Conservative compact coupled space-time constancy-preserving schemes for vertical transport in oceanic and atmospheric models

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Abstract

The state-of-the-art coupled schemes, fail to retain the formal order of accuracy when used for nonlinear transport. This problem is fixed by developing a space-time generalization of the fourth-order semi-implicit conservative compact schemes in this study. The scheme is obtained in flux-form by correcting the successive error terms in Taylor series expansion in time and space. Space-time order of accuracy is successfully maintained for the solution of nonlinear scalar transport. The scheme is coupled in space and time, conservative in flux-form, semi-implicit in time, and compact due to its tri-diagonal matrix. The approach includes variable velocities and non-uniform grids on the adapted 1-D coordinate. For example, vertical grid refinement for oceanic models remains an issue to consider in particular because it requires very robust numerics to properly integrate vertical advective terms in a stable way. Traditionally, centered advection schemes are used in the vertical to avoid excessive numerical mixing. It is shown that the coupled space-time schemes behave very well as expected with the increase of spatial and/or temporal resolutions. With this approach, high-order accuracy is retained for the variable velocity. An important feature of the scheme is that the high-order solution is obtained in one step in contrast to the decoupled schemes where the higher-order solutions are obtained in multiple stages (e.g. RK4). The scheme is very efficient due to the tri-diagonal matrix and stable in the range of Courant number up to 1 which is the domain of this study. Constancy of the scalar is preserved through the agreement of the advection and continuity equations or imposed by the pseudo-density approach. It ensures the consistency of the solution and the order of accuracy. Furthermore, the stability and efficiency of the matrix inversion associated to the scheme is studied. For the application of the schemes for larger Courant numbers, there exist different methods that are being applied for the upcoming studies. A general form of 1D nonlinear advection along with continuity equation is shown for oceanic vertical transport, which is very important in the representation of physical processes in the ocean.

Key words: Scalar advection, Nonlinear Transport, Coupled space-time, Compact Padé schemes, Conservation and constancy preservation, Atmospheric/Oceanic models

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