Environment and Climate Change Canada
Environnement et Changement climatique Canada
Canada

Global Environmental Multiscale model with a new terrain-following vertical coordinate based on height

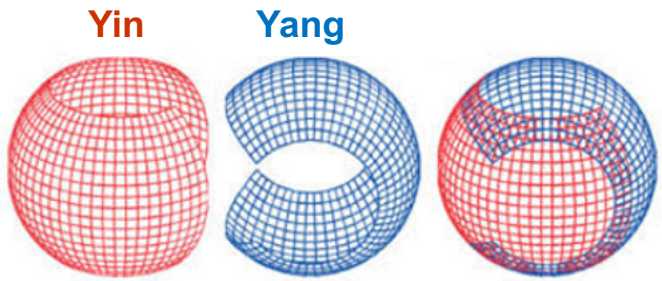
Syed Zahid Husain, Claude Girard, Abdessamad Qaddouri, André Plante

RPN-A, Meteorological Research Division
Environment and Climate Change Canada, Dorval, QC, Canada

The Global Environmental Multiscale (GEM) Model

- Principal features of the GEM dynamical core include
  - log-hydrostatic-pressure-type terrain-following vertical coordinate,
  - two time-level iterative implicit temporal discretization,
  - semi-Lagrangian advection,
  - Charney-Phillips grid in the vertical, and
  - Arakawa C-grid in the horizontal.

- The global horizontal discretization is based on an Yin-Yang type grid.



Motivations for the New Core

- Pressure-based vertical coordinate allows use of fast a direct solver for the discretized elliptic problem. However, it
  - loses scalability for large number of processor cores, and therefore,
  - necessitates development of efficient iterative solvers.

- The pressure-based dynamical core exhibits prohibitive numerical instability for steep orography (slope > 45°) .
  - Adversely affects the sub-kilometer forecasting systems over complex orography.

- Overall, a **height-based dynamical core** is expected to be
  - more amenable to iterative solvers, and
  - may improve steep orography-related instability.

Height-based Vertical Coordinate in GEM

- A height-based **SLEVE-**like (Schär et al., 2002) terrain-following vertical coordinate, defined as

$$z = \zeta + B_1 z_{sL} + B_2 (z_s - z_{sL})$$

  is adopted, where ζ is the terrain-following coordinate, z is the true height, z_s is the height of the surface, and z_{sL} stands for large-scale orography.

- The terms B_1 and B_2 determine the rate of flattening of the large- and fine-scale orographic imprints on the terrain-following coordinate with increasing altitude, defined as

$$B = \left(\frac{\zeta_T - \zeta}{\zeta_T}\right)^r = \lambda^r \text{ where } r = [r_{max} - (r_{max} - r_{min})\lambda].$$

Schär, C., Leuenberger, D., Fuhrer, O., Lüthi, D., and Girard, C. (2002): A new terrain-following vertical coordinate formulation for atmospheric prediction models. Mon. Wea. Rev., 130: 2459-2480.

Quick Implementation Strategy

- From here on, we will refer to the new and the existing dynamical cores as **GEM-H** and **GEM-P**.

- In order to facilitate a **quick implementation** of the **GEM-H** dynamical core, we have

  - the same horizontal and vertical grids and discretization as in GEM-P,

  - employed the same (or corresponding) prognostic variables,

  - the same semi-Lagrangian approach for advection, and

  - applied the same two-time-level implicit iterative temporal discretization.

Model Formulation: GEM-H

- **GEM-H** formulation utilizes an isothermal basic state, defined by temperature T_* and pressure p_*, connected through the hydrostatic relation, i.e., $\frac{\partial \ln p_*}{\partial z} = -g/RT_*$.

- We have **five prognostic equations for five variables**, namely, $\mathbf{V} = (u, v, w)$, T, and q [where $q = RT_* \ln(p/p_*)$].

- **Transformation from z to ζ coordinate** requires

$$\frac{\partial}{\partial x_z} \equiv \frac{\partial}{\partial x} - \frac{\mathrm{J}_X}{\mathrm{J}_\zeta}\frac{\partial}{\partial \zeta}; \quad \frac{\partial}{\partial y_z} \equiv \frac{\partial}{\partial y} - \frac{\mathrm{J}_Y}{\mathrm{J}_\zeta}\frac{\partial}{\partial \zeta}; \quad \frac{\partial}{\partial z} \equiv \frac{1}{\mathrm{J}_\zeta}\frac{\partial}{\partial \zeta}; \quad \frac{\partial}{\partial t_z} \equiv \frac{\partial}{\partial t}$$

where

$\mathrm{J}_X = \frac{\partial z}{\partial X}$, $\mathrm{J}_Y = \frac{\partial z}{\partial Y}$, and $\mathrm{J}_\zeta = \frac{\partial z}{\partial \zeta}$.

Dynamical System of Eqn.

$$\frac{du}{dt} - \left(f + \frac{\tan\theta}{a}u\right)v + \frac{T}{T_*}\left(\frac{\partial q}{\partial X} - \frac{\mathrm{J}_X}{\mathrm{J}_\zeta}\frac{\partial q}{\partial \zeta}\right) = 0$$

$$\frac{dv}{dt} + \left(f + \frac{\tan\theta}{a}u\right)u + \frac{T}{T_*}\left(\frac{\partial q}{\partial Y} - \frac{\mathrm{J}_Y}{\mathrm{J}_\zeta}\frac{\partial q}{\partial \zeta}\right) = 0$$

$$\frac{dw}{dt} + \frac{T}{T_*}\left(\frac{1}{\mathrm{J}_\zeta}\frac{\partial q}{\partial \zeta} - g\frac{T'}{T}\right) = 0$$

Momentum equations

$$\frac{d}{dt}\left[\ln\left(\frac{T}{T_*}\right) - \frac{q}{c_p T_*}\right] + \frac{N_*^2}{g}\mu w = 0$$

Thermodynamic equation

$$\frac{d}{dt}\left(\frac{q}{c_*^2} + \ln \mathrm{J}_\zeta\right) + \frac{\partial u}{\partial X} + \frac{1}{\cos\theta}\frac{\partial(\cos\theta\, v)}{\partial Y} + \frac{\partial \dot{\zeta}}{\partial \zeta} - \frac{g}{c_*^2}w = 0$$

Continuity equation

$$\frac{d}{dt}(z - \zeta) + \dot{\zeta} - w = 0$$

Kinematic equation

where, $T' = T - T_*$, $N_*^2 = g^2/c_p T_*$, and $c_*^2 = RT_*/(1-\kappa)$.

Temporal Discretization

- Individual equations have the form

$$\frac{dF}{dt} + G = 0,$$

and are discretized in time using a two-time level Crank-Nicholson scheme as

$$\frac{F_A^+ - F_D^-}{\Delta t} + \underbrace{bG_A^+ + (1-b)G_D^-}_{\text{off-centered averaging}} = 0. \qquad [b \geq 0.5]$$

A: arrival points
D: depart. locations

- **Differential off-centering:** Three off-centering parameters:
  i. b_m for the horizontal momentum equations,
  ii. b_h for continuity and thermodynamic equations, and
  iii. b_{nh} for the vertical momentum equation.

Solution Approaches

- Individual equations are rearranged as

$$\left(\frac{F_A^+}{b\Delta t} + G_A^+\right)_{linear} = \frac{F_D^-}{b\Delta t} - \left(\frac{1-b}{b}\right) G_D^- - \left(\frac{F_A^+}{b\Delta t} + G_A^+\right)_{nonlinear}$$

- Through elimination of variable the system of equations is reduced to a single elliptic boundary value (EBV) problem.

- **General form of the EBV** (for both **GEM-P** and **GEM-H**):

$$\nabla_H^2 P + M_{VH} P = R$$

where, the subscripts H and V indicate coefficients that vary in the horizontal and the vertical, and M_{VH} includes difference and mean operators in the horizontal and the vertical.

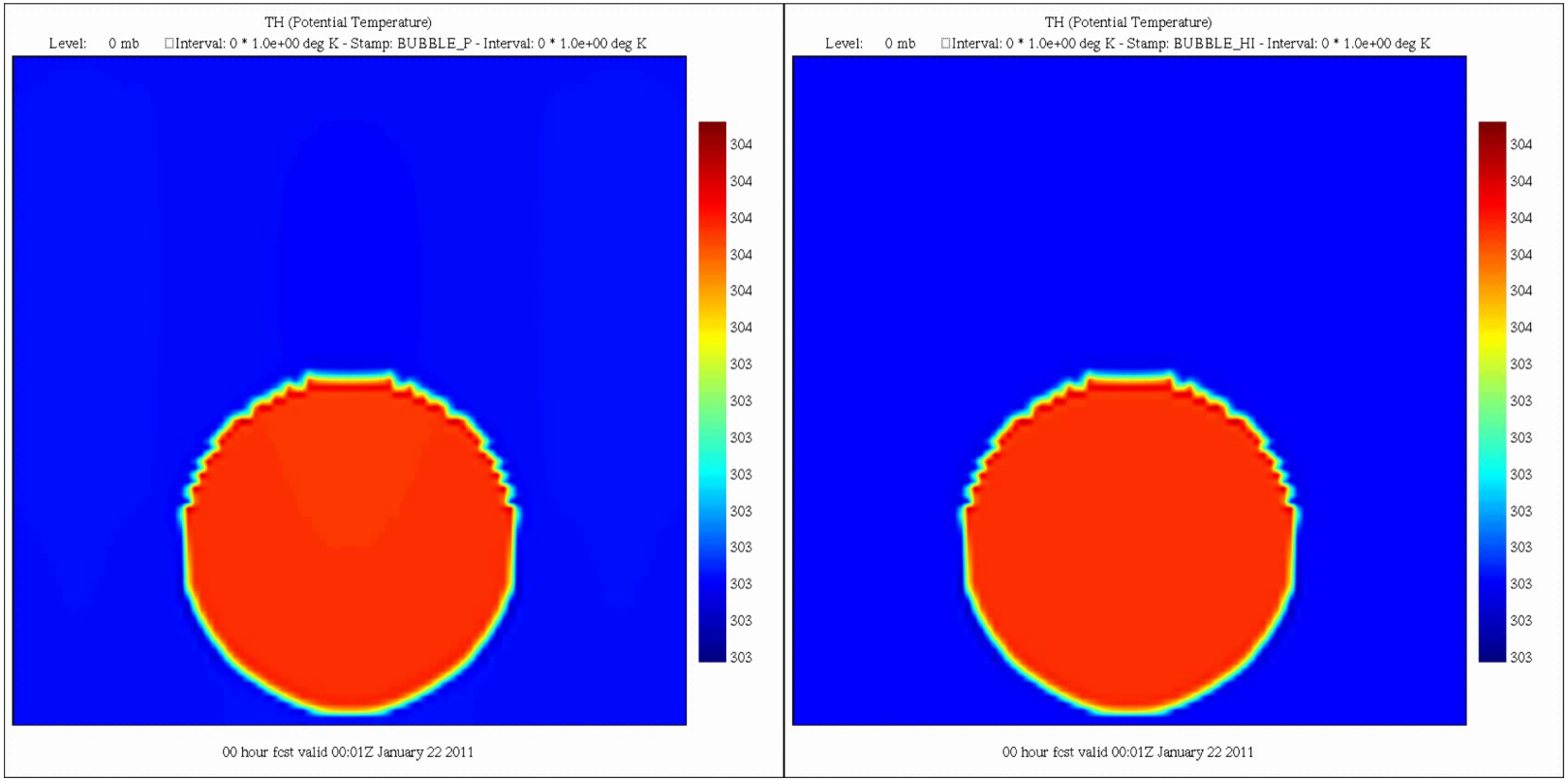
- **Simplified approach:** Metric terms in the nonlinear part. Allows use of the fast **direct solver**.

Results

Robert's Bubble Convection Test

- Evolution of potential temperature distribution for a 500 m diameter bubble with an initial potential temperature excess of 0.5°C over an isentropic environment (Robert, 1993).

GEM-P | **GEM-H**



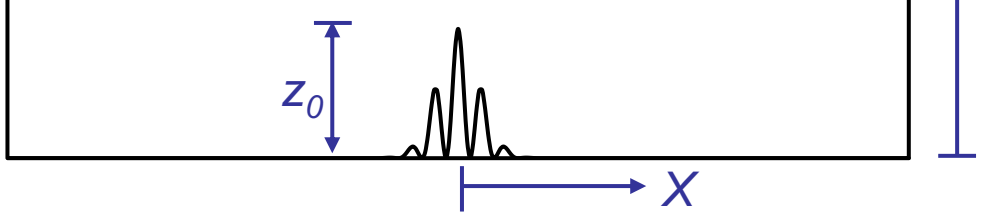
Robert, A. (1993): Bubble convection experiments with a semi-implicit formulation of the Euler equations. J. of Atmos. Sciences, 50: 1865-1873.

The Schär Mountain Case

A Theoretical Non-Hydrostatic Case

Schär case: Dry flow past idealized topography

$$z_s = z_0 \exp\left[-\left(\frac{x}{a}\right)^2\right] \cos^2\left(\frac{\pi x}{l_x}\right)$$



$U = 10\ \mathrm{ms}^{-1}$
$N = 0.01\ \mathrm{s}^{-1}$
$T_0 = 288\ \mathrm{k}$

$L = 200\ \mathrm{km}$ $H = 19.5\ \mathrm{km}$
$N_i = 401$ $N_k = 65$
$\Delta x = 500\ \mathrm{m}$ $\Delta z = 300\ \mathrm{m}$
$z_0 = 250\ \mathrm{m}$ $a = 5\ \mathrm{km}$
$l_x = 4\ km = 8\Delta x$

Schär, C., Leuenberger, D., Fuhrer, O., Lüthi, D., and Girard, C. (2002): A new terrain-following vertical coordinate formulation for atmospheric prediction models. Mon. Wea. Rev., 130: 2459-2480.

The Schär Mountain Case

- A theoretical nonhydrostatic case.
- Dry adiabatic flow past idealized topography given by

$$z_s = z_0 \exp\left[-\left(\frac{x}{a}\right)^2\right] \cos^2\left(\frac{\pi x}{l_x}\right)$$

where N_i=401, N_k=65, Δx=500 m, Δz=300 m, $z_0 = 250$ m, $a = 5$ km, and $l_x = 4\ k\text{m} = 8\Delta x$.

Also $U = 10$ ms^{-1}, $N = 0.01$ s^{-1}, and $T_0 = 288$ k.

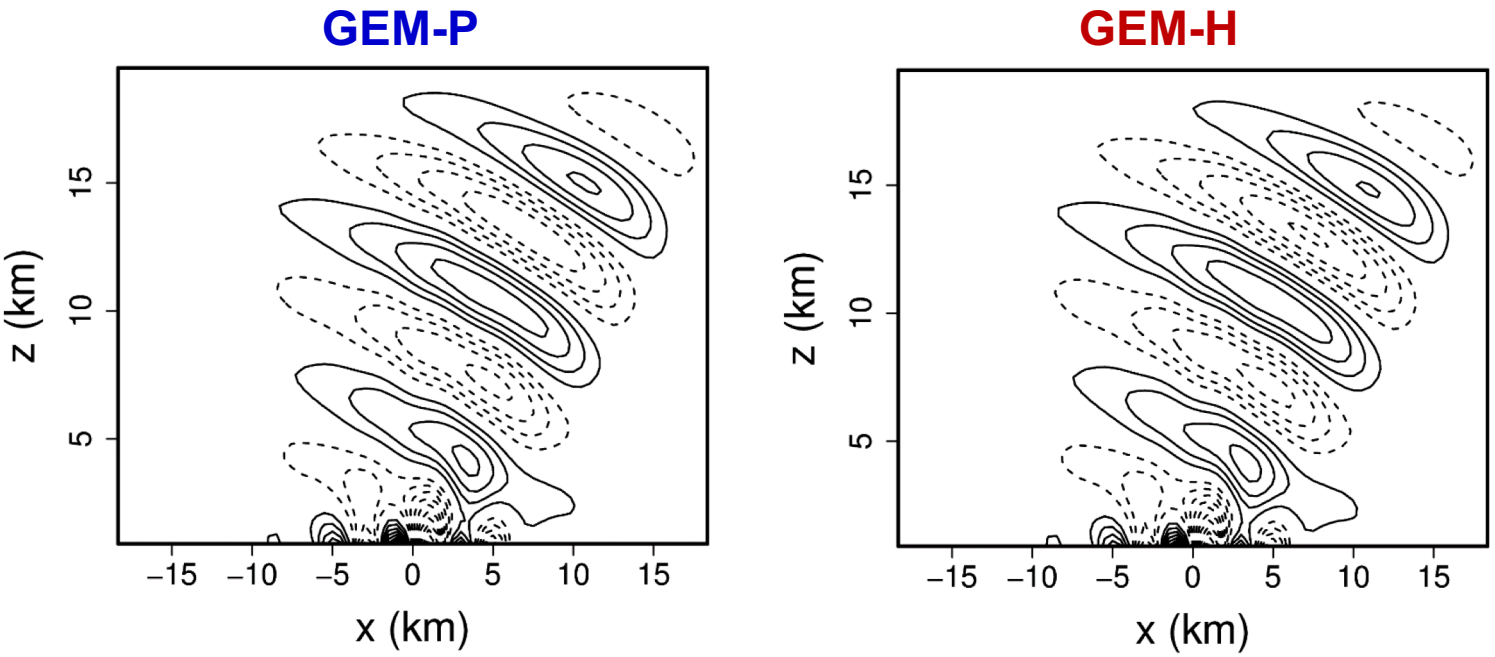


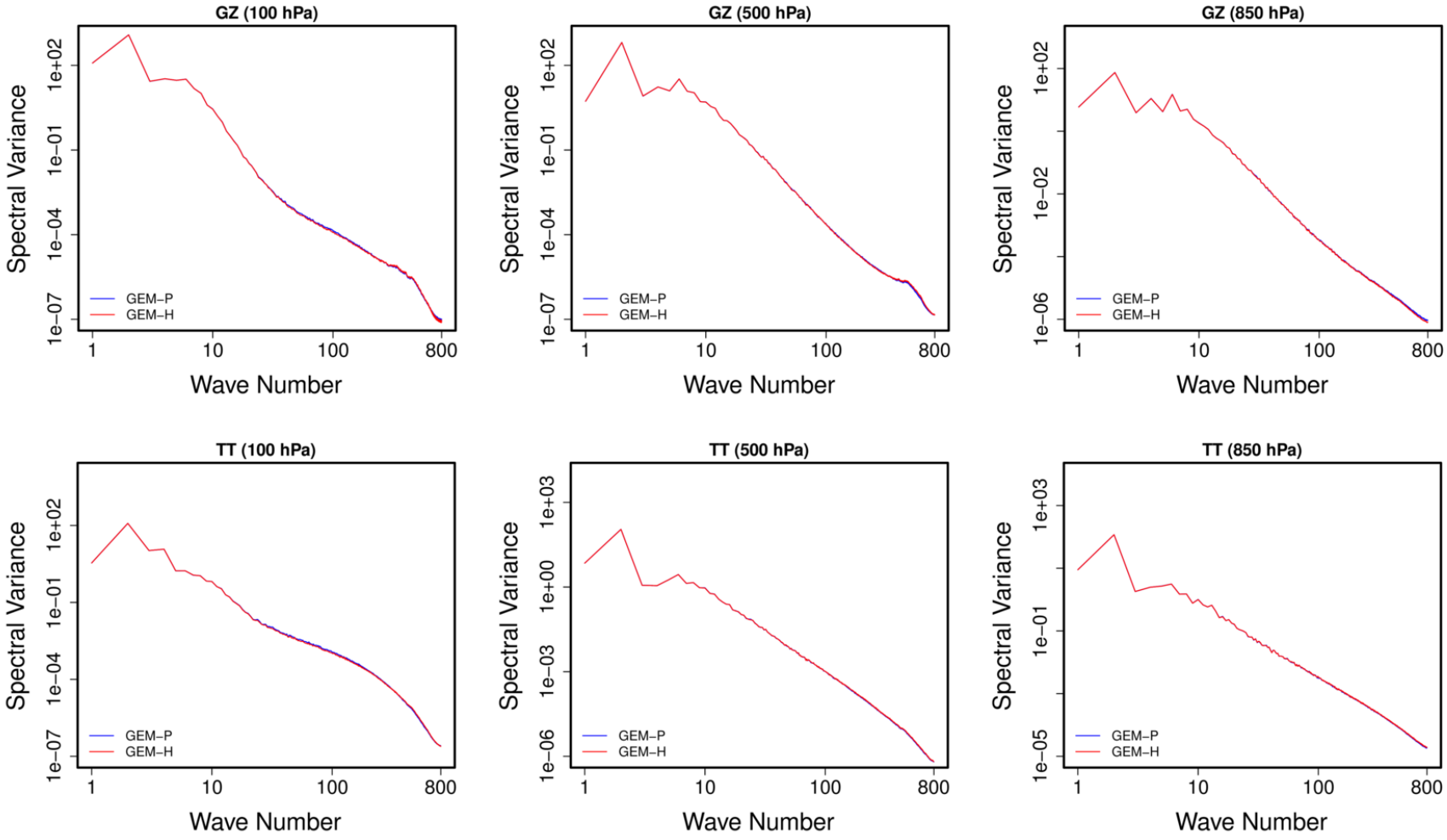
Fig: Vertical velocity contours with interval of 0.1 m s^{-1}

NWP Tests

- At present, we have a functional **GEM-H** dynamical core for both global and limited-area prediction systems.
- The RPN physics package has been coupled to the new dynamical core (without any additional calibration).
- Both direct and iterative solvers are available for **GEM-H**.
- At this point, our objective is to first produce forecasts with **GEM-H** that are equivalent of **GEM-P** for ECCC's different operational prediction system configurations.
- Tests have been conducted for the 25-km GDPS (Global Deterministic Prediction System) setup.
- Results are available for northern hemisphere winter and summer cases.

Variance Spectra

GEM-P vs **GEM-H** (T+120 hr)

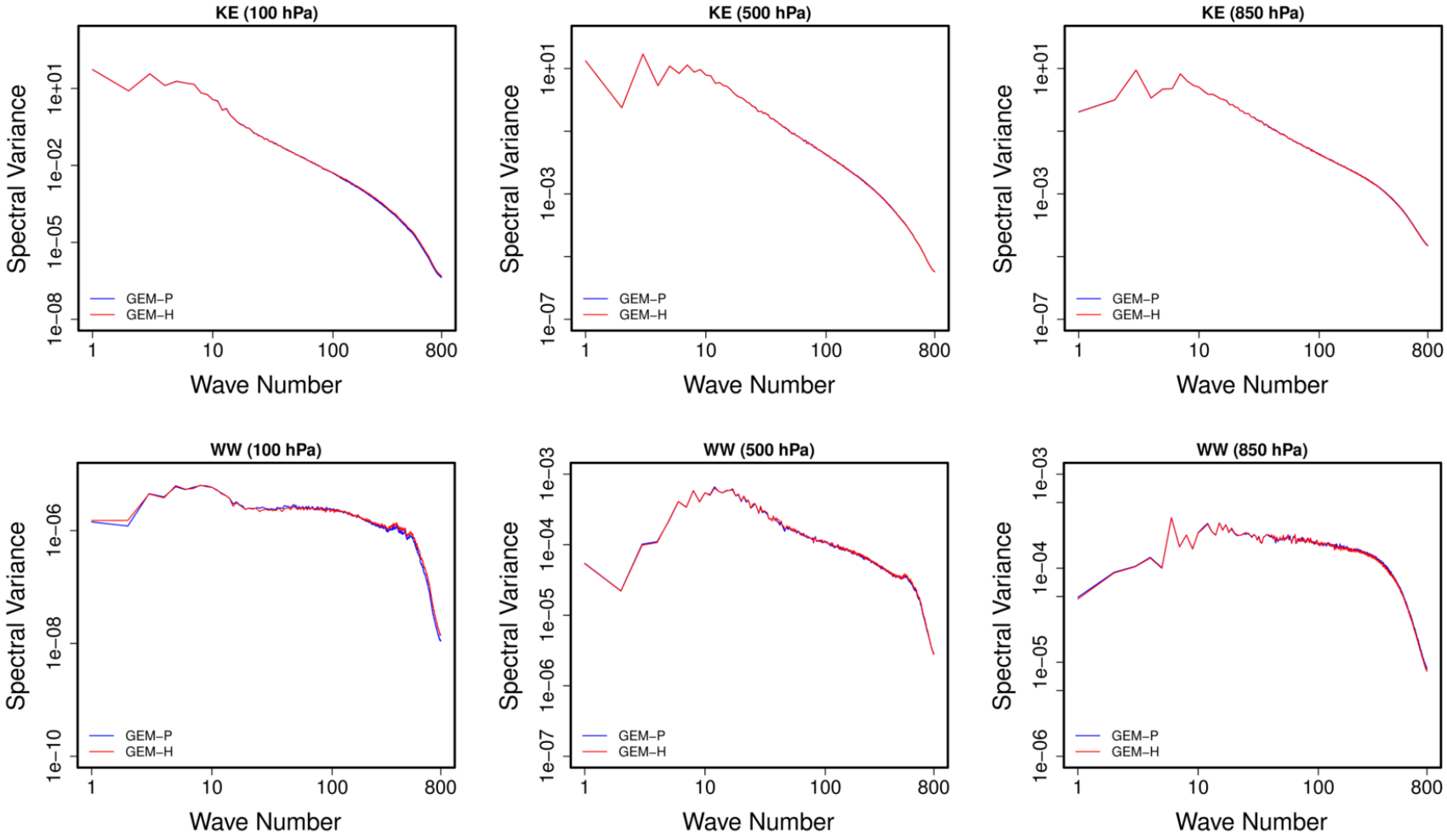


GZ: Geopotential height

TT: Temperature

Variance Spectra

GEM-P vs **GEM-H** (T+120 hr)



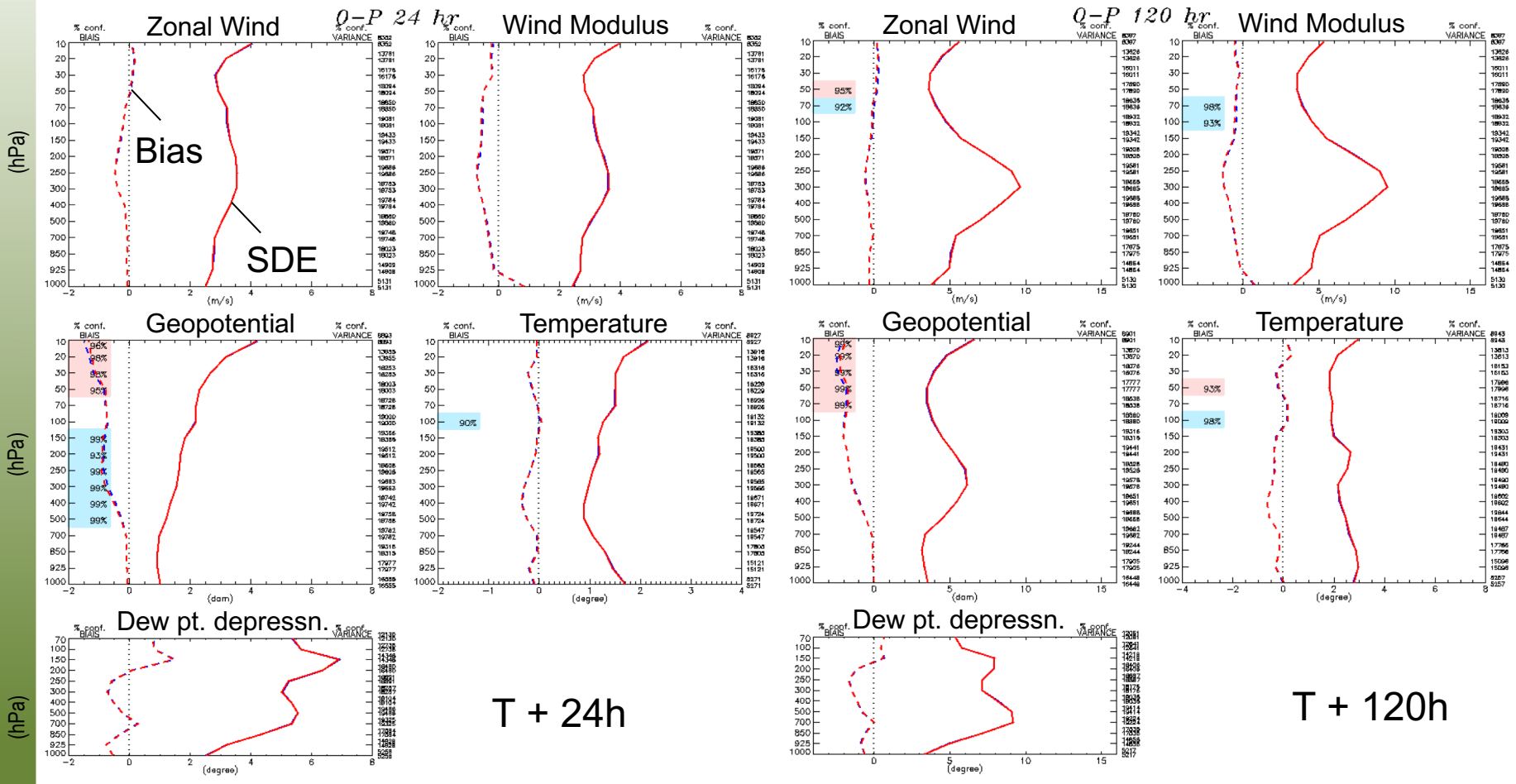
KE: Kinetic Energy

WW: Vertical Motion

Upper Air Scores

GEM-P vs GEM-H

Winter 2015 (Global average of 44 cases)

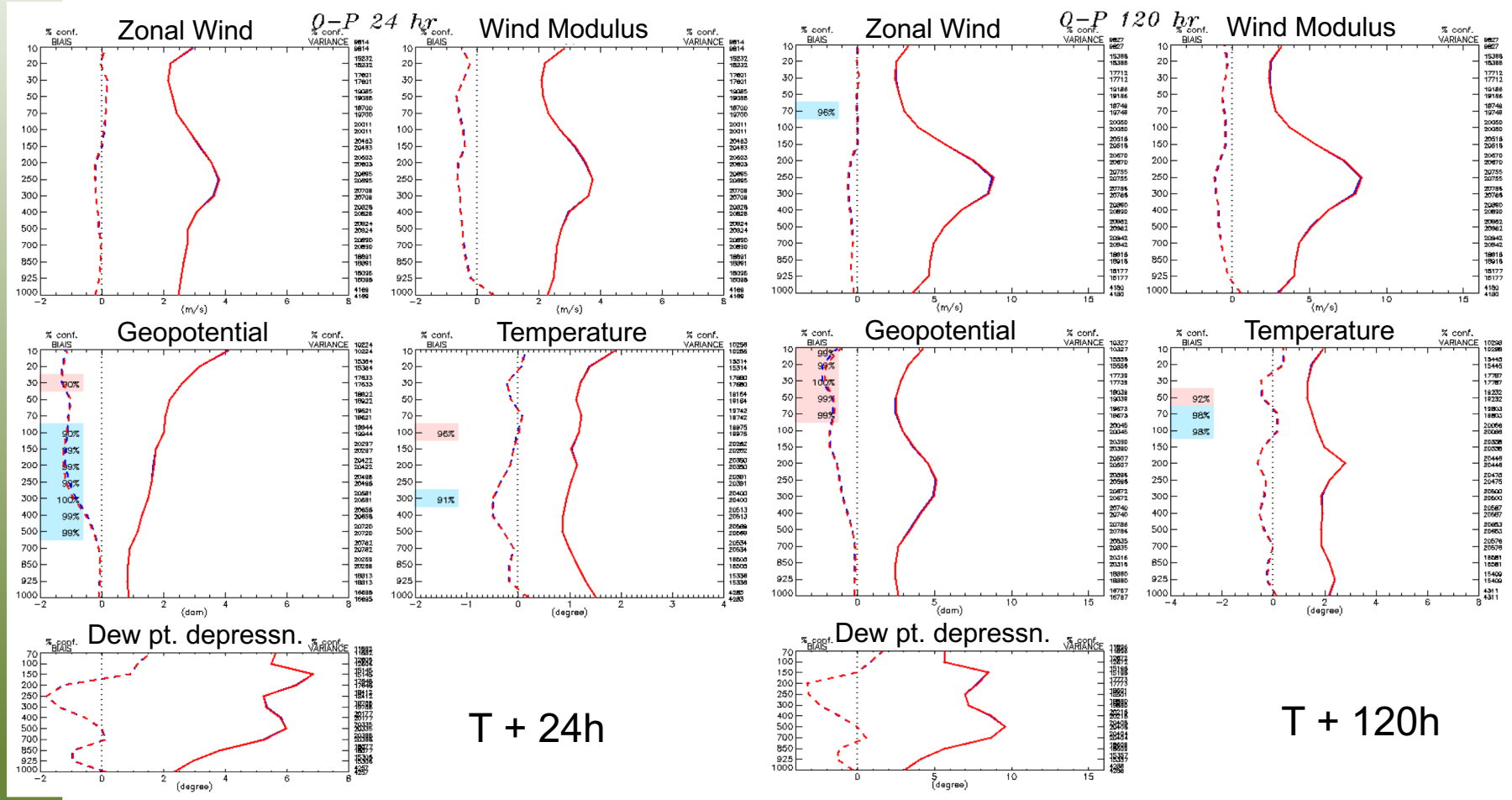


*SDE: Standard Deviation of Error

Upper Air Scores

GEM-P vs GEM-H

Summer 2014 (Global average of 44 cases)



Stability Comparison

GEM-P

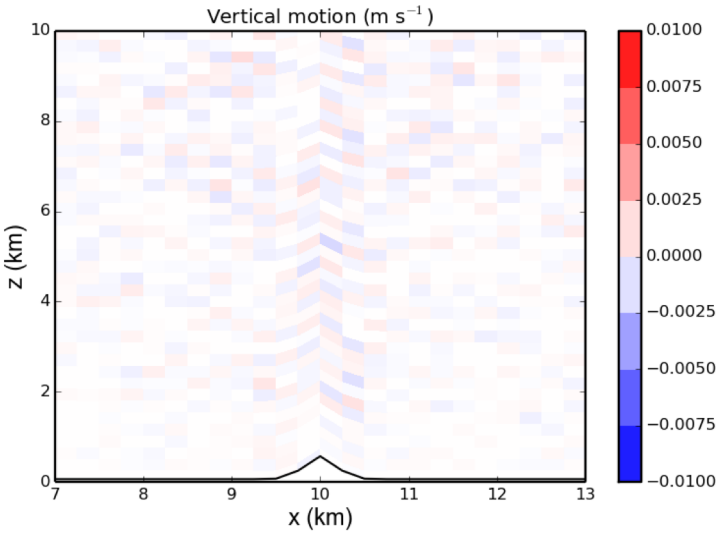
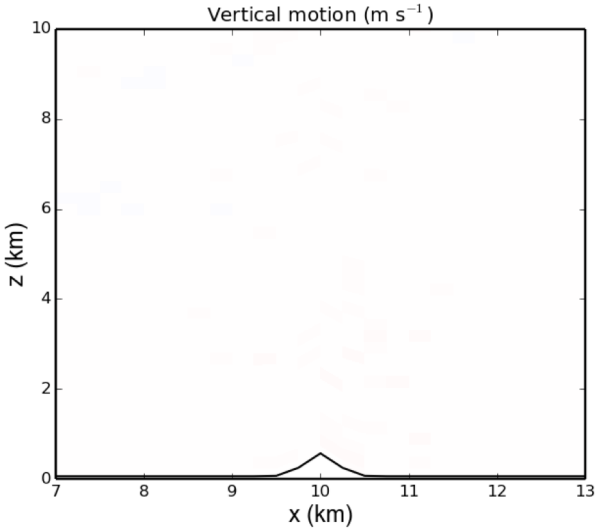


Fig. Vertical motion

H=500 m
Max slope = 51.7°

$b_m=b_h=b_{nh}=0.6$

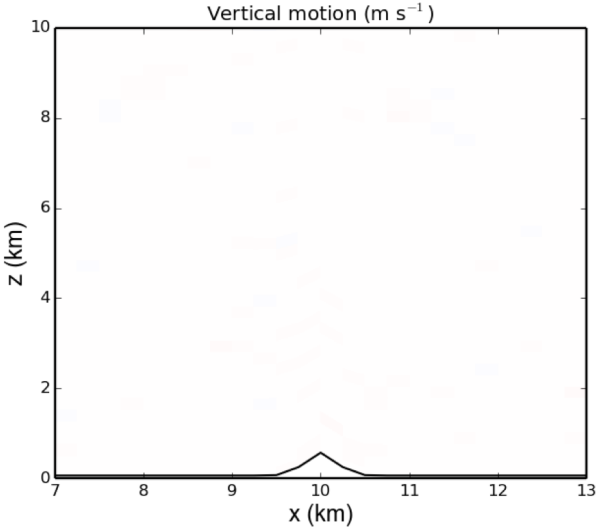
GEM-H (direct)



No flow case:

Δx=Δz=250 m; Δt=10 s
81 x 80 domain (N_i x N_k)
Mountain height, H
$z_s=H\ exp[-\{(x-x_c)/a\}^2]$
where a = 1 km
Stationary flow (u=w=0)
Initial temp. T_0=260 K
Ref. basic state temp. T_*=240 K

GEM-H (iterative)



Stability Comparison

GEM-P

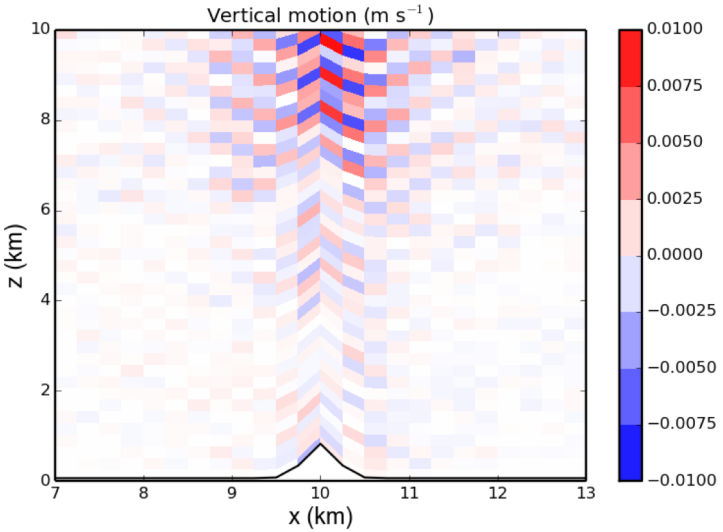
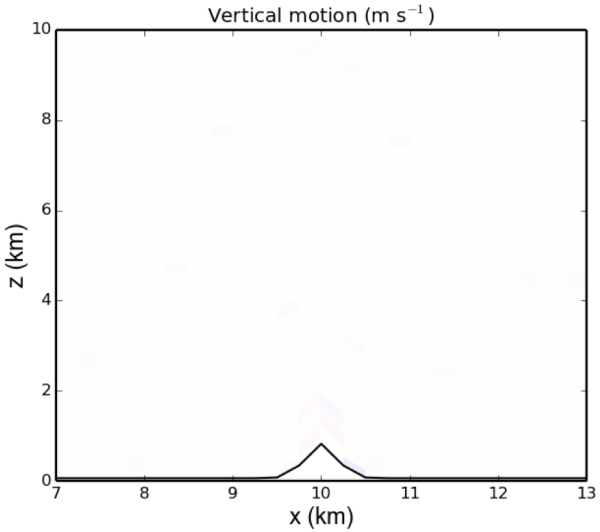


Fig. Vertical motion

H=750 m
Max slope = 62.2°

b_m=b_h=b_{nh}=0.6

GEM-H (iterative)



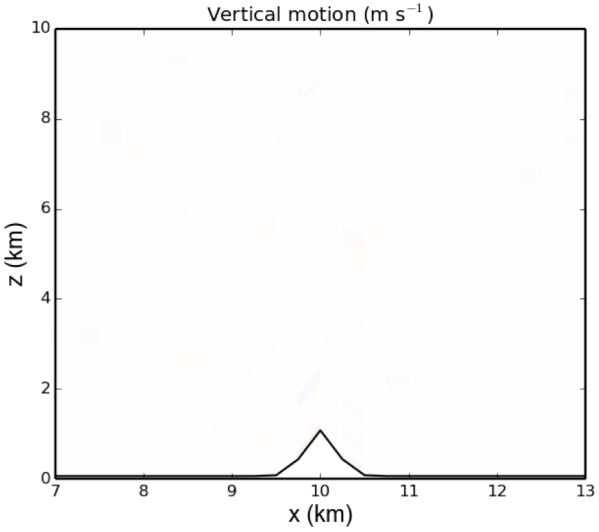
GEM-P

UNSTABLE

H=1000 m
Max slope = 68.4°

b_m=b_h=0.6
b_{nh}=0.75

GEM-H (iterative)



Summary

- The newly developed **GEM-H** dynamical core works for both global and limited-area prediction systems.
- Both direct and iterative solvers are functional for **GEM-H**.
- The RPN physics package has been coupled to the new dynamical core (without any additional calibration).
- For the theoretical cases as well as for 25-km global forecasts with **GEM-H** are found to be equivalent of **GEM-P**.
- Initial tests reveal the potential for **GEM-H** to improve stability over steep orography.
- The simplified approach for **GEM-H** that permits the use of the direct solver works as long as the slope is not too steep.

Appendix

Vertical Coordinates and Solvers: GEM-P vs GEM-H

General form of the EBV (for both **GEM-P** and **GEM-H**):

$$\nabla_H^2 P + M_{VH} P = R$$

where, the subscripts H and V indicate coefficients that vary in the horizontal and the vertical, and M_{VH} includes difference and mean operators in the horizontal and the vertical.

GEM-P

M_{VH} does not include horizontally variable coefficients. The EBV is vertically separable and solve using the fast **direct solver**.

GEM-H

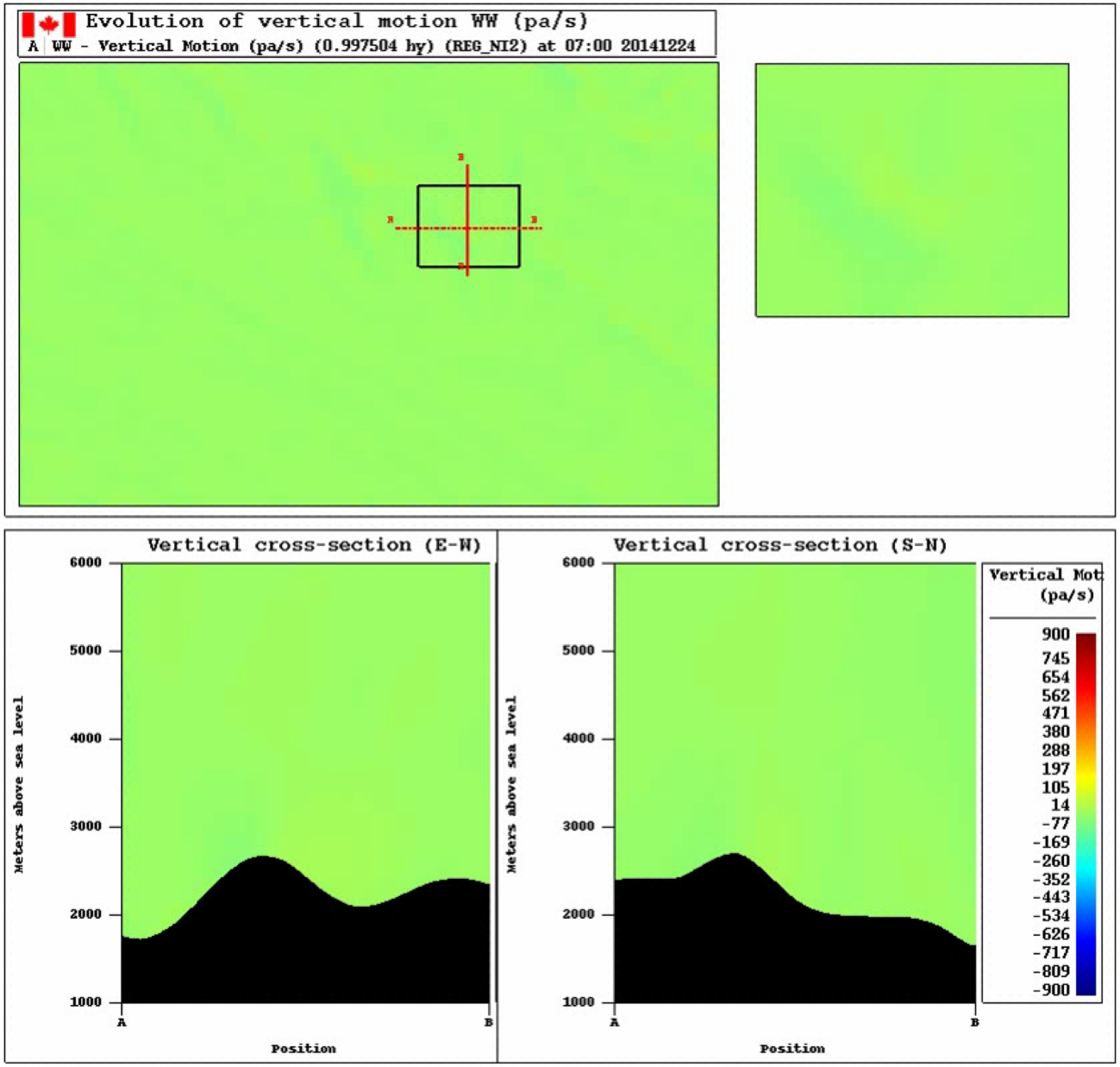
M_{VH} includes horizontally variable coefficients due to the metric terms (unless simplified). The EBV is not vertically separable and solve with an **iterative solver** (unless simplified).

The Solvers

- **Direct solver** is based on the generalized eigenvalue problem (Qaddouri and Lee, 2010).
  - The EBV problem is separated vertically in terms of the vertical eigenvalues of M_{VH}.
  - For N vertical levels, the resulting N number of vertically-decoupled 2D Helmholtz problems are solved using FFT to obtain the full 3D solution.
- **Iterative solver** is based on FGMRES (Qaddouri and Lee, 2010).
  - Solution vector minimizes the residual in a Krylov subspace.
  - Only needs to account for the action of the operator $(\nabla_H^2 + M_V)$ on the vector P (instead of generating the coefficient matrix).
  - Preconditioner is based on the block Jacobi solution of the elliptic problem in the absence of the metric terms.

Qaddouri, A., and Lee, V. (2010): The elliptic solvers in the Canadian limited area forecasting model GEM-LAM. In Shkelzen Cakaj, Editor, Modeling Simulation & Optimization – Tolerance & Oper. Control, 1-17.

Steep Orography-Induced Instability



Height-Based Vertical Coordinate

- The traditional definition of terrain-following height-based vertical coordinate ζ is given by

$$\zeta(z) = H\frac{z - z_s}{z_T - z_s}$$

where, z is the true height, the subscripts S and T denote surface and model top, respectively, and H is a normalizing constant.

- Assuming, $H = z_T = \zeta_T$, we get

$$z = A(\zeta) + B(\zeta)z_s$$

where, $A = \zeta$ and $B = \left(\frac{\zeta_T - \zeta}{\zeta_T}\right) = \lambda$.

- We have, $B = (\lambda)^r$, where $r = [r_{max} - (r_{max} - r_{min})\lambda]$.

Height-Based Vertical Coordinate

- This allows B to decrease more rapidly with height (if needed).
- **SLEVE-**like coordinate (Schär et al., 2002), defined as

$$z = \zeta + B_1 z_{sL} + B_2 (z_s - z_{sL})$$

  is available, where z_{sL} stands for large-scale orography.

- **SLEVE** coordinate permits faster decrease of the effect of small-scale orography, i.e., $(z_s - z_{sL})$, on the model vertical coordinate with increasing height, and helps controlling computational noise in the upper troposphere and stratosphere.

Schär, C., Leuenberger, D., Fuhrer, O., Lüthi, D., and Girard, C. (2002): A new terrain-following vertical coordinate formulation for atmospheric prediction models. Mon. Wea. Rev., 130: 2459-2480.

Pressure-based Vertical Coordinate

- The current definition of vertical coordinate gives

$$\ln\pi = \zeta + Bs$$

where π is the hydrostatic pressure, ζ is the vertical coordinate, $s = \ln(\pi_s/p_{ref})$ with $p_{ref} = 10^5\text{Pa}$ and B is a metric term to determine the flattening of the coordinate with height. Additionally,

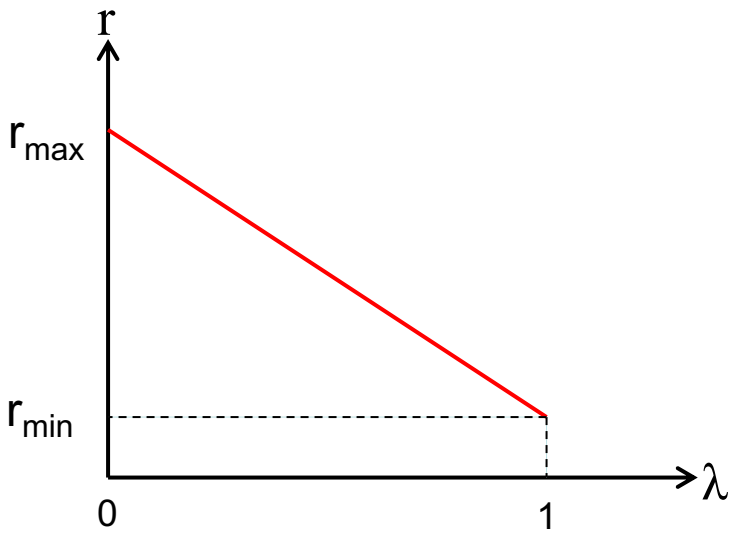
$B = \left(\frac{\zeta-\zeta_T}{\zeta_S-\zeta_T}\right)^r$ where $r = r_{max} - (r_{max} - r_{min})[(\zeta - \zeta_T)/\zeta_S - \zeta_T]$.

- Flattening is controlled by adjusting the values of r_{min} and r_{max}.

Height-Based Vertical Coordinate

- We have, $B = (\lambda)^r$, where $\lambda = \frac{\zeta_T - \zeta}{\zeta_T}$ and.
- This implies that at $\zeta = \zeta_S$, $\lambda = 1$ and at $\zeta = \zeta_T$, $\lambda = 0$.
- We want r to vary in such a way that when $\lambda = 1$, $r = r_{min}$ and $\lambda = 0$, $r = r_{max}$. This gives

$$r = [r_{max} - (r_{max} - r_{min})\lambda]$$



Final Dynamical System of Eqn.

Discretization in space leads to –

$$\frac{du}{dt} - \left(f + \frac{\tan\theta}{a}u\right)\bar{v}^{XY} + \frac{\bar{T}^{X\zeta}}{T_*}\left(\delta_X q - \mathrm{J}_X\overline{\mathrm{J}_\zeta^{-1}\delta_\zeta q}^{X\zeta}\right) = 0$$

$$\frac{dv}{dt} + \left(f + \frac{\tan\theta}{a}\bar{u}^{XY}\right)\bar{u}^{XY} + \frac{\bar{T}^{Y\zeta}}{T_*}\left(\delta_Y q - \mathrm{J}_Y\overline{\mathrm{J}_\zeta^{-1}\delta_\zeta q}^{Y\zeta}\right) = 0$$

$$\frac{dw}{dt} + \frac{T}{T_*}\left(\mathrm{J}_\zeta^{-1}\delta_\zeta q - g\frac{T'}{T}\right) = 0$$

$$\frac{d}{dt}\left[\ln\left(\frac{T}{T_*}\right) - \frac{\bar{q}^\zeta}{c_p T_*}\right] + \mu w = 0$$

$$\frac{d}{dt}\left(\frac{q}{c_*^2} + \ln\mathrm{J}_\zeta\right) + \delta_X u + \frac{1}{\cos\theta}\delta_Y(\cos\theta\, v) + \delta_\zeta\dot{\zeta} - \varepsilon\bar{w}^\zeta = 0$$

$$\frac{d}{dt}(z-\zeta) + \ddot{\zeta} - w = 0$$

where δ and $\overline{(\cdot)}$ denote difference and averaging operators.

Vertical Grid

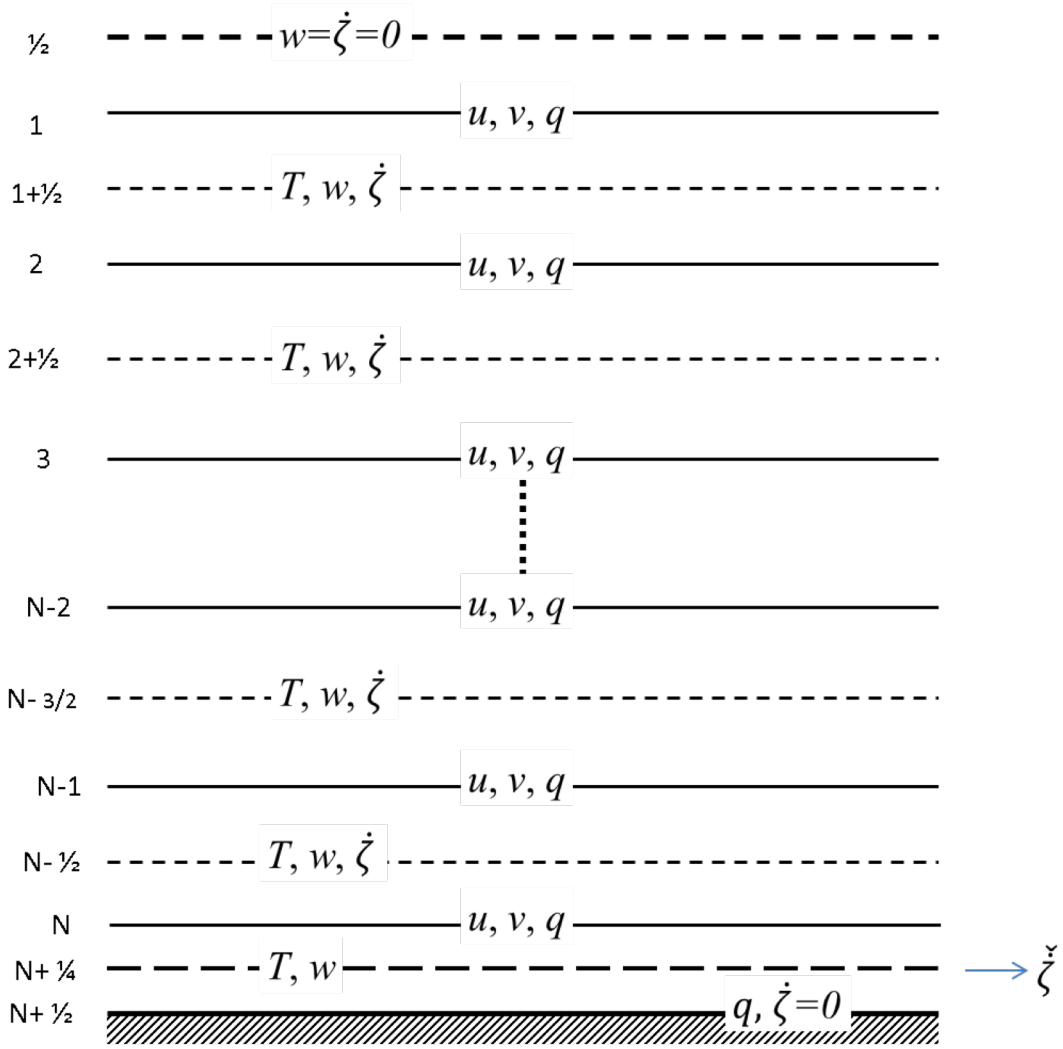


Fig. Vertical Charney-Phillips grid.

Horizontal Grid

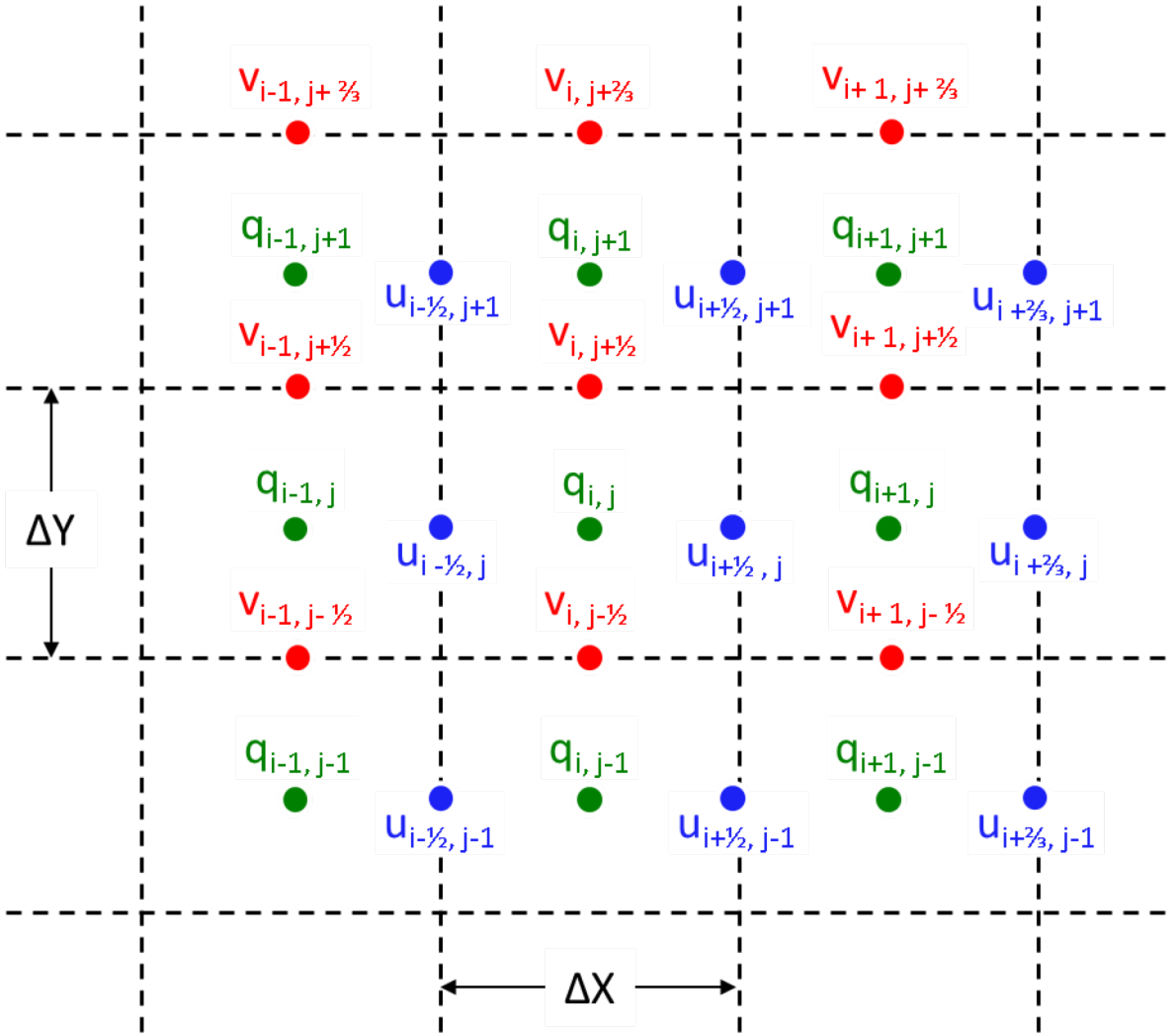


Fig. Horizontal Arakawa C grid.

Theoretical Test Cases

- At the early stage of development, the GEM-H dynamical core was tested against GEM-P for a number of theoretical cases to identify

  - possible issues around advection, and

  - its response to orography and nonhydrostatic scenario.

- Two of the theoretical cases were

  - Robert's bubble convection test (Robert, 1993) and
  - Schar's mountain wave case (Schär et al., 2002).

Robert, A. (1993): Bubble convection experiments with a semi-implicit formulation of the Euler equations. J. of Atmos. Sciences, 50: 1865-1873.

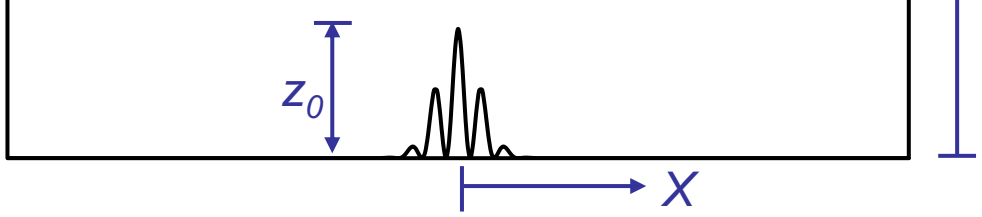
Schär, C., Leuenberger, D., Fuhrer, O., Lüthi, D., and Girard, C. (2002): A new terrain-following vertical coordinate formulation for atmospheric prediction models. Mon. Wea. Rev., 130: 2459-2480.

The Schär Mountain Case

A Theoretical Non-Hydrostatic Case

Schär case: Dry flow past idealized topography

$$z_s = z_0 \exp\left[-\left(\frac{x}{a}\right)^2\right] \cos^2\left(\frac{\pi x}{l_x}\right)$$



$U = 10\ \mathrm{ms}^{-1}$
$N = 0.01\ \mathrm{s}^{-1}$
$T_0 = 288\ \mathrm{k}$

$L = 200\ \mathrm{km}$ $H = 19.5\ \mathrm{km}$
$N_i = 401$ $N_k = 65$
$\Delta x = 500\ \mathrm{m}$ $\Delta z = 300\ \mathrm{m}$
$z_0 = 250\ \mathrm{m}$ $a = 5\ \mathrm{km}$
$l_x = 4\ km = 8\Delta x$

Schär, C., Leuenberger, D., Fuhrer, O., Lüthi, D., and Girard, C. (2002): A new terrain-following vertical coordinate formulation for atmospheric prediction models. Mon. Wea. Rev., 130: 2459-2480.

2D Mountain Wave: Nonhydrostatic

- **Off-centering** leads to **perturbations** in both GEM-H and GEM-P.

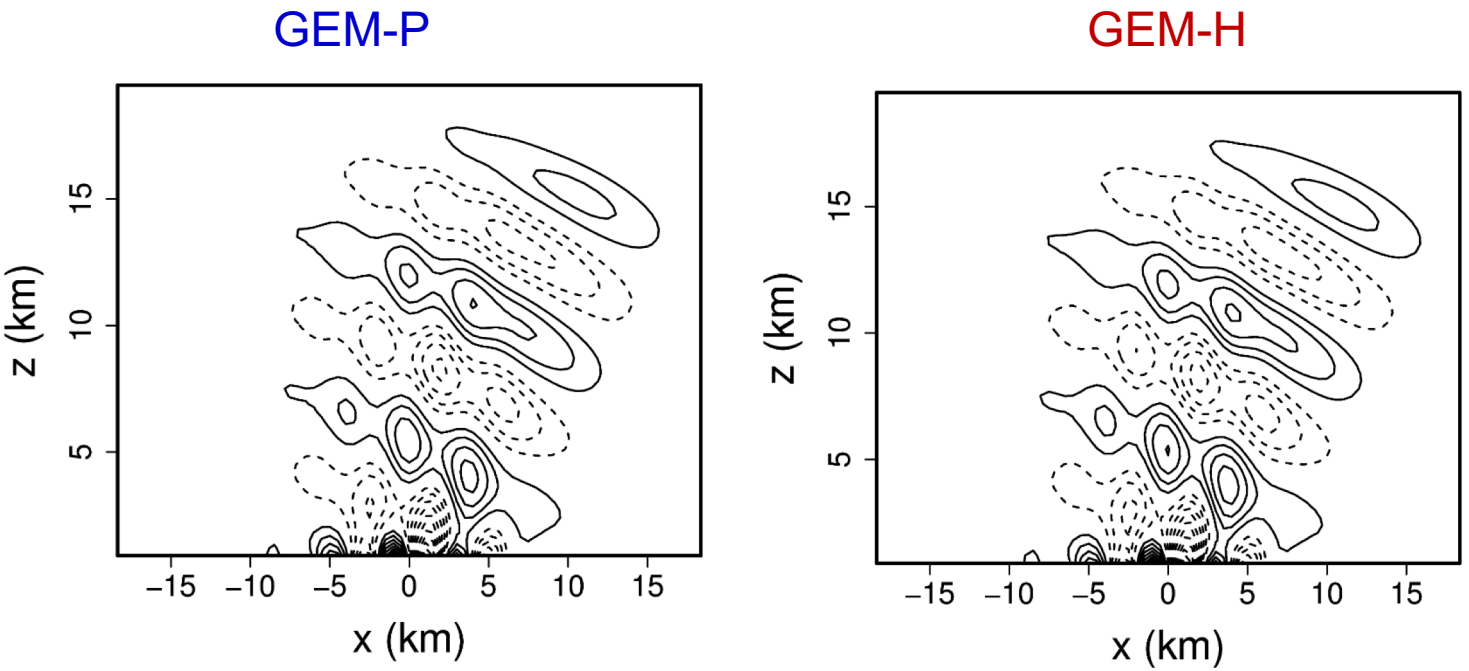


Fig: Vertical velocity contours with interval of 0.1 m s^{-1} ($b_A = 0.6$) .

- Perturbations result from **inconsistent trajectory calculations** (Husain and Girard, 2017).

Husain, S.Z.., and Girard, C. (2017): Impact of consistent semi-Lagrangian trajectory calculations on numerical weather prediction performance. Mon. Wea. Rev., 145: 4127-4150.

2D Mountain Wave: Nonhydrostatic

- Problem is resolved with similar off-centering employed in the discretized trajectory equations.

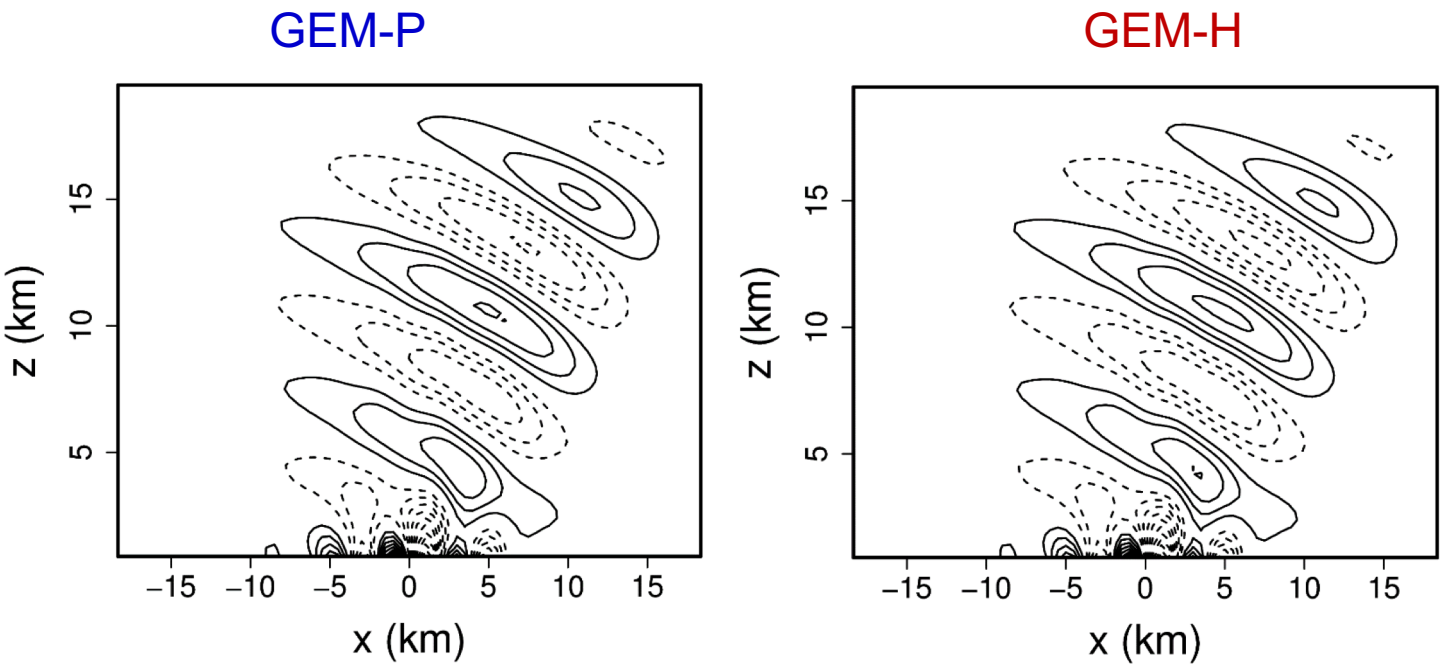
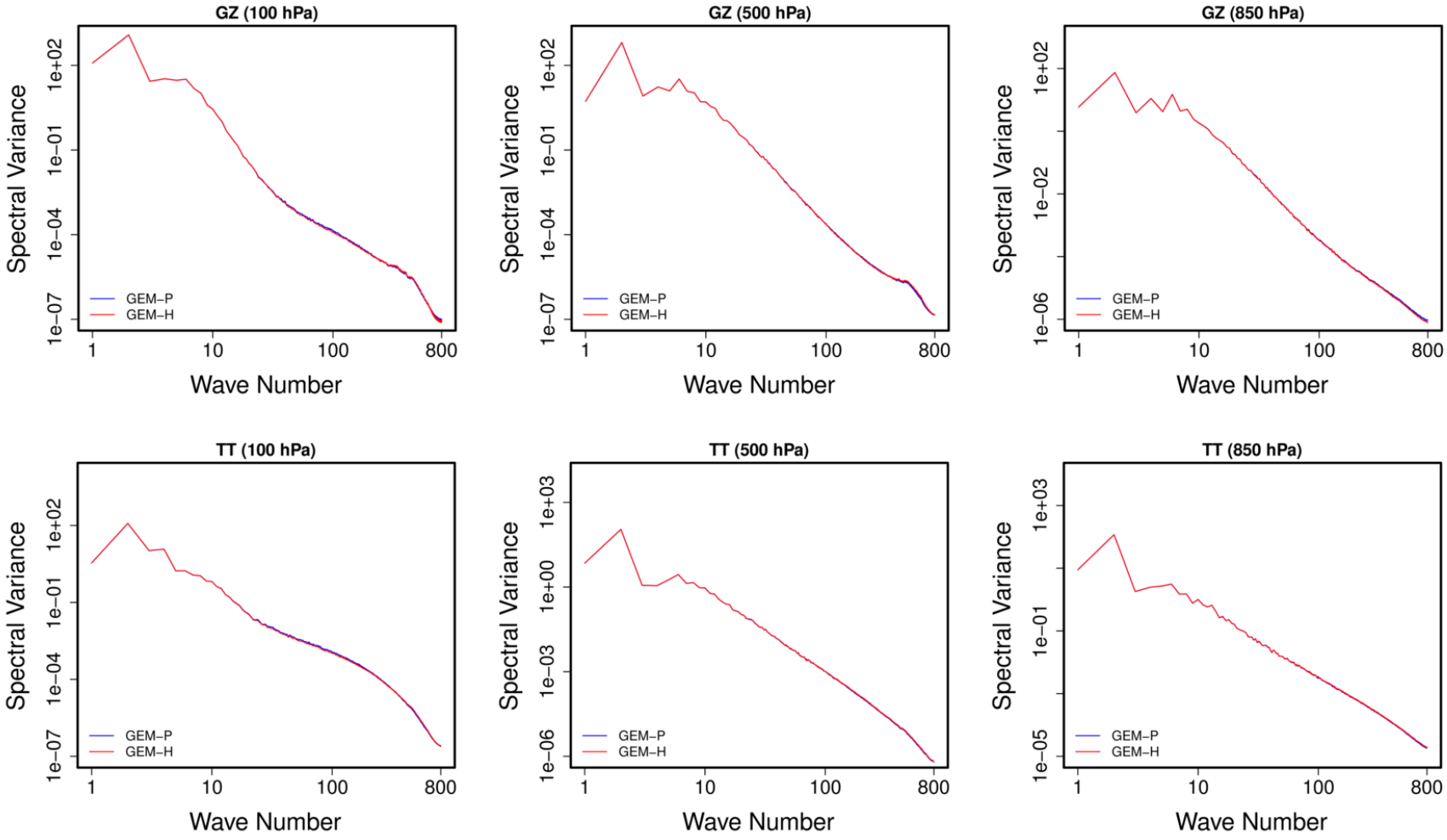


Fig: Vertical velocity contours with interval of 0.1 m s^{-1} ($b_A = 0.6$) with consistent trajectory calculations (Schm_advec = 2).

Variance Spectra

GEM-P vs **GEM-H** (T+120 hr)

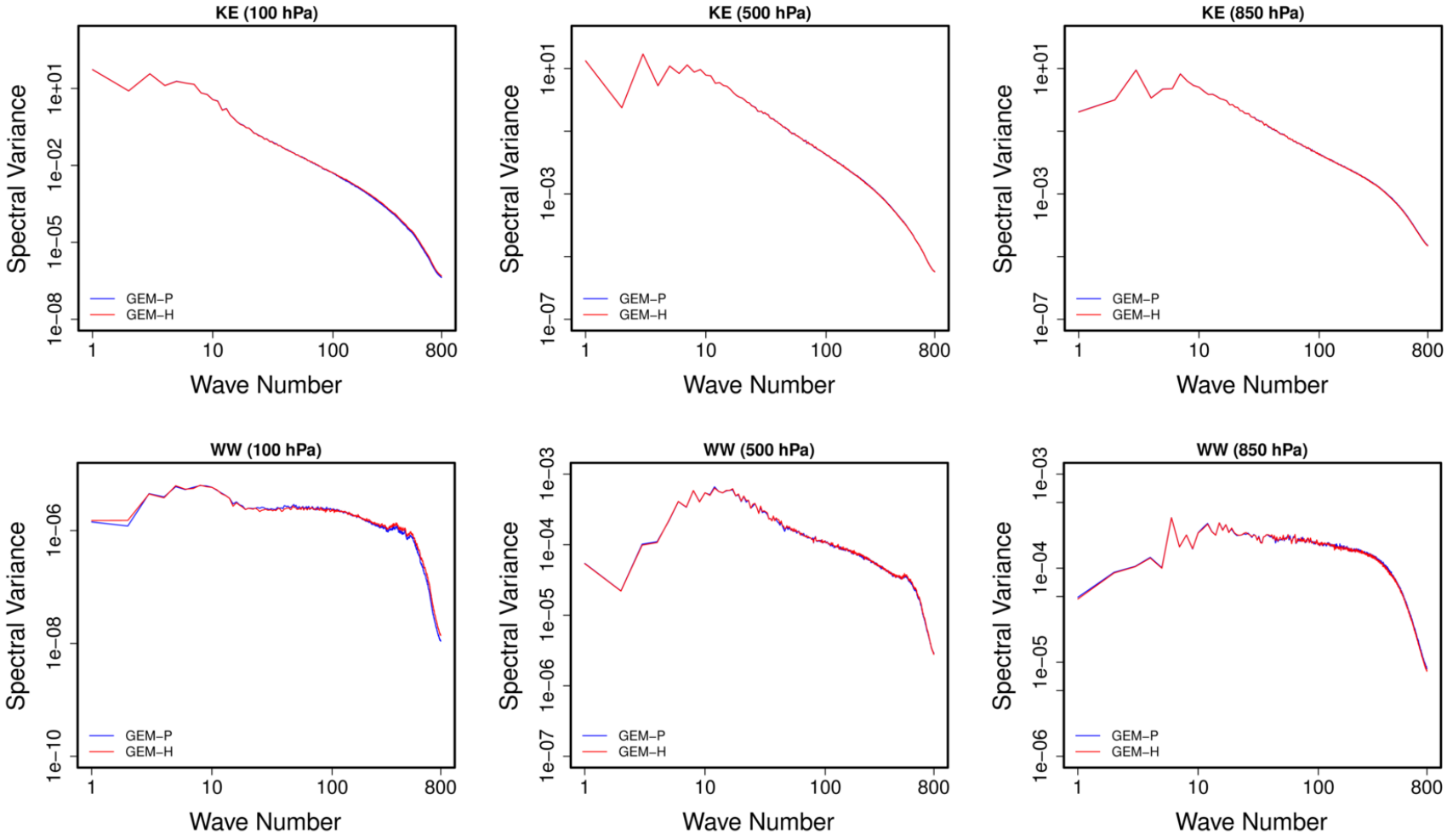


GZ: Geopotential height

TT: Temperature

Variance Spectra

GEM-P vs **GEM-H** (T+120 hr)



KE: Kinetic Energy

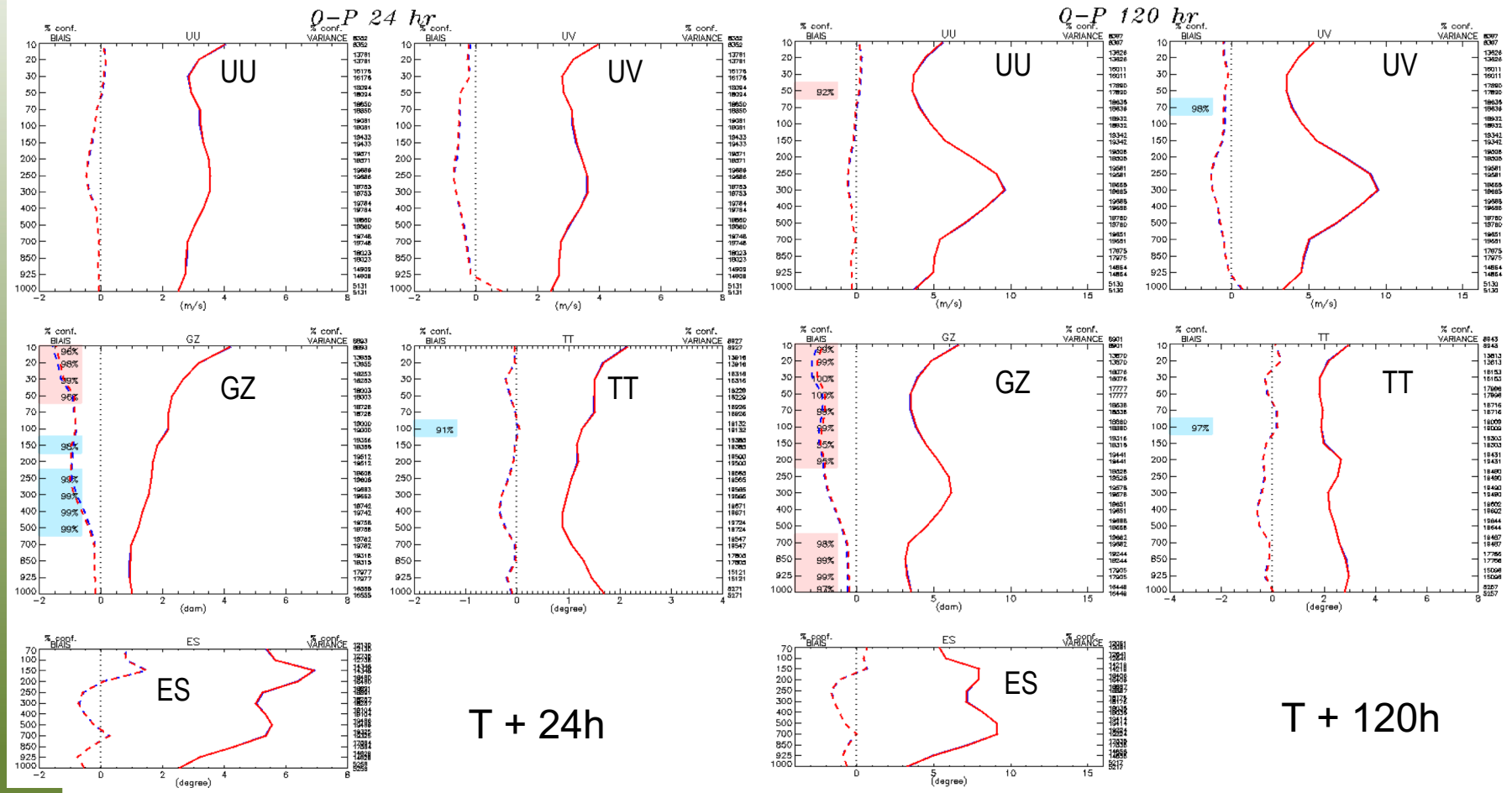
WW: Vertical Motion

Comparison Against Observations

GEM-P vs GEM-H

Winter 2015 (Global average of 44 cases)

Without mass fixer



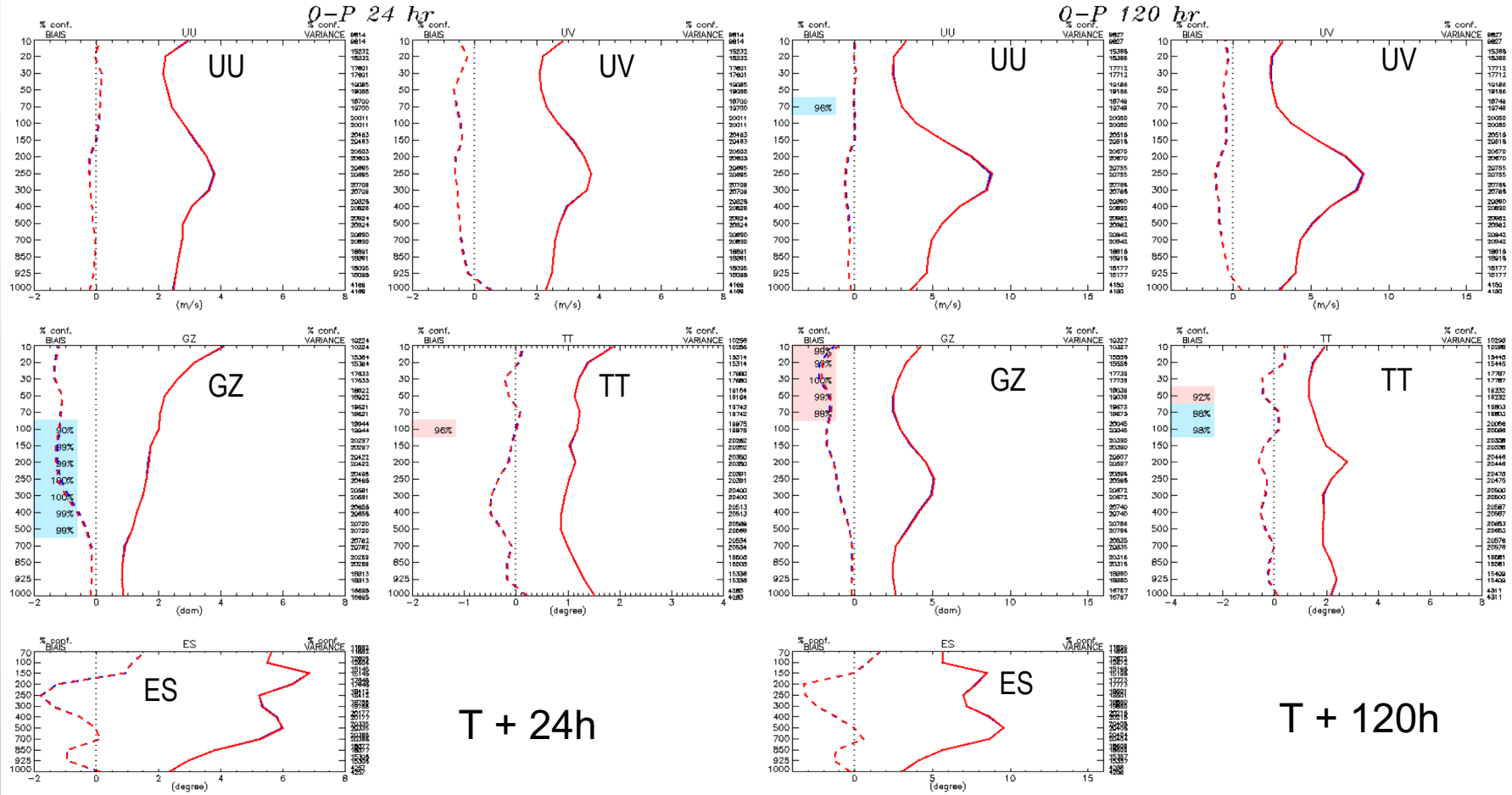
UU: Zonal Wind, UV: Wind Speed, GZ: Geopotential Height, TT: Temperature, ES: Dew Point Depression

Comparison Against Observations

GEM-P vs GEM-H

Summer 2014 (Global average of 44 cases)

Without mass fixer



UU: Zonal Wind, UV: Wind Speed, GZ: Geopotential Height, TT: Temperature, ES: Dew Point Depression

Work in Progress

- **NWP:** Modifying and adapting the model code to permit GEM-H work with IAU, digital filter, etc.
- **Solver:** Developing highly optimized 3D iterative solvers for both GEM-P and GEM-H.
- **Stability:** Exploring strategies to improve stability of GEM-H over steep orography.

  - Initial tests revealed stability improvements with GEM-H.
  - Improved numerical approximation of horizontal gradients in the terrain-following coordinate (Zängl, 2012; Mahrer, 1984) may also be explored in GEM-H.

Zängl, G. (2012): Extending the numerical stability limit of terrain-following coordinate models over steep slopes. Mon. Wea. Rev., 140: 3722-3733.
Mahrer, Y. (1984): An improved numerical approximation of the horizontal gradients in a terrain-following coordinate system. Mon. Wea. Rev., 112: 918-922