Global simulations of the solar convection zone using perturbed MHD equations

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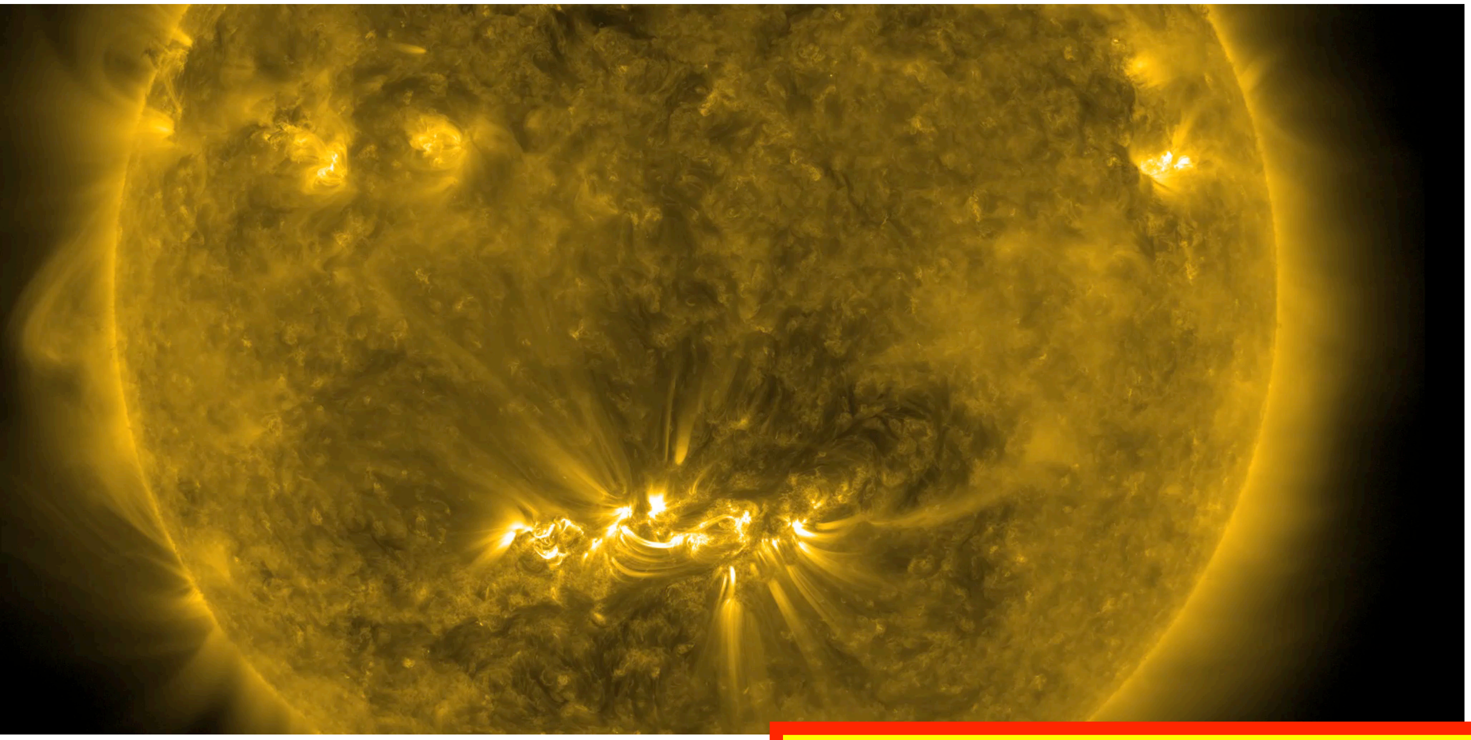
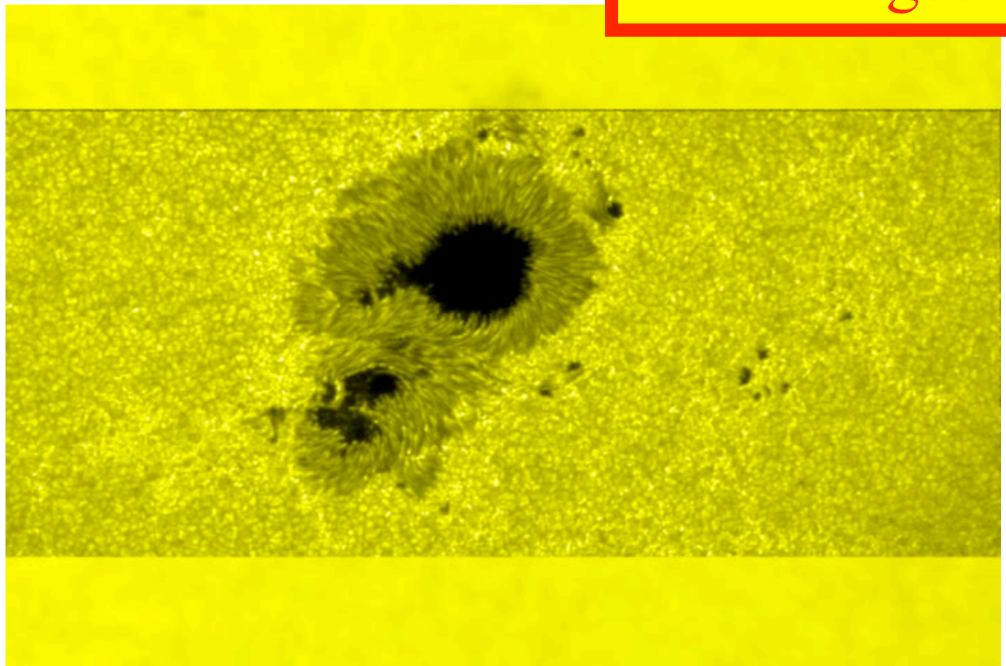
The many scales of solar convection



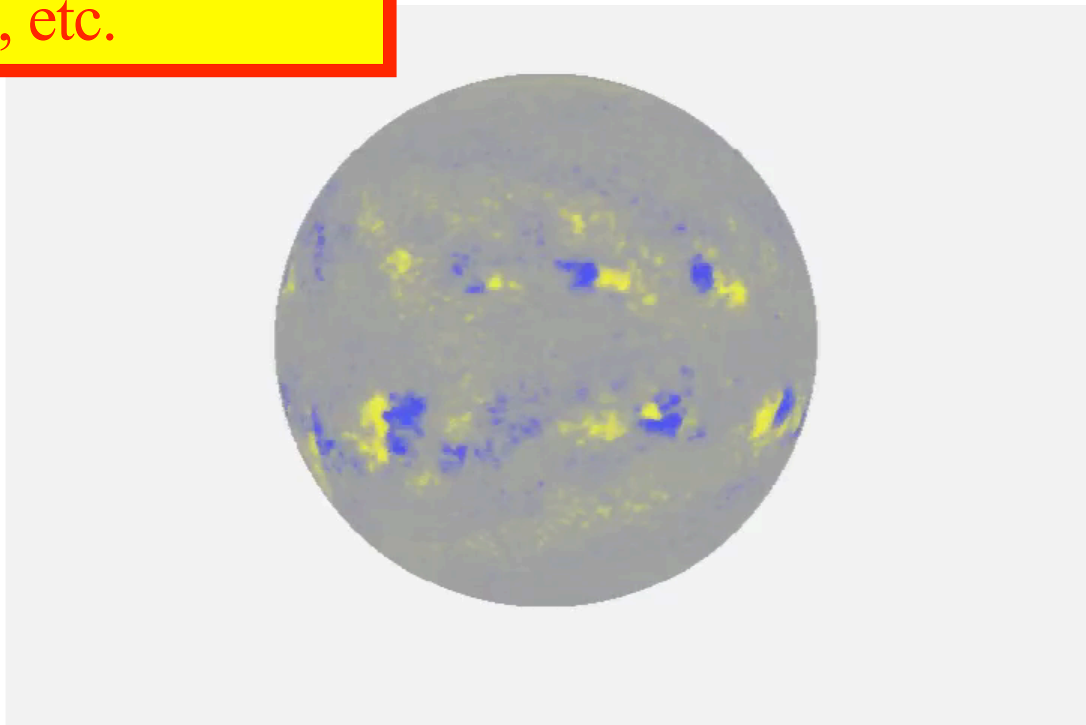
Sun (~700Mm)

visible light

The solar cycle drives all solar activity including flares, coronal mass ejections, geomagnetic storms, etc.



Sunspot (~10Mm) , Granulation (~1Mm)



11 year solar cycle (Credit: David Hathaway)

MHD (*magnetohydrodynamic*) equations are applicable to a collisionnally dominated, nonrelativistic and globally neutral plasma obeying Ohm’s law.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \ ,$$

$$\boxed{\nabla \cdot \mathbf{B} = 0}$$

Lorentz force

$$\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} = -\frac{1}{\rho}\nabla p - 2\boldsymbol{\Omega} \times \mathbf{u} + \mathbf{g} \boxed{+ \frac{1}{\mu_0 \rho}(\nabla \times \mathbf{B}) \times \mathbf{B}} + \frac{1}{\rho}\nabla \cdot \boldsymbol{\tau} \ ,$$

$$p = f(\rho, T, \ldots) \ ,$$

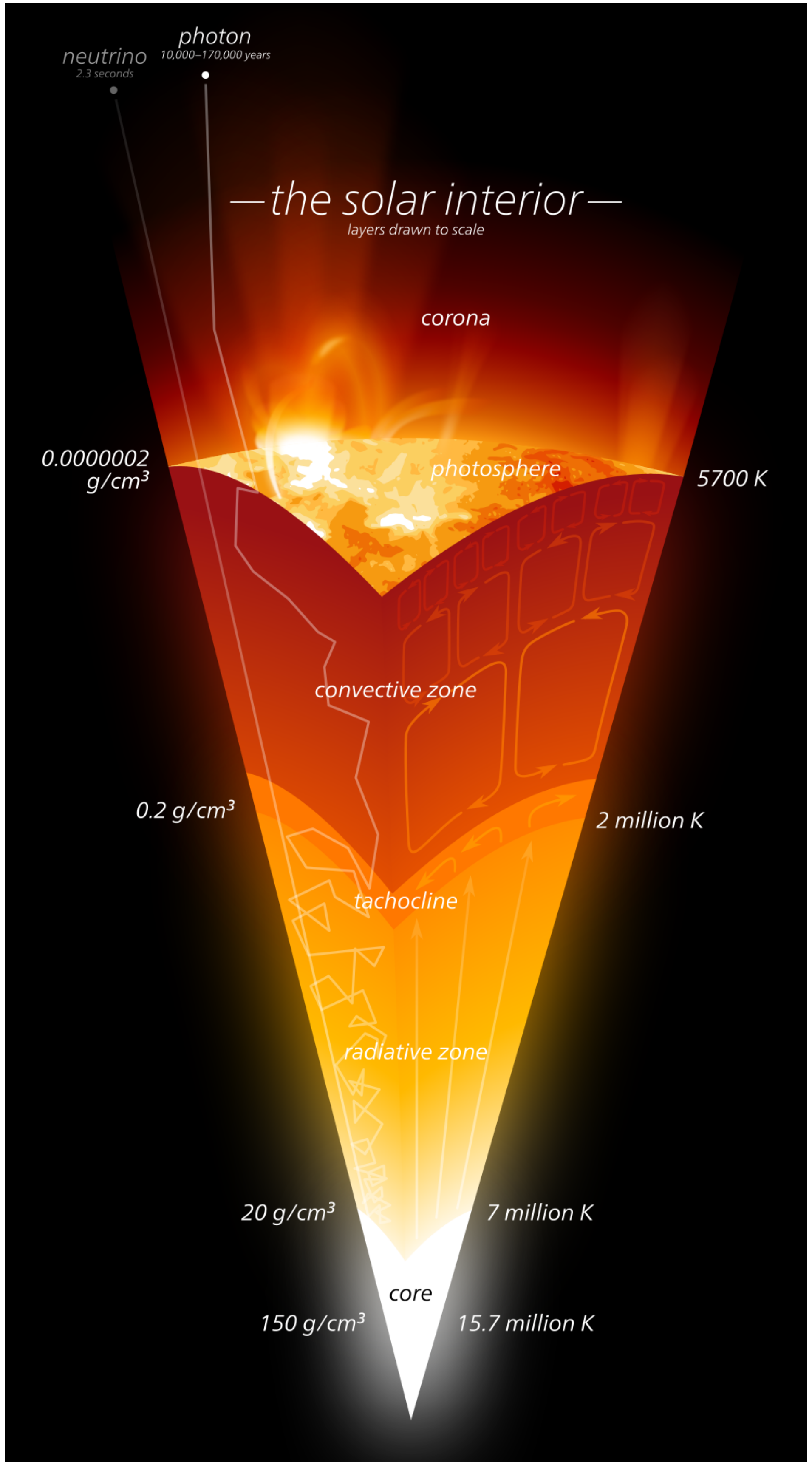
$$\frac{\mathrm{D}e}{\mathrm{D}t} + (\gamma - 1)e\nabla \cdot \mathbf{u} = \frac{1}{\rho}\Big[\nabla \cdot \Big((\chi + \chi_r)\nabla T\Big) + \phi_\nu \boxed{+ \phi_B\Big]} \ ,$$

Ohmic heating

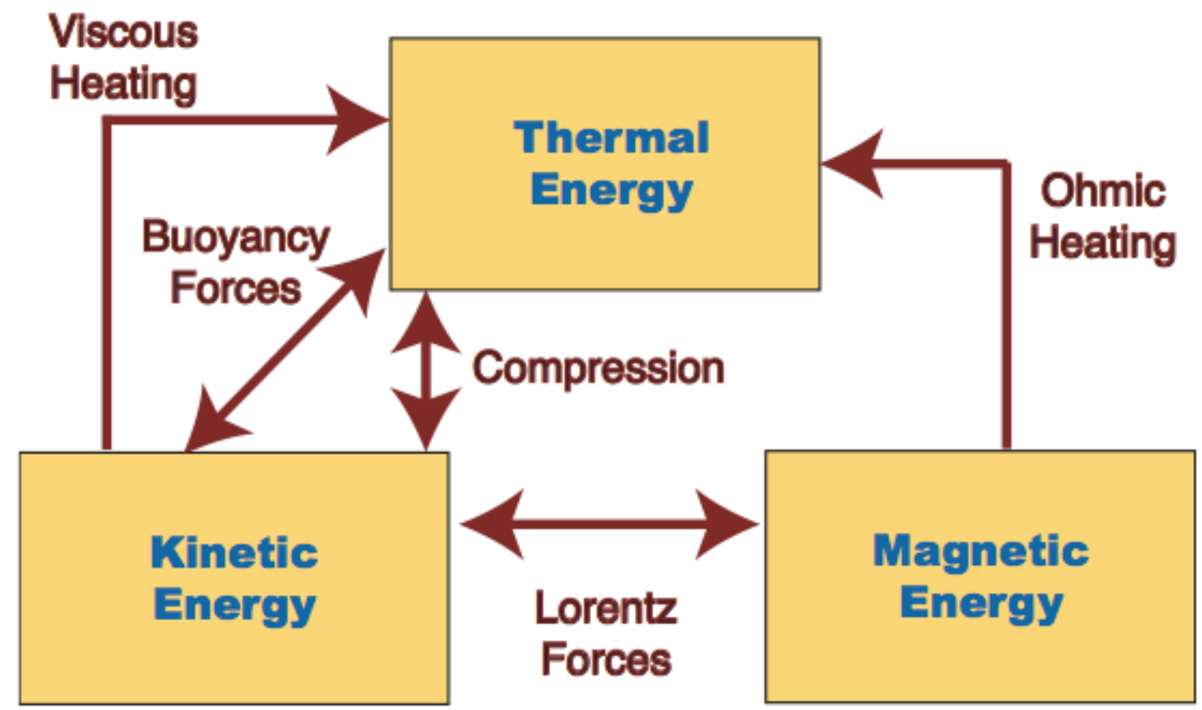
$$\boxed{\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\underbrace{\mathbf{u} \times \mathbf{B}}_{\text{amplification}} - \underbrace{\eta \nabla \times \mathbf{B}}_{\text{destruction}})}$$

Magnetic induction equation

Energetics of the Sun in a nutshell



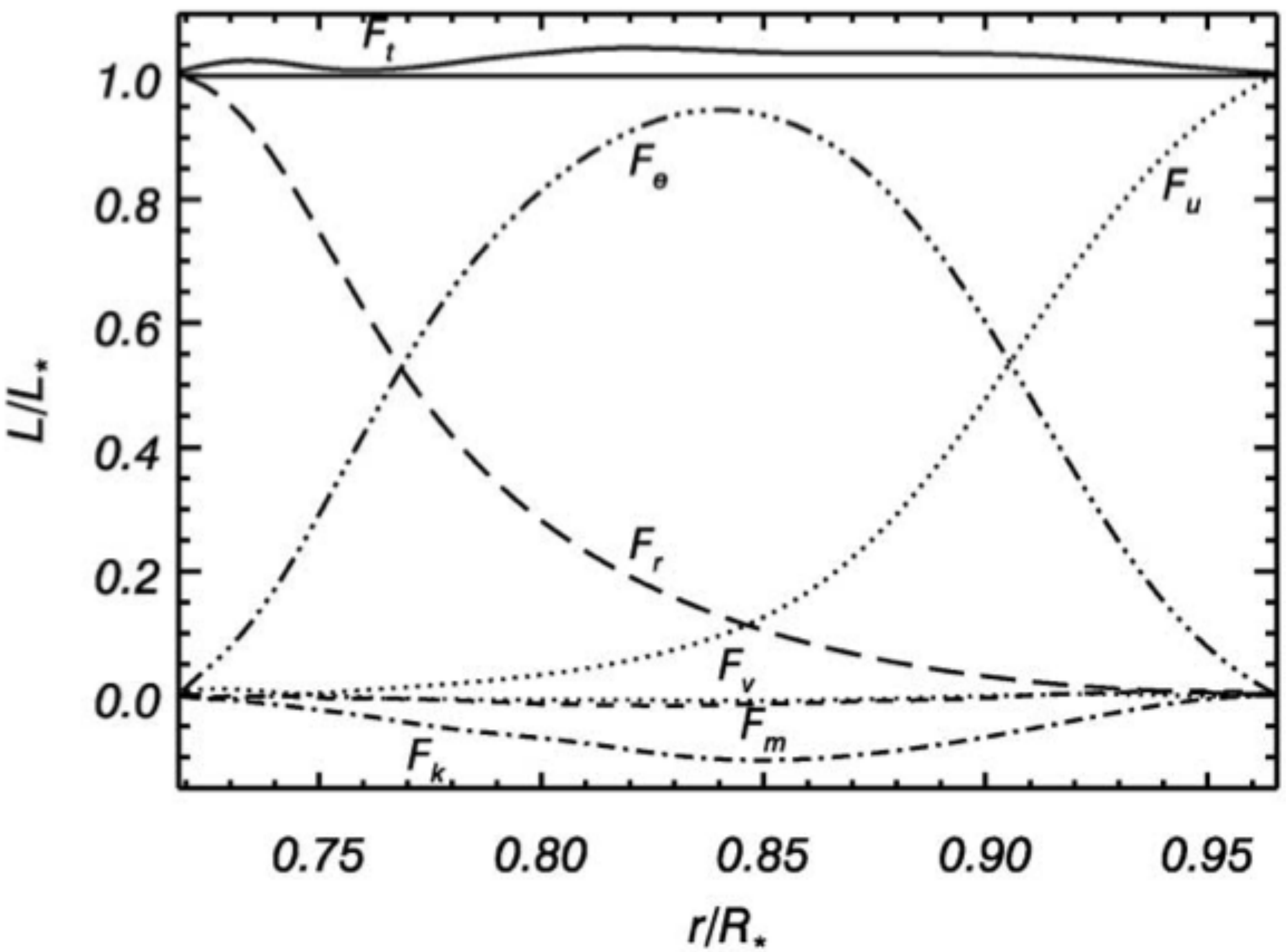
Miesch 2005, LRSP



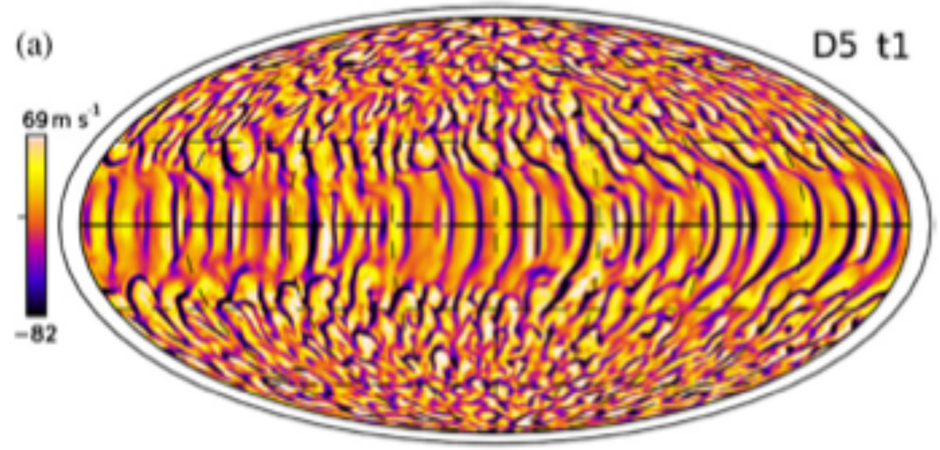
- **Turbulent convection** is driven by radiative heating at the bottom of the convection zone and by radiative cooling in a thin surface layer.
- Kinetic energy is converted into magnetic energy via **small and large-scale flow shear** acting on the magnetic field (induction equation).
- Magnetic energy is converted into kinetic energy (and vice-versa) via the Lorentz-force and into thermal energy via Ohmic heating.

Modeling global solar convection

- Most astrophysical convection codes drive convection by means of large amplitude heating/cooling terms at bottom/top boundaries.



Anelastic **S**pherical **H**armonic code: Heating and cooling and bottom and top boundaries to drive convection inside a spherical shell (e.g. Brun et al. 2004, ApJ, 614, 1073)



$$F_e = \bar{\rho} c_p \overline{v_r T'},$$

$$F_k = \frac{1}{2} \bar{\rho} \overline{v^2 v_r},$$

$$F_r = -\kappa_r \bar{\rho} c_p \frac{d\bar{T}}{dr},$$

$$F_u = -\kappa \bar{\rho} \bar{T} \frac{d\bar{S}}{dr},$$

$$F_v = -\overline{\boldsymbol{v} \cdot \boldsymbol{\mathcal{D}}},$$

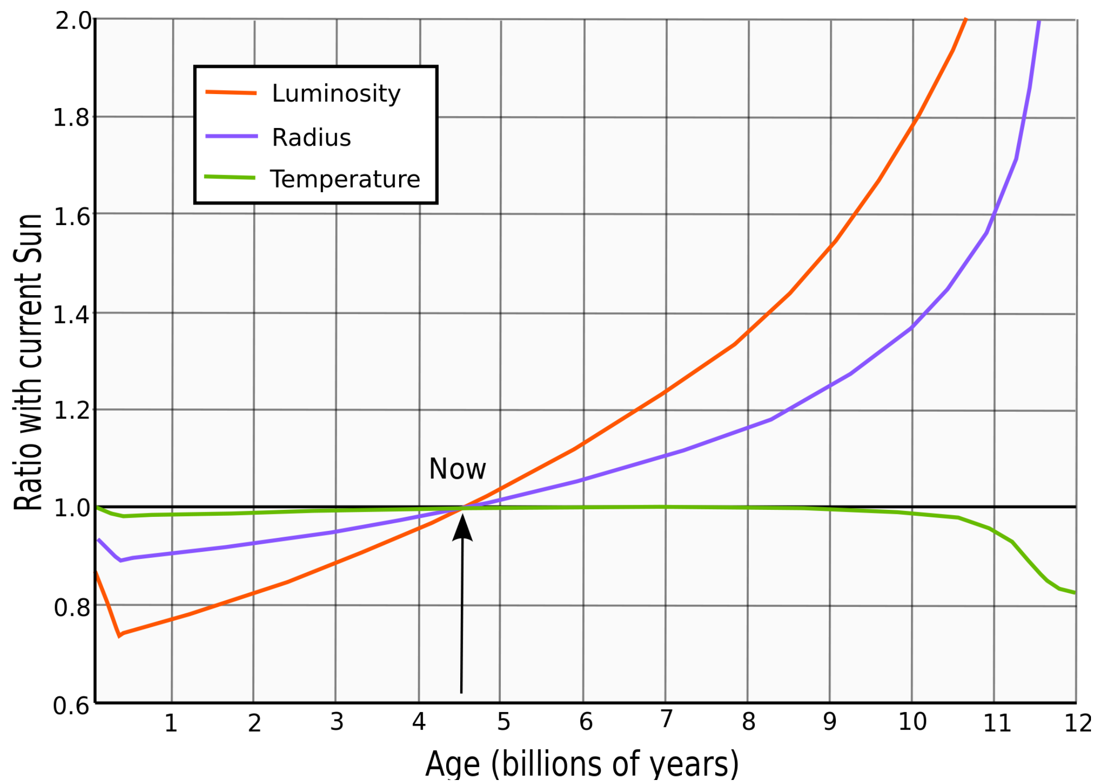
$$F_m = \frac{c}{4\pi} \overline{E_\theta B_\phi - E_\phi B_\theta},$$

$$F_e + F_k + F_r + F_u + F_v + F_m = \frac{L(r)}{4\pi r^2},$$

- The transport of energy near the top boundary is modeled via a flux proportional to the entropy gradient and a large thermal diffusivity.

Modeling global solar convection

- An alternate approach is based on the fact that the Sun exists in a state of **global thermodynamic equilibrium:**



$$\Theta \approx \langle \Theta \rangle^t$$

- The Sun's luminosity increases steadily on evolutionary time scales due to change in nuclear burning rate (has increased by ~30% over the last 3 billion years).
- We are interested in modeling magnetic activity over decadal/multi-decadal time scales.
- The Sun's thermal structure is constant on timescales $\boldsymbol{t}$ << evolutionary timescale (e.g. Kelvin-Helmoltz.)

Goal: solve for thermodynamic perturbations evolving about this state of equilibrium.

- Expressed in terms of the potential temperature, the anelastic (sound-proof) MHD equations take the following form:

$$\frac{D\boldsymbol{u}}{Dt} = -\nabla\left(\frac{p - p_o}{\rho_o}\right) - \boldsymbol{g}\frac{\Theta - \Theta_o}{\Theta_o} + 2\boldsymbol{u} \times \boldsymbol{\Omega} + \frac{1}{\mu\rho_o}\boldsymbol{B} \cdot \nabla\boldsymbol{B} + \mathcal{D}_{\mathbf{u}},$$

$$\frac{D\Theta}{Dt} = \frac{\Theta_o}{\rho_o T_o}\left[\boxed{\nabla \cdot (\kappa_r \rho_o \nabla T)} + \boxed{\nabla \cdot \left(\kappa \frac{\rho_o T_o}{\Theta_o}\nabla\Theta\right)}\right]$$

$$\frac{D\boldsymbol{B}}{Dt} = \boldsymbol{B} \cdot \nabla\boldsymbol{u} - \boldsymbol{B}\nabla \cdot \boldsymbol{u} + \mathcal{D}_{\mathbf{B}},$$

Radiative heating/cooling

$$\nabla \cdot (\rho_o \boldsymbol{u}) = 0,$$

$$\nabla \cdot \boldsymbol{B} = 0.$$

Thermal conduction

Subtract a globally and temporally averaged solution from the generic set of MHD equations to obtain the equations governing the thermodynamic perturbations about this averaged state (Talk by Piotr Smolarkiewicz)

Seek solutions with respect to horizontal & temporal average. (solar cycle period << t << KH time scale)

$$\mathbf{u} \equiv \langle \mathbf{u} \rangle^t + \tilde{\mathbf{u}}$$

$$\Theta \equiv \langle \Theta \rangle^t + \tilde{\Theta}$$

$$\frac{D\Theta}{Dt} = \frac{\Theta_o}{\rho_o T_o}\left[\nabla\cdot\left(\kappa_r \rho_o \nabla T\right) + \nabla\cdot\left(\kappa \frac{\rho_o T_o}{\Theta_o}\nabla\Theta\right)\right]$$

$$\frac{\partial \rho_o \Theta}{\partial t} + \nabla\cdot(\rho_o \mathbf{u}\Theta) = \frac{\Theta_o}{T_o}\left[\nabla\cdot\left(\kappa_r \rho_o \nabla\Theta\frac{T_o}{\Theta_o}\right) + \nabla\cdot\left(\kappa \frac{\rho_o T_o}{\Theta_o}\nabla\Theta\right)\right]$$

$$\langle\Theta\rangle^t \frac{d}{dr}\rho_o\langle u_r\rangle^t + \frac{d}{dr}\rho_o\langle\widetilde{u_r}\widetilde{\Theta}\rangle^t = \frac{\Theta_o}{T_o}\left[\frac{d}{dr}\left(\kappa_r\rho_o\frac{d}{dr}\frac{T_o}{\Theta_o}\langle\Theta\rangle^t\right) + \frac{d}{dr}\left(\kappa\frac{\rho_o T_o}{\Theta_o}\frac{d}{dr}\langle\Theta\rangle^t\right)\right]$$

$$\Theta_e \equiv \langle\Theta\rangle^t$$

is the ambient state

$$\boxed{0 = \frac{\Theta_o}{\rho_o T_o}\left[\frac{d}{dr}\left(\kappa_r\rho_o\frac{d}{dr}\frac{T_o}{\Theta_o}\Theta_e\right) + \frac{d}{dr}\left(\kappa\frac{\rho_o T_o}{\Theta_o}\frac{d}{dr}\Theta_e\right)\right] + \mathcal{H}^\star}$$

$$\mathcal{H}^\star \equiv -\rho_o^{-1} d/dr(\rho_o\langle\tilde{u}_r\widetilde{\Theta}\rangle^t)$$

Global thermodynamic equilibrium

Reynolds heat flux

Subtract thermodynamically balanced-state from the generic entropy equation:

$$\frac{\partial \rho_o \Theta}{\partial t} + \nabla \cdot (\rho_o \mathbf{u} \Theta) = \frac{\Theta_o}{T_o} \left[\nabla \cdot \left(\kappa_r \rho_o \nabla \Theta \frac{T_o}{\Theta_o} \right) + \nabla \cdot \left(\kappa \frac{\rho_o T_o}{\Theta_o} \nabla \Theta \right) \right]$$

$$\boxed{-}$$

$$0 = \frac{\Theta_o}{\rho_o T_o} \left[\frac{d}{dr} \left(\kappa_r \rho_o \frac{d}{dr} \frac{T_o}{\Theta_o} \Theta_e \right) + \frac{d}{dr} \left(\kappa \frac{\rho_o T_o}{\Theta_o} \frac{d}{dr} \Theta_e \right) \right] + \mathcal{H}^\star$$

$$\boxed{=}$$

$$\frac{D\Theta'}{Dt} = -\mathbf{u} \cdot \nabla \Theta_e + \mathcal{H}(\Theta') - \mathcal{H}^\star$$

Newtonian cooling

Approximate Reynolds heat flux as $\mathcal{H}^\star \equiv -\rho_o^{-1} d/dr(\rho_o \langle \widetilde{u}_r \widetilde{\Theta} \rangle^t) \approx \alpha \Theta'$.

$$\frac{D\Theta'}{Dt} = -\mathbf{u} \cdot \nabla \Theta_e \boxed{- \alpha \Theta'} + \mathcal{D}_\Theta$$

Modeling global solar convection

- We solve the anelastic MHD equations using the EULAG-MHD model, which relies upon either Eulerian or semi-Lagrangian NFT transport operators to perform numerical integration.

$$
\begin{aligned}
\frac{d\mathbf{u}}{dt} &= -\nabla\pi' - \mathbf{g}\frac{\Theta'}{\Theta_o} + 2\mathbf{u}\times\mathbf{\Omega} + \frac{1}{\mu\rho_o}\mathbf{B}\cdot\nabla\mathbf{B} + \mathcal{D}_{\mathbf{v}} \ , \\
\boxed{\frac{d\Theta'}{dt} = \mathbf{u}\cdot\nabla\Theta_e - \alpha\Theta' + \mathcal{D}_\Theta} \ , \\
\frac{d\mathbf{B}}{dt} &= \mathbf{B}\cdot\nabla\mathbf{u} - \mathbf{B}\nabla\cdot\mathbf{u} + \mathcal{D}_{\mathbf{B}} \ , \\
\nabla &\cdot (\rho_o\mathbf{u}) = 0 \ , \\
\nabla &\cdot \mathbf{B} = 0 \ .
\end{aligned}
$$

Newtonian cooling term relaxes entropy perturbations toward a superadiabatic ambient state.

Implicit **L**arge **E**ddy **S**imulation: All dissipation is handled implicitly using MPDATA algorithm.
(Paul Charbonneau's talk)

These equations are a special case of the generalised perturbation equations for all-scale atmospheric dynamics (Piotr Smolakiewicz's talk)

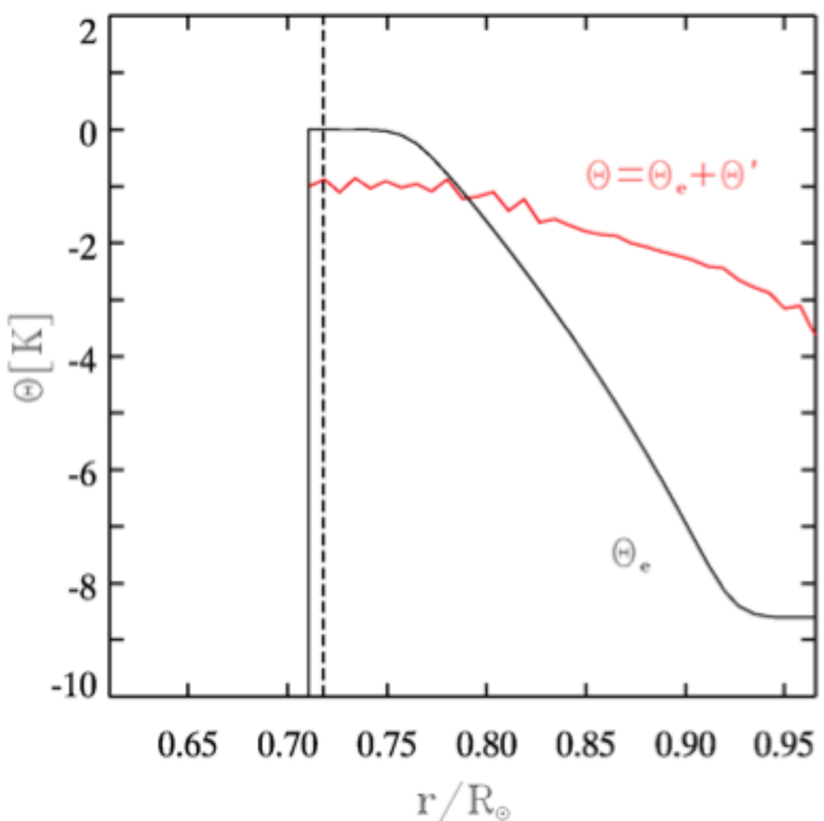
Modeling global solar convection

- Helioseismology provides a constraint on the superadiabaticity of the convection zone.

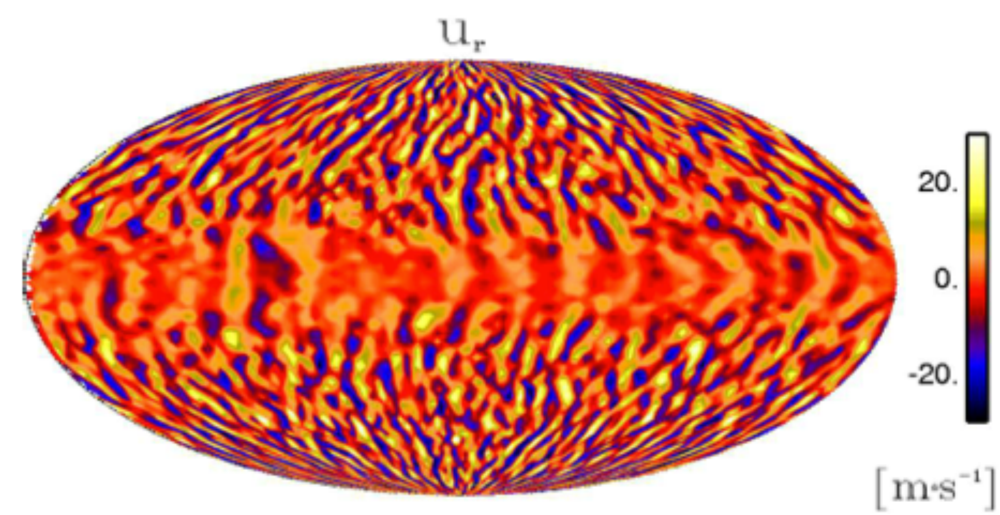
$$\epsilon = \frac{\partial T/\partial r - (\partial T/\partial r)_{\mathrm{ad}}}{\partial T/\partial r} = -\frac{H_T}{C_P}\frac{\partial S}{\partial r} \quad \sim 10^{-4} - 10^{-6}$$

- Construct a solution consistent with the solar stratification:

$$p = K\rho^{1+\frac{1}{m}}\ , \quad p = \rho R T\ , \quad \frac{dp}{dr} = -\rho g \longrightarrow \boxed{\Theta_e \equiv T_e\left(\frac{\rho_b T_b}{\rho_e T_e}\right)^{1-1/\gamma}}$$



EULAG-MHD



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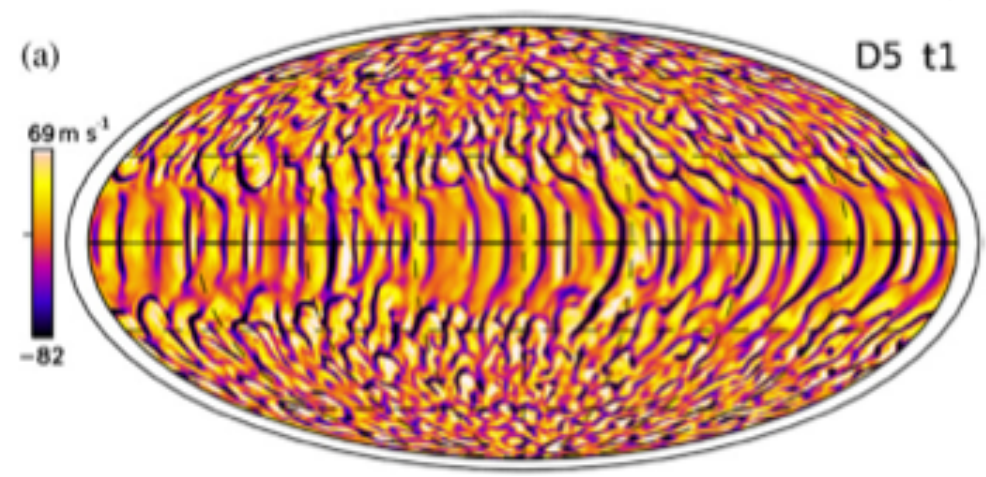
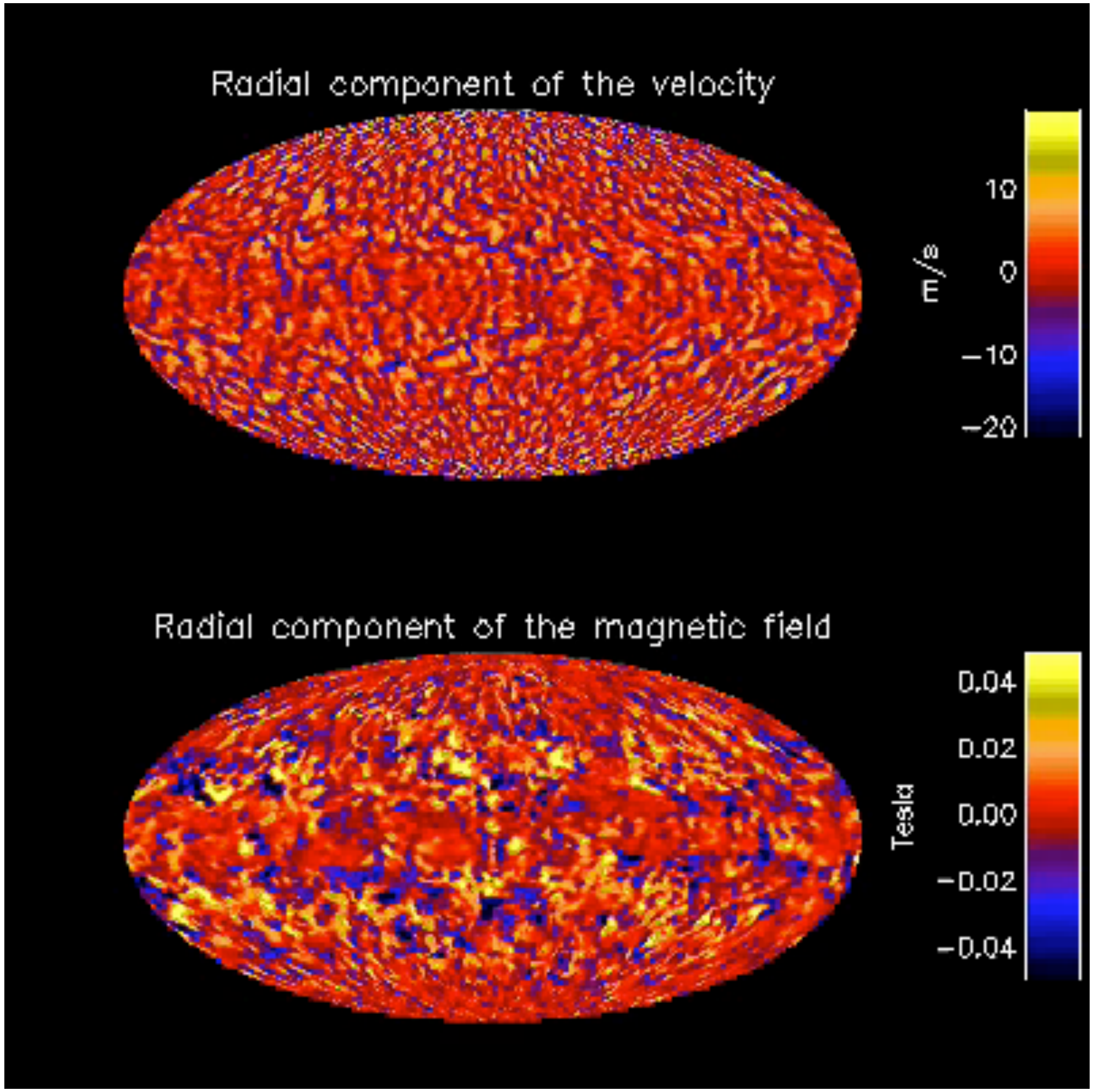


A convectively unstable mean state is achieved by relaxing the potential temperature toward a slightly superadiabatic ambient state

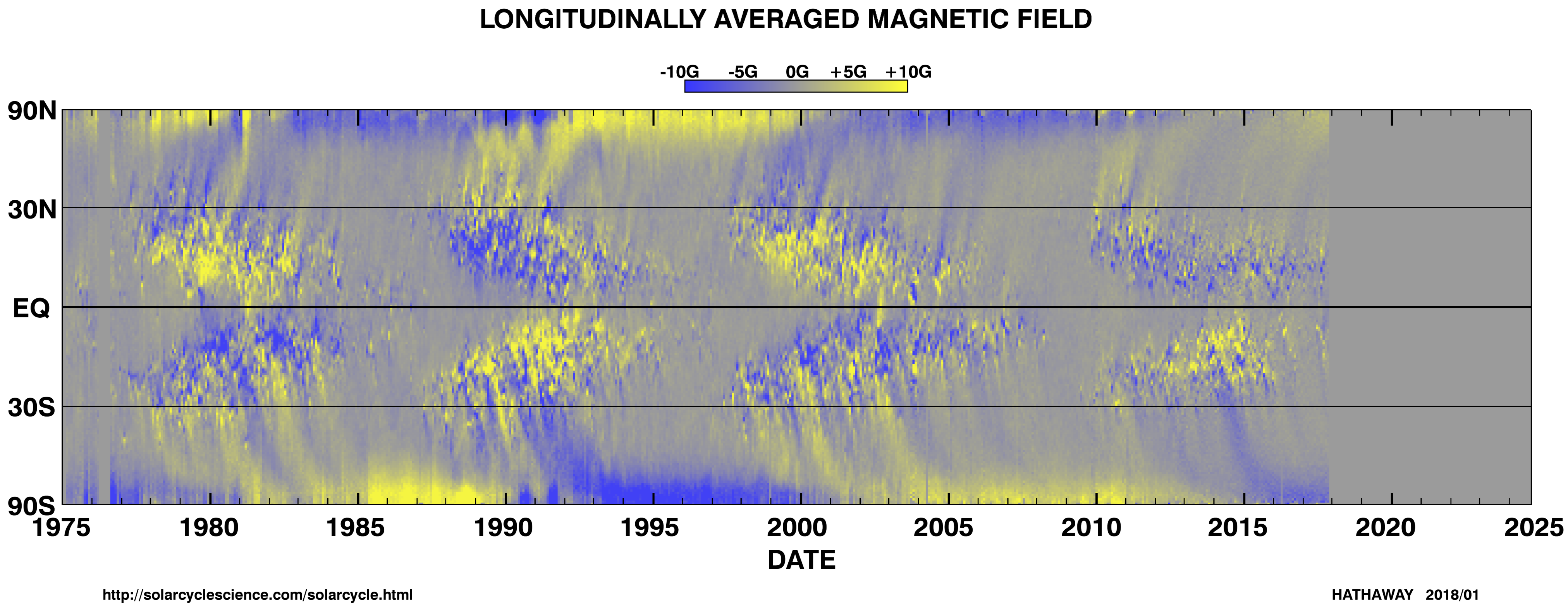
Magnetic cycles

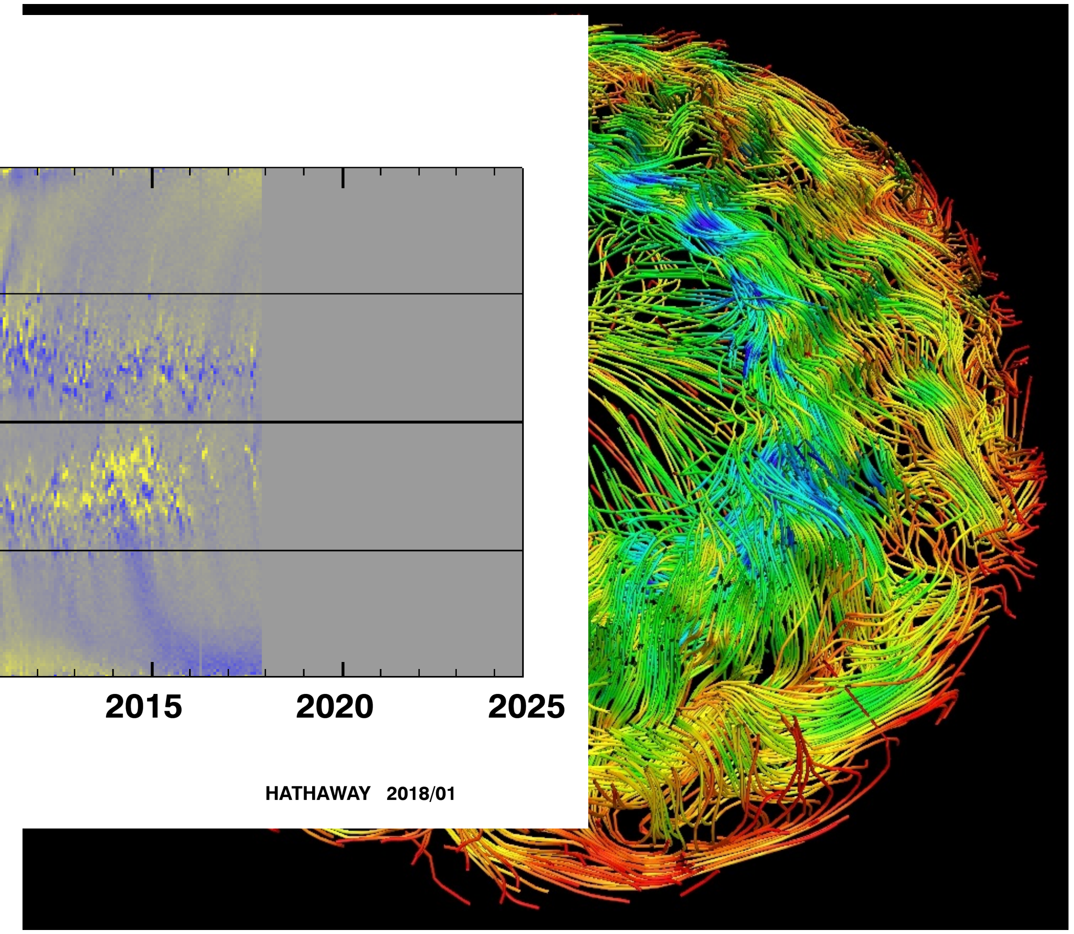
- A spatially and temporally complex small-scale magnetic field of mixed polarity is produced by the simulation:



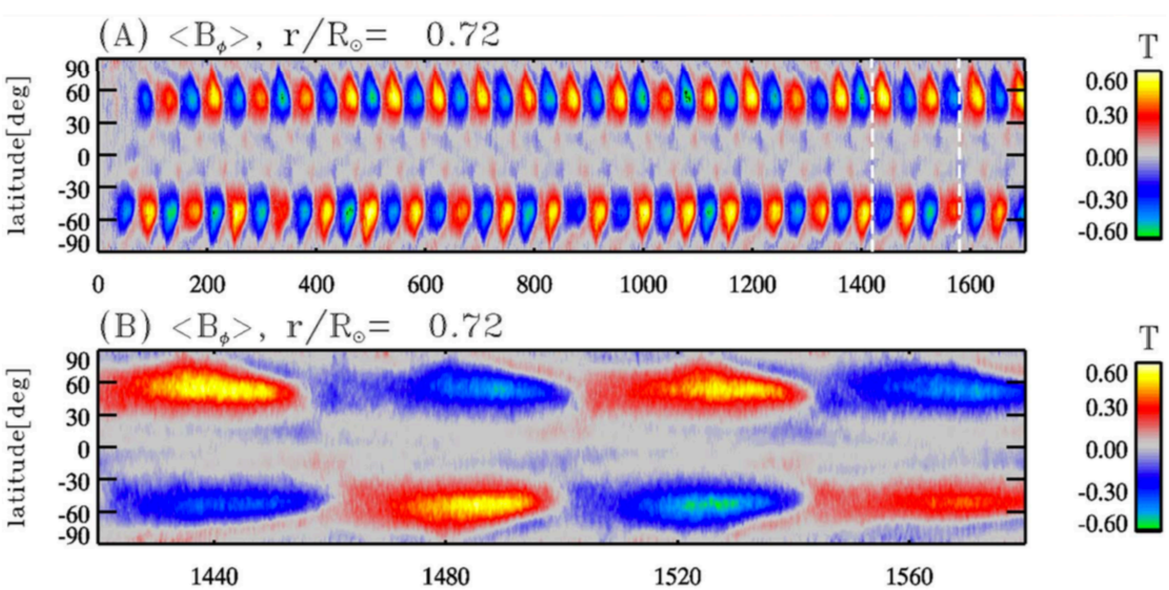
Magnetic cycles

- The most remarkable aspect of the simulation is the generation of a spatially well-organized antisymmetric large-scale magnetic field undergoing polarity regular reversals about the equator on a 40 yr period.





(maximum) (minimum) (maximum)



Longitudinally averaged longitudinal magnetic field component in our simulation:

- Antisymmetric about the equator.
- Undergoes regular polarity reversals on a decadal time scale.
- Migrates toward the equator over the course of the cycle.

Entropy equation

What is the role of each term on the right-hand-side of the entropy equation?

$$\frac{\partial \Theta'}{\partial t} = -\frac{1}{\rho_o}\nabla\cdot(\mathbf{F}_\Theta) - \alpha\Theta' + \mathcal{H}(\Theta') \qquad \boldsymbol{F}_\Theta \equiv \rho_o \boldsymbol{u}\Theta$$

$$\mathcal{S}^u_\Theta(r) \equiv -\frac{c_p}{\Theta_o}\int_{\partial\Omega(u)} \nabla\cdot\mathbf{F}_\Theta d\sigma \ , \qquad \mathcal{Q}^u_\Theta(r) \equiv T^\star_r \int_{r_b}^{r} \mathcal{S}^u_\Theta dr$$

$$\mathcal{S}^d_\Theta(r) \equiv -\frac{c_p}{\Theta_o}\int_{\partial\Omega(d)} \nabla\cdot\mathbf{F}_\Theta d\sigma \ , \qquad \mathcal{Q}^d_\Theta(r) \equiv T^\star_r \int_{r_b}^{r} \mathcal{S}^d_\Theta dr$$

$$\mathcal{S}^u_\alpha(r) \equiv -\alpha\frac{c_p}{\Theta_o}\int_{\partial\Omega(u)} \rho_o\Theta' d\sigma \ , \qquad \mathcal{Q}^u_\alpha(r) \equiv T^\star_r \int_{r_b}^{r} \mathcal{S}^u_\alpha dr \ ,$$

$$\mathcal{S}^d_\alpha(r) \equiv -\alpha\frac{c_p}{\Theta_o}\int_{\partial\Omega(d)} \rho_o\Theta' d\sigma \ . \qquad \mathcal{Q}^d_\alpha(r) \equiv T^\star_r \int_{r_b}^{r} \mathcal{S}^d_\alpha dr \ .$$

$$\mathcal{S}_\Theta \equiv \mathcal{S}^u_\Theta + \mathcal{S}^d_\Theta \qquad \mathcal{S}_\alpha \equiv \mathcal{S}^u_\alpha + \mathcal{S}^d_\alpha \qquad \mathcal{Q}_\Theta \equiv \mathcal{Q}^u_\Theta + \mathcal{Q}^d_\Theta \qquad \mathcal{Q}_\alpha \equiv \mathcal{Q}^u_\alpha + \mathcal{Q}^d_\alpha$$

$$\int_{\partial\Omega} c_p \frac{\rho_o}{\Theta_o}\frac{\partial\Theta'}{\partial t} d\sigma = \mathcal{S}_\Theta(r) + \mathcal{S}_\alpha(r)$$

Total rate of internal energy deposition on a spherical surface (infinitesimal shell).

$$\frac{\partial Q_r}{\partial t} = T^\star_r \frac{\partial S_r}{\partial t} = \mathcal{Q}_\Theta(r) + \mathcal{Q}_\alpha(r)$$

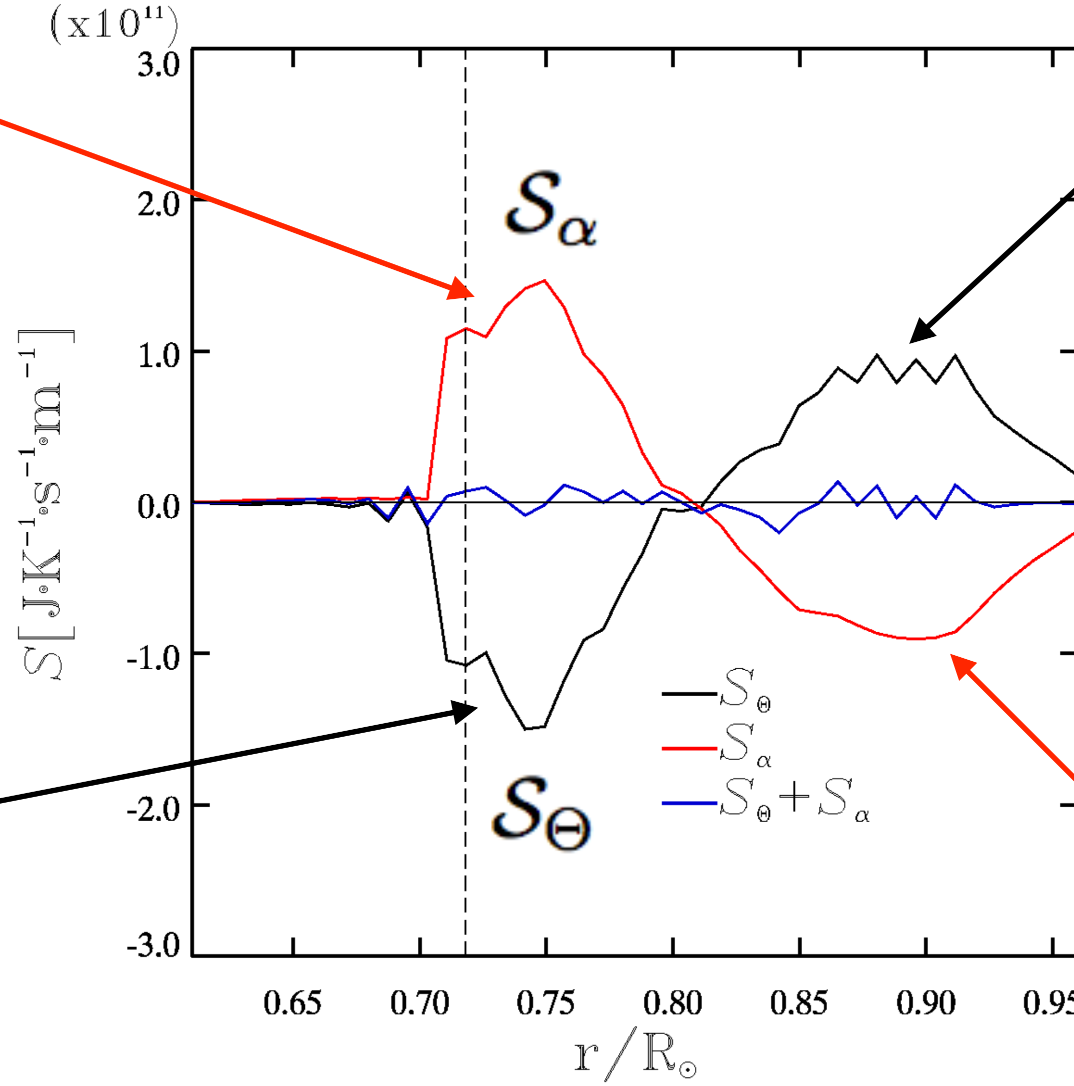
Total rate of internal energy deposition by convective motions inside a spherical shell extending from the base of the convection zone up to some radius “r”.

Entropy equation

$$\int_{\partial\Omega} c_p \frac{\rho_o}{\Theta_o} \frac{\partial \Theta'}{\partial t} \, d\sigma = \mathcal{S}_\Theta(r) + \mathcal{S}_\alpha(r)$$

Newtonian cooling (represented by $\mathcal{S}_\alpha$) balances the action of the mean resolved heat transfer (represented by $\mathcal{S}_\Theta$), thus taking the role of $-\mathcal{H}^\star = \mathcal{H}(\Theta^\star)$, which is positive near the base of the convection zone and negative near the top as a result of radiative heating and cooling.

The positive $\mathcal{S}_\Theta$ near the top represents the deposition of the energy carried by the fluid, which is then radiated into space.



The negative $\mathcal{S}_\Theta$ near the base of the convection zone represents the transfer of the energy deposited by radiative heating into convective motions

Newtonian cooling (represented by $\mathcal{S}_\alpha$) removes the energy carried by convective motions.

$$\boxed{\mathcal{S}_\Theta(r) + \mathcal{S}_\alpha(r) \approx 0}$$

A quasi-stationary state is achieved by the near cancellation of $\mathcal{S}_\Theta$ and $\mathcal{S}_\alpha$.

- Convective motions homogenize entropy field (leads to an adiabatic stratification) whereas Newtonian cooling relaxes entropy toward superadiabatic stratification of the ambient state.

Magnetically modulated heat transport

- This simulation shows an in-phase variation of the convective heat flux with total magnetic energy:

$$\mathcal{Q}^u_\Theta(r) \equiv T^\star_r \int_{r_b}^{r} S^u_\Theta dr$$

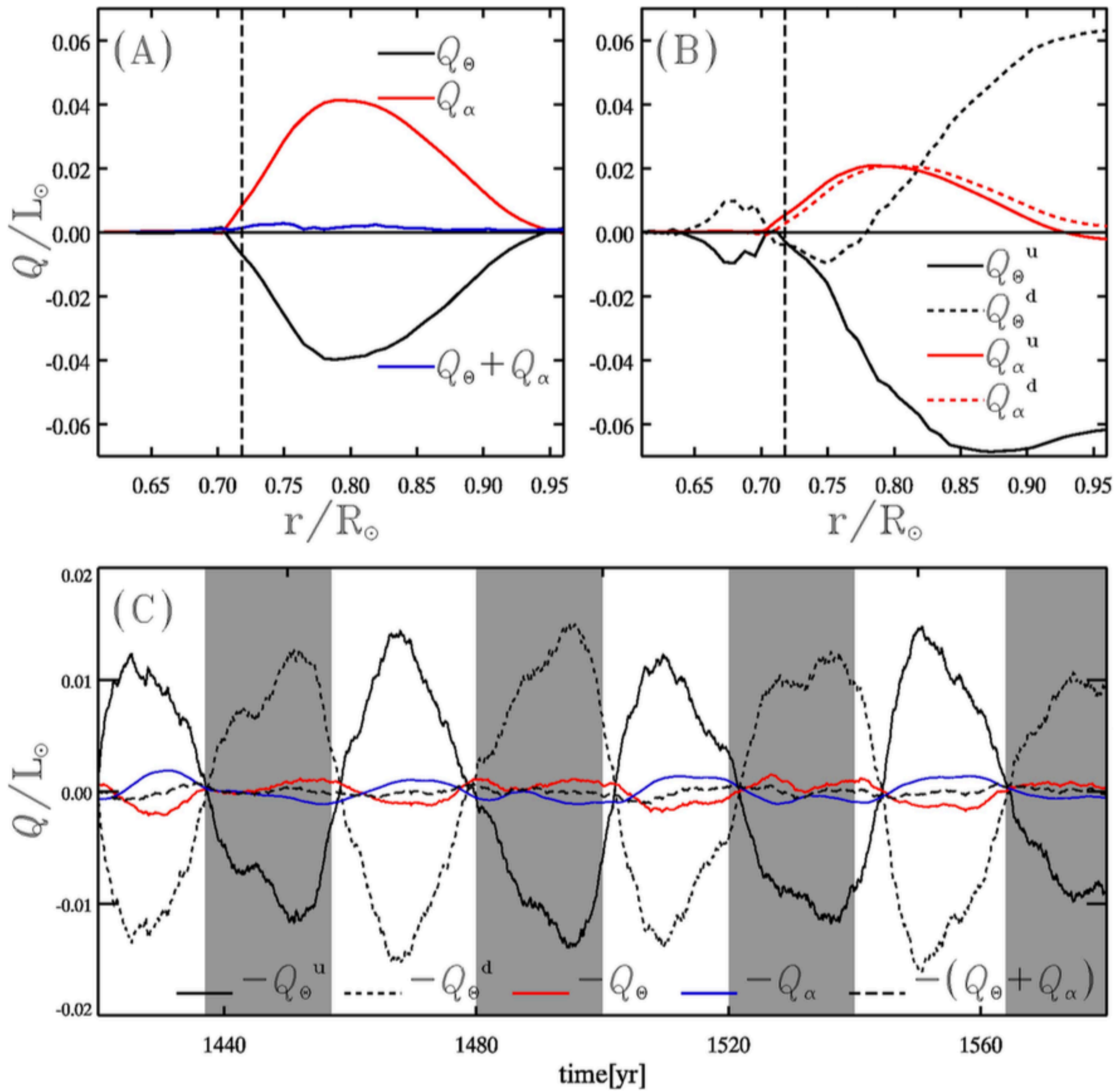
$$\mathcal{Q}^d_\Theta(r) \equiv T^\star_r \int_{r_b}^{r} S^d_\Theta dr$$

$$\mathcal{Q}^u_\alpha(r) \equiv T^\star_r \int_{r_b}^{r} S^u_\alpha dr\ ,$$

$$\mathcal{Q}^d_\alpha(r) \equiv T^\star_r \int_{r_b}^{r} S^d_\alpha dr\ .$$

$$\mathcal{Q}_\Theta \equiv \mathcal{Q}^u_\Theta + \mathcal{Q}^d_\Theta \qquad \mathcal{Q}_\alpha \equiv \mathcal{Q}^u_\alpha + \mathcal{Q}^d_\alpha$$

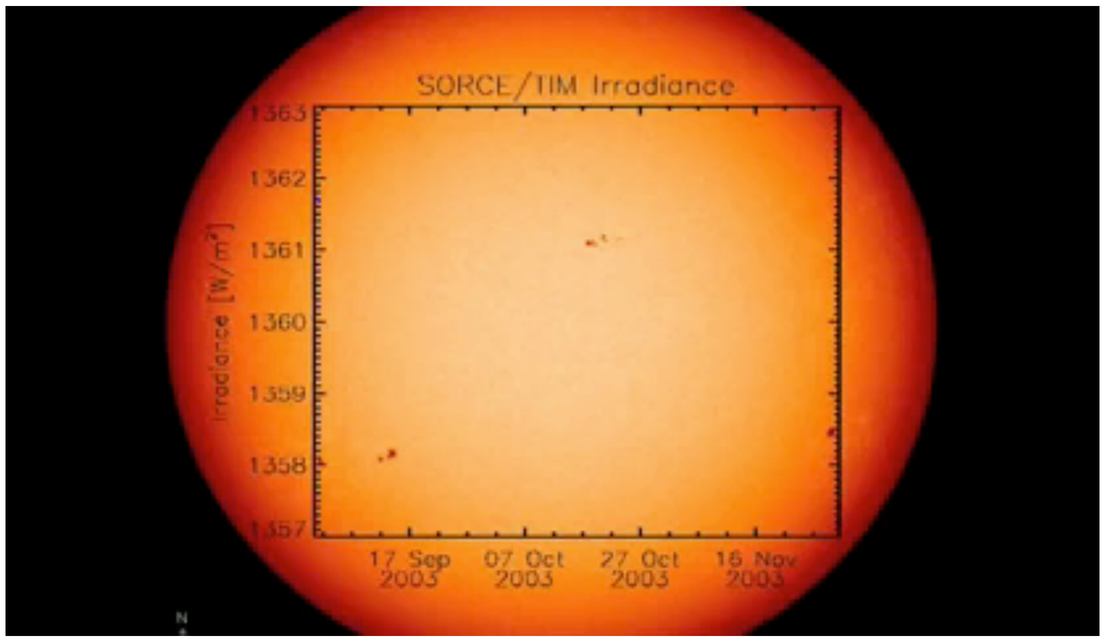
$$\begin{aligned}\mathcal{Q}_\Theta &= -\frac{c_p T_r}{\Theta_o}\int_{r_b}^{r}\int_{\partial\Omega} \nabla\cdot \boldsymbol{F}_\Theta dV \\ &= -\frac{c_p T_r}{\Theta_o}\Big(\int_{\partial\Omega} (\rho_o(r) u_r(r,\theta,\phi)\Theta'(r,\theta,\phi) \\ &\quad - \rho_o(r_b) u_r(r_b,\theta,\phi)\Theta'(r_b,\theta,\phi))d\sigma \\ &= -\frac{c_p \rho_o(r) T_r}{\Theta_o}\int_{\partial\Omega} u_r(r,\theta,\phi)\Theta'(r,\theta,\phi)d\sigma,\end{aligned}$$



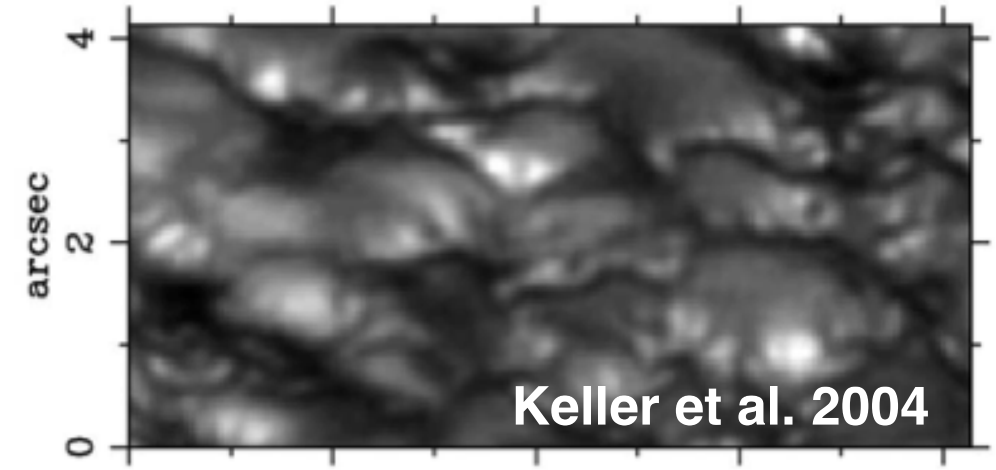
Magnetic activity and irradiance variations

Varying surface coverage by magnetic structures at the surface leads to variations in the total irradiance.

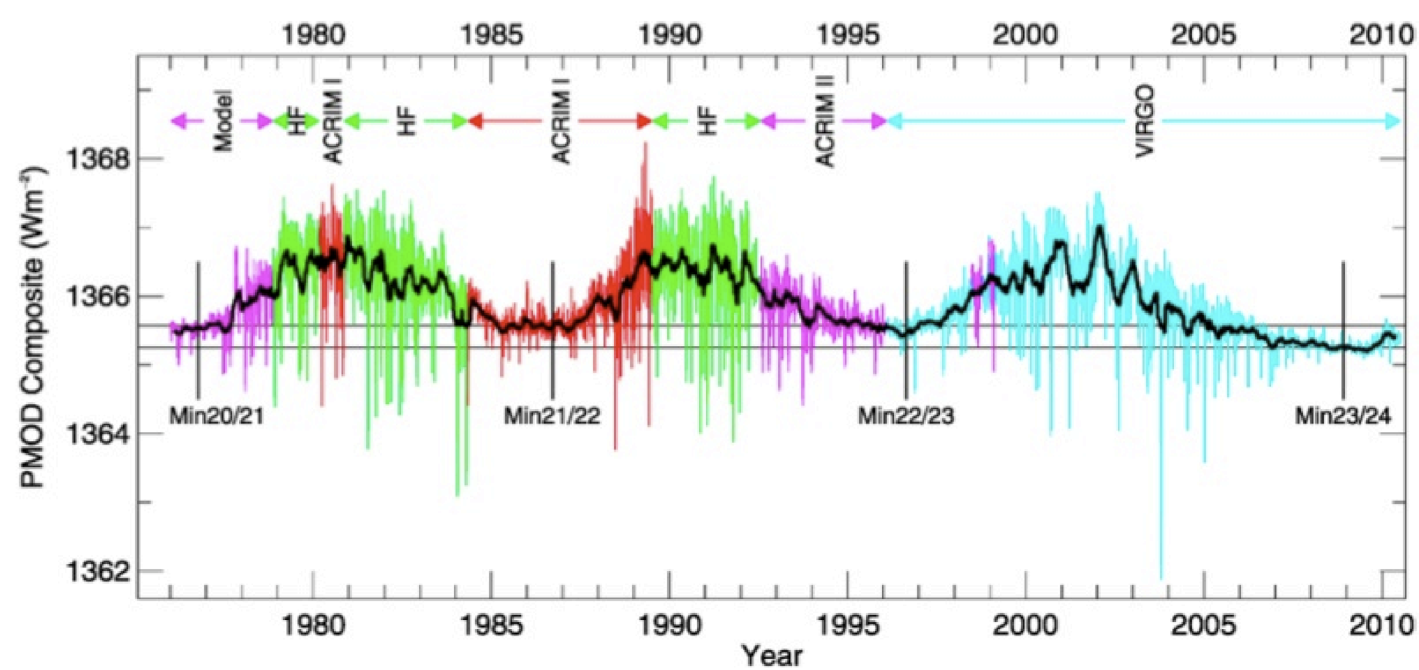
Passage of a sunspot at the surface leads to darkening



Brightening by small scale magnetic structures (faculae, network)



Overcompensation of sunspot darkening by brightening from small-scale magnetic structures leads to 0.1% increase in the total irradiance at cycle maximum.



Summary/Conclusions

- Global MHD simulation produces solar-like large-scale antisymmetric magnetic field undergoing regular polarity reversals about the equator on a decadal time scale.
- We solve for perturbations evolving about a slightly superadiabatic ambient state consistent with the mean solar stratification.
- Convection is driven internally by relaxing the entropy of fluid parcels toward the superadiabatic ambient state.
- Because solutions are sought in terms of fluctuations about a superadiabatic Θ_e ambient profile in the convection zone, the solution's dependence on dissipation/diffusion is greatly reduced. This enables dynamic equilibria that might have been unreachable on dissipative paths of integrations starting with $\Theta_e \equiv \Theta_o$ and a large amplitude heating/cooling forcing applied at the model lower/upper boundaries.
- The in-phase modulation of the convective heat flux with the magnetic cycle present in our simulation suggests that a similar mechanism could be operating in the Sun and also contribute to solar irradiance variations.

Summary/Conclusions

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Magnetically Modulated Heat Transport in a Global Simulation of Solar Magneto-convection

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Abstract

We present results from a global MHD simulation of solar convection in which the heat transported by convective flows varies in-phase with the total magnetic energy. The purely random initial magnetic field specified in this experiment develops into a well-organized large-scale antisymmetric component undergoing hemispherically synchronized polarity reversals on a 40 year period. A key feature of the simulation is the use of a Newtonian cooling term in the entropy equation to maintain a convectively unstable stratification and drive convection, as opposed to the specification of heating and cooling terms at the bottom and top boundaries. When taken together, the solar-like magnetic cycle and the convective heat flux signature suggest that a cyclic modulation of the large-scale heat-carrying convective flows could be operating inside the real Sun. We carry out an analysis of the entropy and momentum equations to uncover the physical mechanism responsible for the enhanced heat transport. The analysis suggests that the modulation is caused by a magnetic tension imbalance inside upflows and downflows, which perturbs their respective contributions to heat transport in such a way as to enhance the total convective heat flux at cycle maximum. Potential consequences of the heat transport modulation for solar irradiance variability are briefly discussed.

Key words: convection – dynamo – magnetohydrodynamics (MHD) – Sun: activity – Sun: interior – turbulence