A total energy error analysis of dynamical cores and physics-dynamics coupling in the Community Atmosphere Model (CAM)

Peter H. Lauritzen & David L. Williamson
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Workshop on the solution to partial differential equations on the sphere, April 29 – May 3, 2019, Montréal, Québec, Canada
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Research question

How large are the spurious total energy sources/sinks in an atmosphere model and where are they coming from?
Total energy (TE) equation - moist atmosphere

\[ \frac{d\widehat{E}}{dt} = \widehat{F}_{\text{net}}, \]

where

\[ \frac{d\widehat{E}}{dt} = \frac{d}{dt} \left\{ \frac{1}{\Delta S} \int_{\eta=0}^{\eta=1} \int_{S} \left( \frac{1}{g} \frac{\partial M^{(d)}}{\partial \eta^{(d)}} \right) \sum_{\ell \in L_{\text{all}}} \left[ m^{(\ell)} \left( K + c_p^{(\ell)} T + \Phi_s \right) \right] dA d\eta^{(d)} \right\}, \]

and

\[ \widehat{F}_{\text{net}} = \frac{1}{\Delta S} \int_{\eta=0}^{\eta=1} \int_{S} \left( \frac{1}{g} \frac{\partial M^{(d)}}{\partial \eta^{(d)}} \right) \sum_{\ell \in L_{\text{all}}} \left[ m^{(\ell)} \right] \widehat{F}_{\text{net}} dA d\eta^{(d)}. \]

where \( \Delta S \) is the surface area of the sphere, \( \Phi_s \) is the surface geopotential and \( \widehat{\cdot} \) refers to the global average.

Lauritzen et al. (2018); https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2017MS001257
Total energy (TE) equation
- moist atmosphere

\[ \frac{d\hat{E}}{dt} = \hat{F}_{net}, \]

The continuous equations of motion on which the dynamical core is based conserve TE globally:

\[ \frac{d\hat{E}}{dt} = 0 \]
Total energy (TE) equation - moist atmosphere

\[
\frac{d\hat{E}}{dt} = \hat{F}_{\text{net}},
\]

Conserving total energy to within \(~0.01\) W/m\(^2\) is considered “good enough” for coupled climate modeling (Boville, 2000; Williamson et al., 2015)

Earth’s energy imbalance is \(~1\) W/m\(^2\)
Total energy (TE) equation
- moist atmosphere

\[ \frac{d \hat{E}}{dt} = \hat{F}_{\text{net}}, \]

Column physics: TE change in column should be balanced by fluxes in/out of the top and bottom

\[ \frac{d \hat{E}}{dt} = \frac{1}{\Delta S} \int \int_S \left( p_{\text{top}} F_{\text{net}} - p_s F_{\text{net}} \right) dA. \]
Potential spurious sources/sinks of total energy in an atmosphere model:

- **Parameterization errors:** Individual parameterizations may not have a closed energy budget. CAM parameterizations are required to have a closed energy budget under the assumption that pressure remains constant during the computation of the subgrid-scale parameterization tendencies. In other words, the TE change in the column is exactly balanced by the net sources/sinks given by the fluxes through the column.

- **Pressure work:** That said, if parameterizations update specific humidity then the surface pressure changes (e.g., moisture entering or leaving the column). In that case the pressure changes which, in turn, changes TE. This is referred to as pressure work [section 3.1.8 in Neale et al., 2012].

- **Continuous TE formula discrepancy:** If the continuous equations of motion for the dynamical core conserve a TE different from the one used in the parameterizations then an energy inconsistency is present in the system as a whole. In CAM this mismatch arose from the evolutionary nature of the model development and not by deliberate design; and should be eliminated in the future.

- **Dynamical core errors:** Energy conservation errors in the dynamical core, not related to physics-dynamics coupling errors, can arise in multiple parts of the algorithms used to solve the equations of motion.

- **Physics-dynamics coupling (PDC):** Assume that physics computes a tendency. Usually the tendency (forcing) is passed to the dynamical core which is responsible for adding the tendencies to the state.
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**TE errors in the CAM spectral-element dynamical core:**

- **Horizontal inviscid dynamics:** Energy errors resulting from solving the inviscid, adiabatic equations of motion.

- **Hyperviscosity:** Filtering errors; Note that we use frictional heating:

\[
\rho c_p \delta T = -\frac{1}{2} \rho \mathbf{v} \cdot \delta \mathbf{v} \Rightarrow \delta T = -\frac{1}{2 c_p} (\mathbf{v} \cdot \delta \mathbf{v}),
\]

(p.71 in ; Neale et al., 2012). As shown in the results section 4.2 this term is rather large and therefore important for good energy conservation characteristics of the dynamical core.

- **Vertical remapping:** The vertical remapping algorithm from Lagrangian to Eulerian reference surfaces does not conserve TE.

- **Near round-off negative values of water vapor which are filled to a minimal value without compensation.**
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Temporal physics-dynamics coupling methods

(a) Initial state & forcing
(b) Apply forcing (ftype=1)
(c) Advection (ftype=1)

State-update method

Mixing ratio

No physics-dynamics coupling error:

(Dry) Energy change due to physics energy increments

\[ \Delta M^{(d)} \Delta T = \Delta M^{(d)} \left[ (\Delta u)^2 + (\Delta v)^2 \right] \Delta m^{(\ell)} \Delta M^{(d)} \]

= Dynamics energy change due to physics forcing
1 year average |dps/dt|; AMIP run

CAM-SE, cpdry, ftype=1 (state-update)

Absolute surface pressure tendency

<table>
<thead>
<tr>
<th>Pa/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02</td>
</tr>
<tr>
<td>0.029</td>
</tr>
<tr>
<td>0.038</td>
</tr>
<tr>
<td>0.047</td>
</tr>
<tr>
<td>0.056</td>
</tr>
<tr>
<td>0.065</td>
</tr>
<tr>
<td>0.074</td>
</tr>
</tbody>
</table>
Temporal physics-dynamics coupling methods

(a) Initial state & forcing
(b) Apply forcing (ftype=1)
(c) Advection (ftype=1)
(d) Initial state & ½ forcing
(e) Apply ½ forcing & advection
(f) Repeat (e)

“Dribbling”
1 year average $|\text{dps/dt}|$; AMIP run
Temporal physics-dynamics coupling methods

- Thermal energy “dribbling” error: Thermal energy increment from physics
  \[ \Delta M^{(d)} \Delta T \]
  does not match thermal energy change in dycore when tendency is added to dycore state.
- Kinetic energy “dribbling” error:
  \[ \Delta M^{(d)} [(\Delta u)^2 + (\Delta v)^2] \]
- Mass “clipping” error: e.g., if logic in dycore to prevent negative mixing ratios
ftype=2: state-updating (type=1) for tracers (i.e. no mass-clipping errors) and “dribbling” (ftype=0) for u, v, and T.
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Fixing spurious sources/sinks of total energy in an atmosphere model:

- **Compensating Energy fixers:** To avoid TE conservation errors which could accumulate and ultimately lead to a climate drift, it is customary to apply an arbitrary energy fixer to restore TE conservation. Since the spatial distribution of many energy errors, in general, is not known, global fixers are used. In CAM a uniform increment is added to the temperature field to compensate for TE imbalance from all processes, i.e. dynamical core, physics-dynamics coupling, TE formula discrepancy, energy change due to pressure work, and possibly parameterization errors if present.
Spurious sources/sinks of total energy in atmosphere model:

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### TE errors in the CAM spectral-element dynamical core (break-down):

- **Horizontal inviscid dynamics:** Energy errors resulting from solving the inviscid, adiabatic equations of motion. **\(-0.01 \text{ W/m}^2\)**

- **Hyperviscosity:** Filtering errors (frictional heating is used!). **\(-0.59 \text{ W/m}^2\)**

  **Frictional heating is \(-0.58 \text{ W/m}^2\)**

  Note that if no frictional heating is used then TE error would be \(-1 \text{ W/m}^2\).

- **Vertical remapping:** The vertical remapping algorithm from Lagrangian to Eulerian reference surfaces does not conserve TE. **\(-0.01 \text{ W/m}^2\)**

- **CAM-SE: PDC errors (“dribbling”):** **\(-0.01 \text{ W/m}^2\)**

- **CAM-FV and CAM-FV3:** **\(-1.1 \text{ W/m}^2\)**
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- **Physics-dynamics coupling (PDC):** Assume that physics computes a tendency. Usually the tendency (forcing) is passed to the dynamical core which is responsible for adding the tendencies to the state. Budget closed in CAM (except for small “clipping” errors) but ...

  - **Pressure work:** ~0.3 W/m²
  - **TE formula discr. (CAM-SE only):** ~0.6 W/m²
  - **CAM-SE:** ~-0.6 W/m² (decreases to ~0.3 W/m² with smoother topography)
  - **CAM-FV and CAM-FV3:** ~ -1.1 W/m²
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Note that there are compensating errors in the system -> need to do detailed TE budget analysis!
Spurious sources/sinks of total energy in atmosphere model:

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**Summary of TE conservation**

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*TE conservation must be assessed with moist physics forcing and ‘real-world’ topography!*
Summary

- Total energy errors in numerical discretizations (dynamical core), physics-dynamics coupling and pressure work errors are \(-0.6 - 0.3\) W/m\(^2\).
- Local errors can be an order of magnitude larger (at least)!

Outlook

- In next-generation models we should consider formulating physics in dry pressure coordinates (so that coordinate surfaces stay fixed during physics updates).
- Can we close the total energy budget locally in models?
- Integrating weather-climate models: parameterizations for weather models are, in general, not formulated to have a closed TE budget. Major challenge?
TE tendencies for the default CAM-SE configuration (AMIP)

- **Total physics TE tendency**
  - Global min = -148.3
  - Global max = 1770

- **Pressure work error TE tendency (pwork)**
  - Global min = -195.3
  - Global max = 32.32

- **Adiabatic dycore TE tendency (adiab)**
  - Global min = -1490
  - Global max = 122.2

- **2D adiabatic dynamics TE tendency (2D)**
  - Global min = -1490
  - Global max = 122.2
Not SPURIOUS locally – should integrate to 0.
TE tendencies for the default CAM-SE configuration (AMIP)

(a) Total physics TE tendency
(b) Pressure work error TE tendency (swork)
(c) Adiabatic core TE tendency (adcore)
(d) 2D adiabatic dynamics TE tendency (2Dad)
(e) Vertical remapping TE tendency (remap)
(f) Frictional heating TE tendency (fricheat)
PARAMETERIZATIONS:

- Last dynamics state received from dynamics
- Output 'pBF'
- Energy fixer
- Output 'pBP'
- Physics updates the state and state saved for energy fixer
- Output 'pAP'
- Pressure work (dry mass correction)
- Output 'pAM'
- Physics tendency (forcing) passed to dynamics

DYNAMICAL CORE

- Output 'dED'
- Do ns=1,nsplit
- Output 'dAF'
- START PHYSICS-DYNAMICS COUPLING
  - Update dynamics state with (1/nsplit) of physics tendency (ftype=2)
  - If (ns=1) Update dynamics state with entire physics tendency (ftype=1)
- DONE PHYSICS-DYNAMICS COUPLING
  - Output 'dBD'
  - Do nr=1,rsplit
    - Advance the adiabatic frictionless equations of motion in floating Lagrangian layer
    - Do ns=1,hypervis subcycle
    - Output 'dDH'
      - Apply hyperviscosity operators
      - Output 'dCH'
        - Add frictional heating to temperature
        - Output 'dAH'
          - End do (ns=1,hypervis subcycle)
          - End do (nr=1,rsplit)
    - Output 'dAD'
      - Vertical remapping from floating Lagrangian levels to Eulerian levels
      - Output 'dAR'
        - End do (ns=1,nsplit)
        - Dynamics state saved for next model time step and passed to physics
        - Output 'dBF'
  - End do (nt=1,ntotal)

Diagnosing TE errors:

- Implemented using CAM history infrastructure by computing column integrals of energy at various places in CAM and outputting the 2D energy fields.
- CAM history internally handles accumulation and averaging in time at each horizontal grid point.