# Structure-preserving models of geophysical fluids

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The dominant features of the large scale dynamics of the atmosphere (and many aspects of the small-scale dynamics and other areas of geophysical fluid dynamics) are balanced states and adjustment processes, wave motions (Rossby, Kelvin, and Inertia-Gravity) and conservation properties (such as total energy). It is therefore desirable that the dynamical core has similar discrete processes and properties, or in other words it is structure-preserving. Underlying these properties at the continuous level is the Hamiltonian formulation, which writes the equations of motion in terms of a Hamiltonian functional and a Poisson bracket. A general framework for structure-preserving discretizations of reversible dynamics then consists of discretizing the Hamiltonian formulation using a mimetic spatial discretization and an energy-conserving Poisson integrator to produce a quasi-Hamiltonian discrete model. This is sufficient to obtain most of the desirable properties, and careful selection of the spatial and temporal discretization gives rest. Specifically, we use the mimetic Galerkin difference element (MGD, a type of compatible Galerkin method) coupled with a second-order, implicit energy-conserving Poisson integrator. The MGD element avoid spectral gaps and other dispersive anomalies found with compatible finite element methods. Concrete examples will be shown of the application of the general framework for two commonly used sets of equations in geophysical fluid dynamics: the thermal shallow water equations and the fully compressible Euler equations. For the latter, this includes several choices of prognostic thermodynamic variable and function space, such as the Galerkin analogues of Lorenz and Charney-Phillips finite-difference grid staggering. Results from planar versions of the commonly used DCMIP test suite will be shown. In both cases, for the first time, models with fully discrete conservation of total mass, buoyancy or entropy and energy for arbitrary equations of state are obtained. There will also be a short discussion of ongoing working on the extension of the general framework to incorporate irreversible processes, including subgrid physics parameterizations.

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