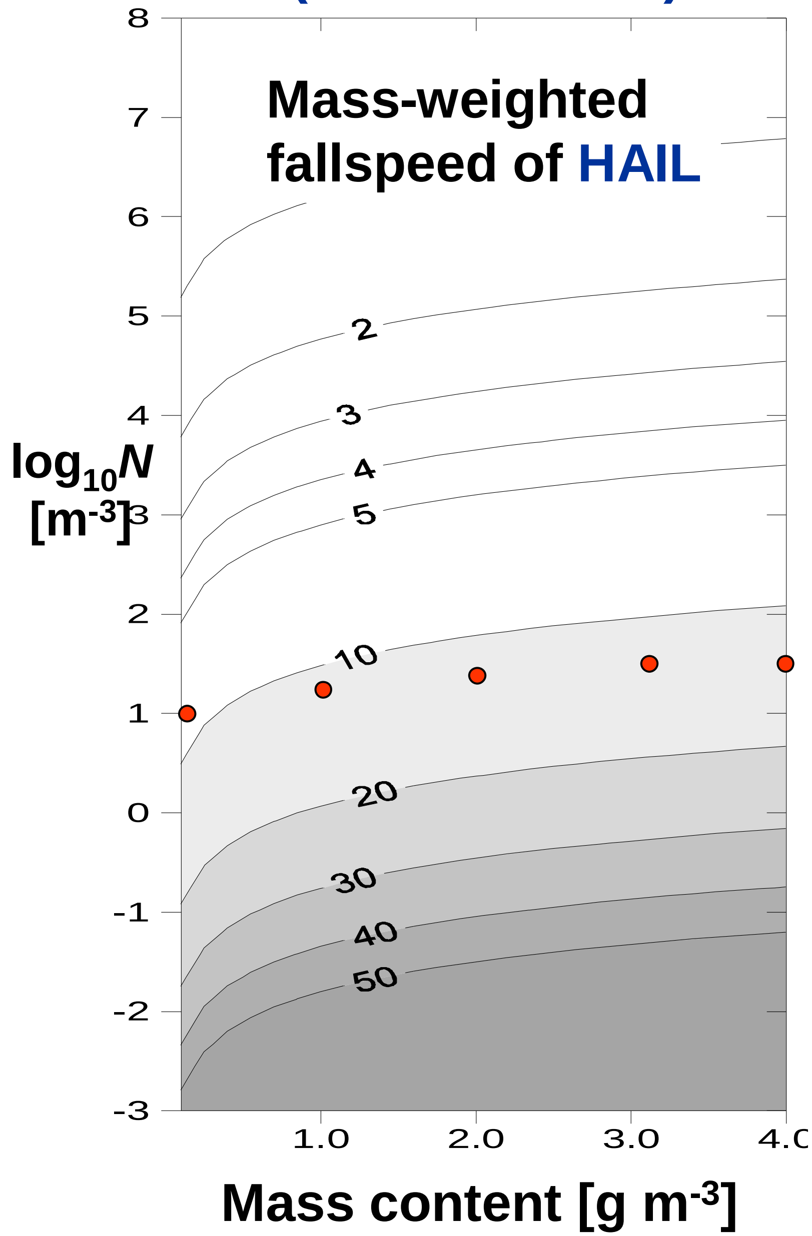
Bin-resolving coalescence model

(from Berry and Reinhardt, 1974)

Single-moment:
(Kong-Yau)



Double-moment:
(Milbrandt-Yau)



Note: $\rho_a = 1$ kg m^{-3}

1. Develop an appropriate technique

Overview of the bulk parameterization method

Standard double-moment method:

$$N_x(D) = N_{0x} D^{\alpha_x} \exp\left(-\lambda_x D\right)$$

Predict changes to Q_x and N_{Tx}

↓

Implies changes to values of the N_{0x} and λ_x

(α_x is held constant)

1. Develop an appropriate technique

Alternative bulk methods:

$$N_x(D) = N_{0x} D^{\alpha_x} \exp(-\lambda_x D)$$

1. **Diagnostic-** α_x (double-moment)

   - $\alpha_x = f\,(Q_x, N_{Tx})$

2. **Prognostic-** α_x (triple-moment)

   - a third moment must be predicted

   e.g. add dZ_x/dt equation

CONCLUSIONS

1. The relative spectral dispersion plays an important role in bulk microphysics schemes

2. For the overall QPF, storm structure, hydrometeor values, and the simulation of hail sizes:

SINGLE-MOMENT $<<$ **DOUBLE-MOMENT Fixed-α** $<$ **DOUBLE-MOMENT Diagnosed-α** $\leq$ **TRIPLE-MOMENT**

THANK YOU