Zonally propagating waves on the spherical Earth classified by formulating approximate Schrödinger equations.

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Abstract

On the unbounded equatorial $\beta$-plane linear wave theory of the Rotating Shallow Water Equations (RSWE) yields a simple and concise classification of zonally propagating waves following the formulation of a Schrödinger eigenvalue equation for the meridional velocity. In contrast, on a sphere the equations are cumbersome, which is why no general classification of waves has been proposed. The combination of exact numerical solutions of the RSWE and analytic solutions of approximate Schrödinger eigenvalue equations yields a clear classification of waves on a sphere. Three Schrödinger equation are derived that approximate the RSWE in different regions of the (Lamb number, frequency, zonal wavenumber) space. The $n=0$ Mixed Rossby-Gravity mode (MRG, AKA Yanai wave) is shown to exist on a sphere but a second $n=0$ westward propagating cannot exist, which replicates the scenario on the equatorial $\beta$-plane. However, the reason for the disappearance of the second westward propagating mode on a sphere differs from that on the equatorial $\beta$-plane: While on the latter the second mode is absent due the singularity of the zonal velocity, on a sphere the zonal velocity is regular everywhere. In addition, the group velocities of low modes of westward propagating, Inertia-Gravity, waves vanish at some low zonal wavenumbers which might be of importance in determining the dominant zonal wavenumbers in the atmosphere and ocean. For eastward propagating waves, the formulation of approximate Schrödinger equations shows that on a sphere, Kelvin waves should be classified as the $n=0$ Inertia-Gravity mode and not as a unique wave-type. The attached figure compares the numerically computed dispersion curves of the MRG mode (blue dots) with analytic solutions derived from the associated approximate Schrödinger equations (orange line) and an ad-hoc asymptotical solution (green line). The orange circles on the ordinate are analytic values derived from an exact Schrödinger equation relevant to zonally symmetric (i.e. $k=0$) waves. In this figure $\epsilon=(2\Omega a)^2/gH$ is Lamb’s parameter (where $\Omega$ and $a$ are earth’s rotation frequency and radius, respectively, and $gH$ is the square of the speed of gravity waves), $k$ is the zonal wavenumber and $\omega$ is the wave’s frequency.