The Global Icosahedral Model with quasi-Lagrangian Vertical Coordinate

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February 2010

Numerical simulation of the atmosphere entering the second century of its development will certainly lead to new numerical models complementing the existing systems based on spectral and finite difference techniques.
The evolution of these new models will likely follow the path which could be summarized briefly in the following way:

- removal of shallow atmosphere approximation
- arbitrary geoid shape
- Euler equations
- spatial discretization based on flexible finite volume schemes
- new time integration schemes
- realistic coupling to world ocean model
- flexible discretization allowing adaptive changes of the model resolution
- grid size selected to resolve convective clouds on a global scale
- redefinition of the concept of parameterizations
- chemical and aerosol processes fully coupled with dynamics
- new forcings related to space weather data

Main trends

Increasing tendency towards unified global non-hydrostatic models

Tendency towards global gridpoint models

Introduction of quasi-homogeneous, quasi-isotropic grids (geodesic grid based on Platonic polyhedra)

New formulation principles based on Hamiltonian formalism, Lie groups and theory of differentiable manifolds

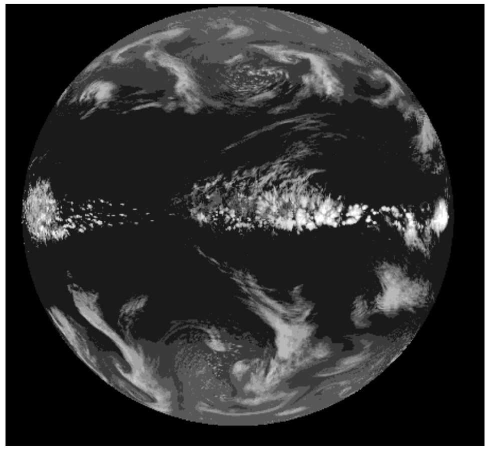
Important examples of these trends

UK Met Office **Unified Model** - emphasis on deep atmosphere nonstandard formulation of the Geophysical Fluid Dynamics

NICAM - global cloud resolving model (Earth Simulator)

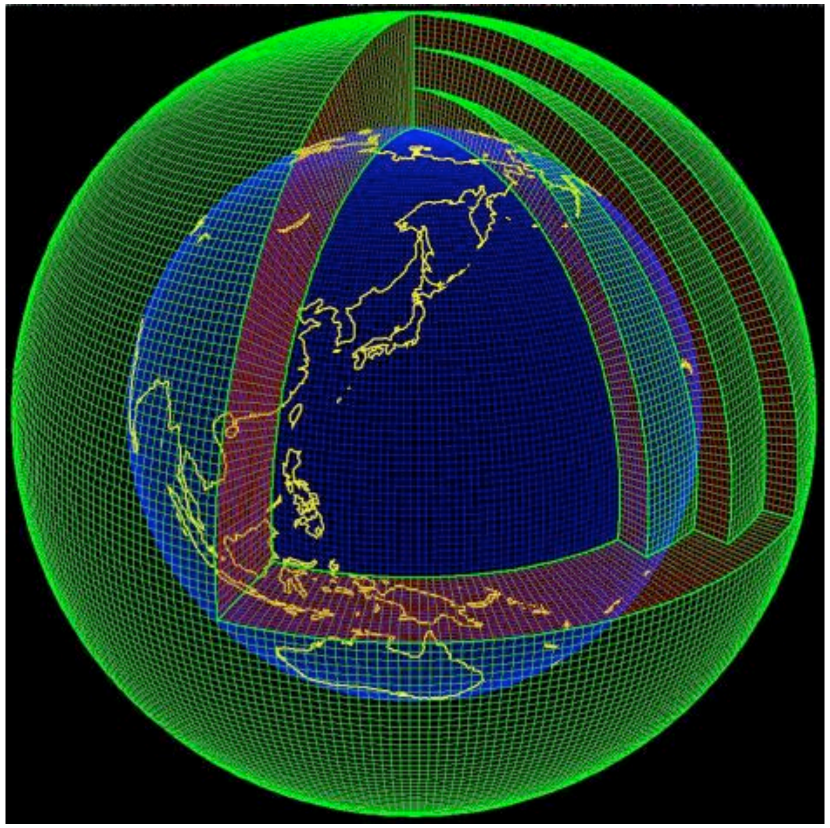
WRF - emphasis on new numerical methods

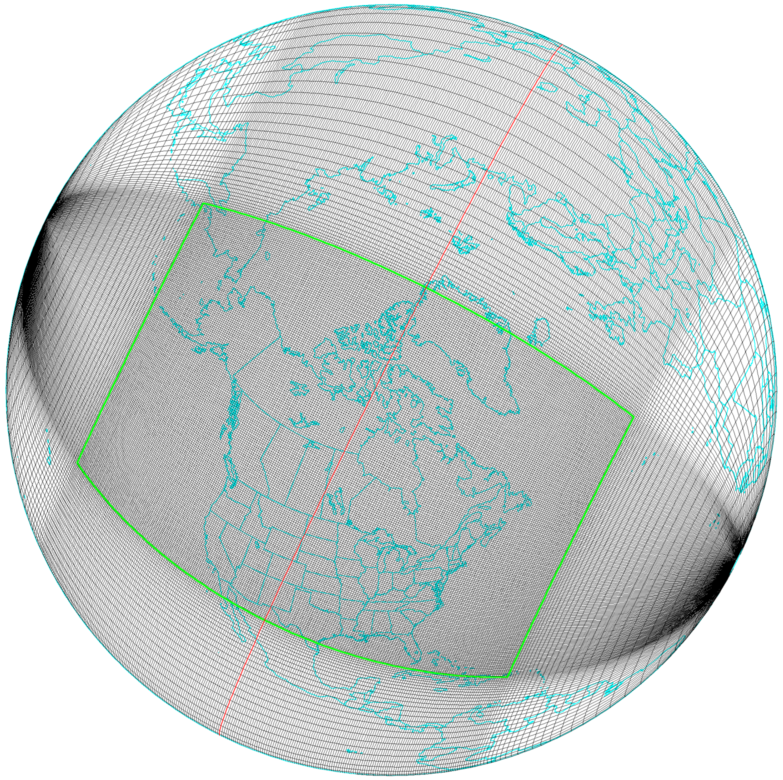
ICON - Hamiltonian formalism (Max Planck Institute)

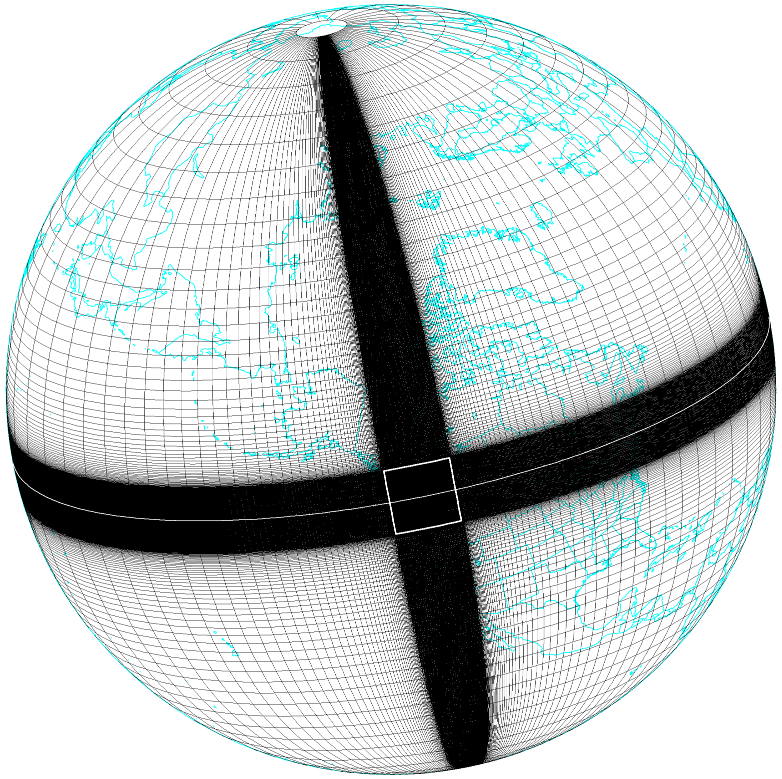


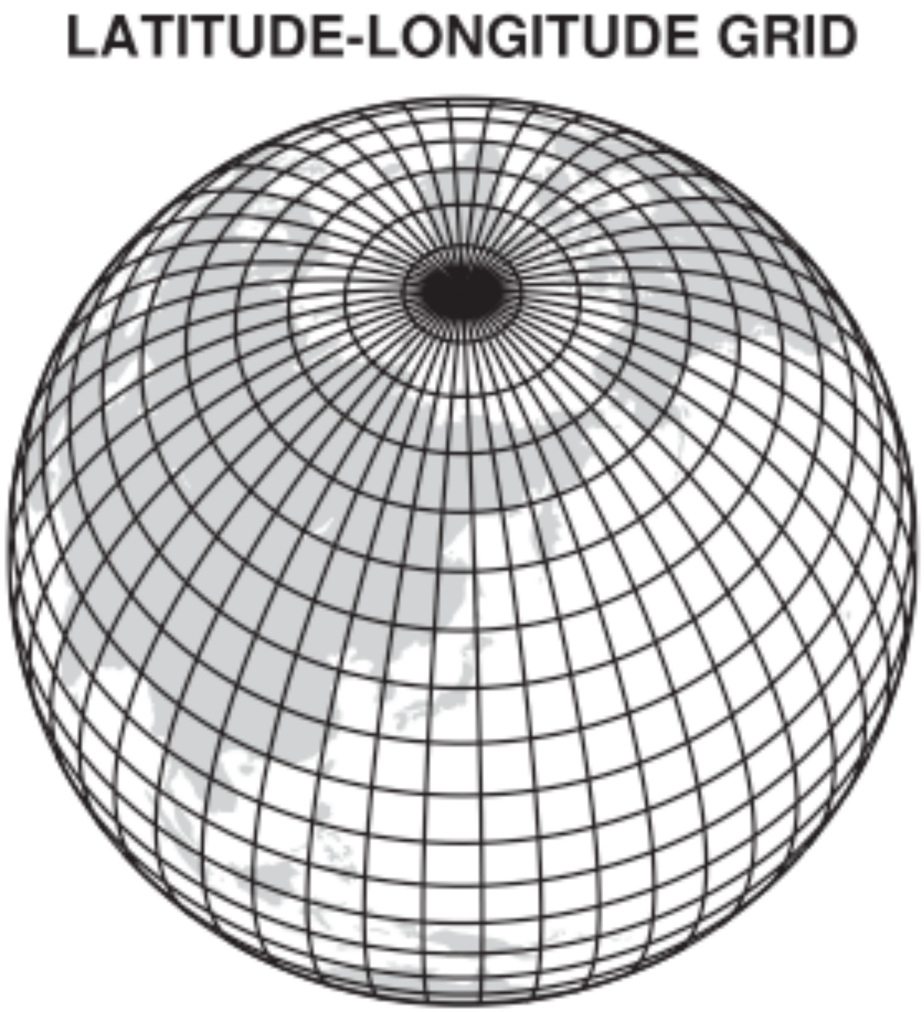
Horizontal discretization

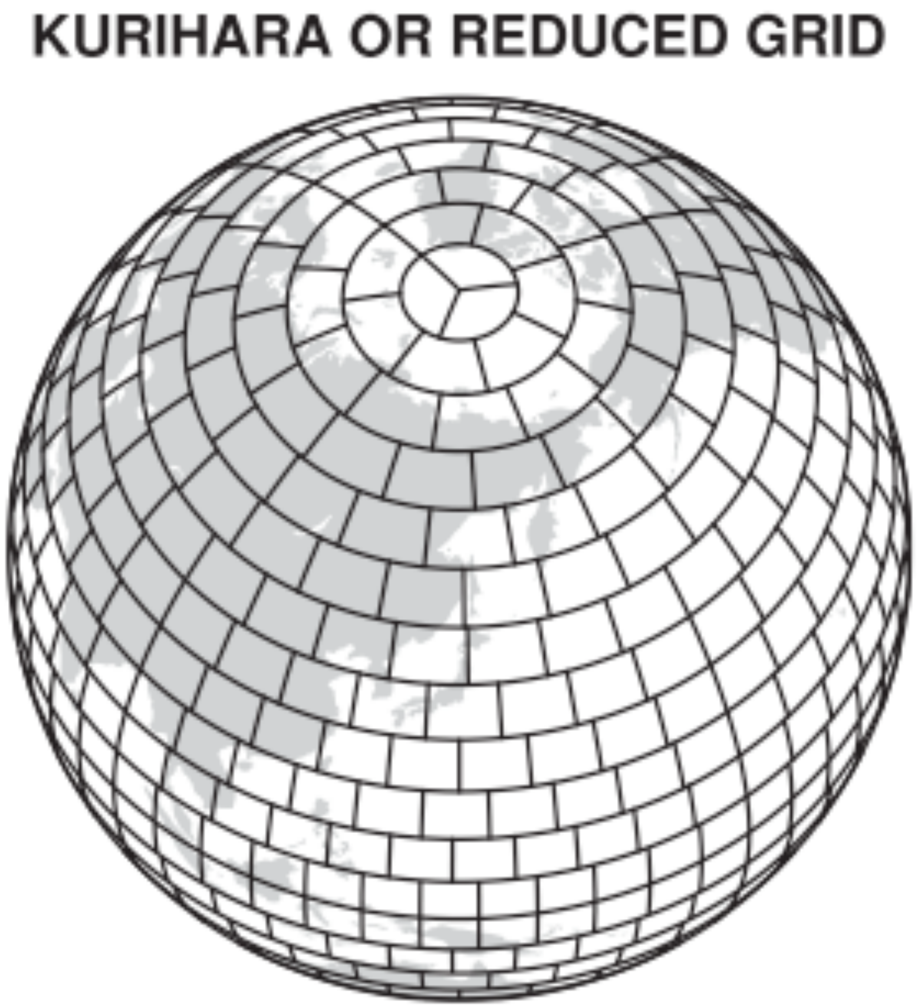
One of the elements of the design of new models is the selection of the horizontal discretization.





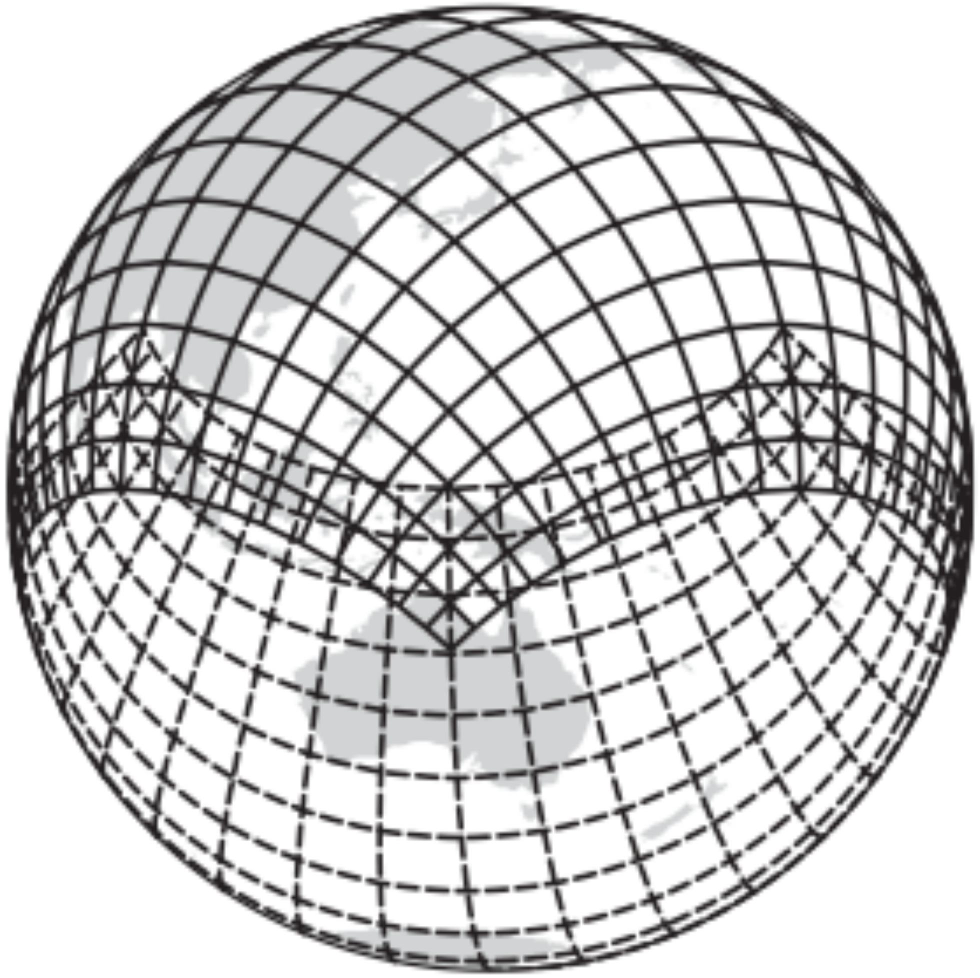




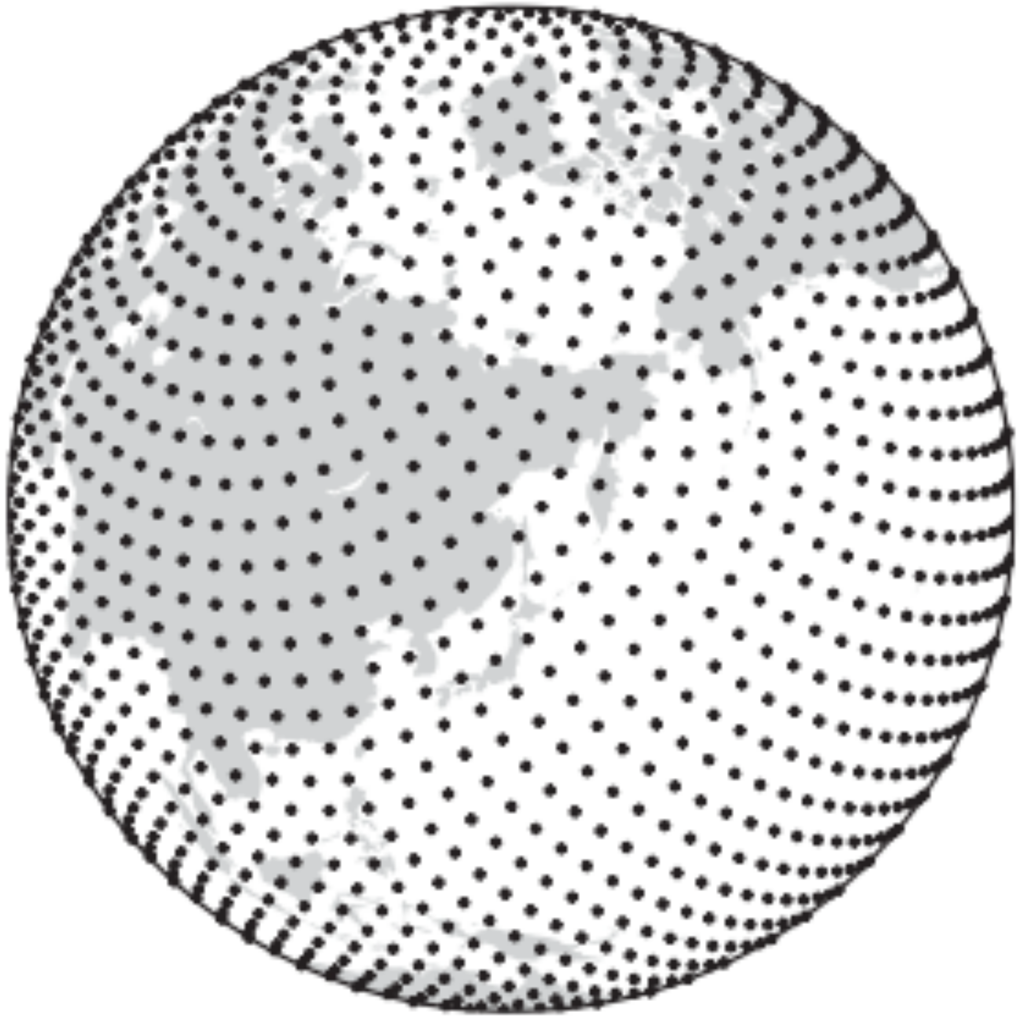


from: Williamson (2007) The Evolution of Dynamical cores for Global Atmospheric Models, J. Met. Soc. Japan, vol 85 B, pp 241-269

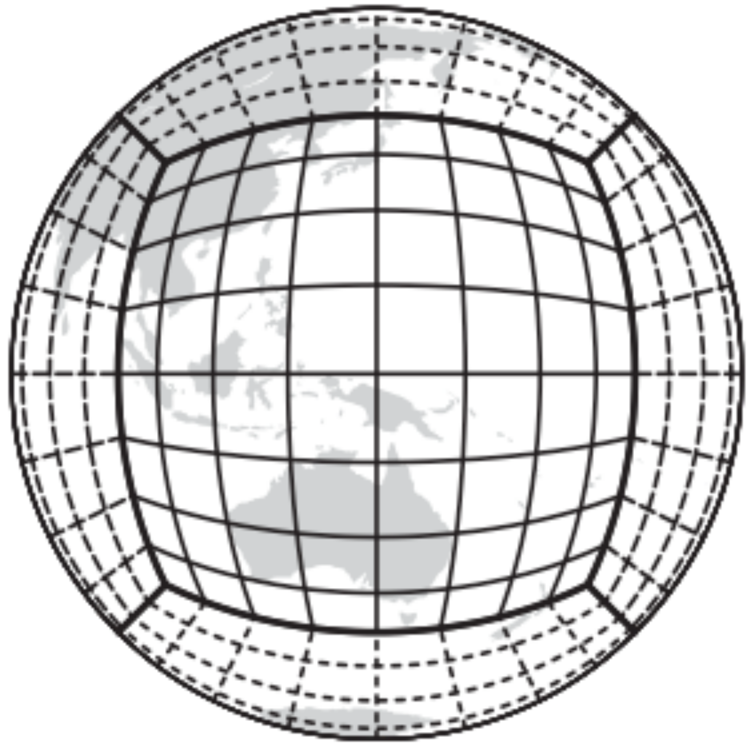
COMPOSITE OR OVERSET GRID



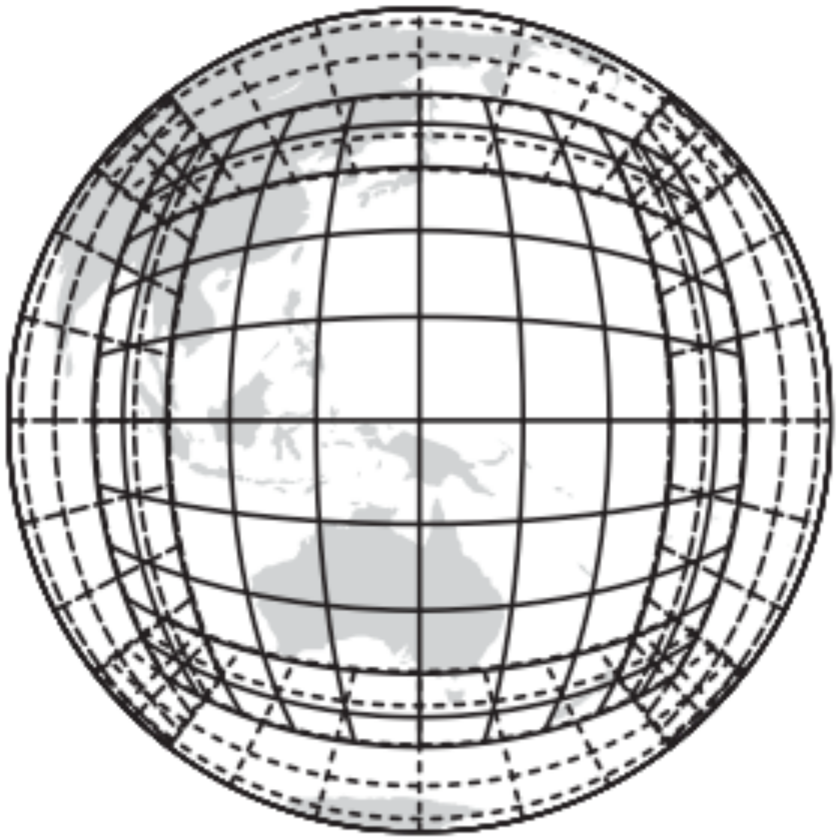
FIBONACCI GRID



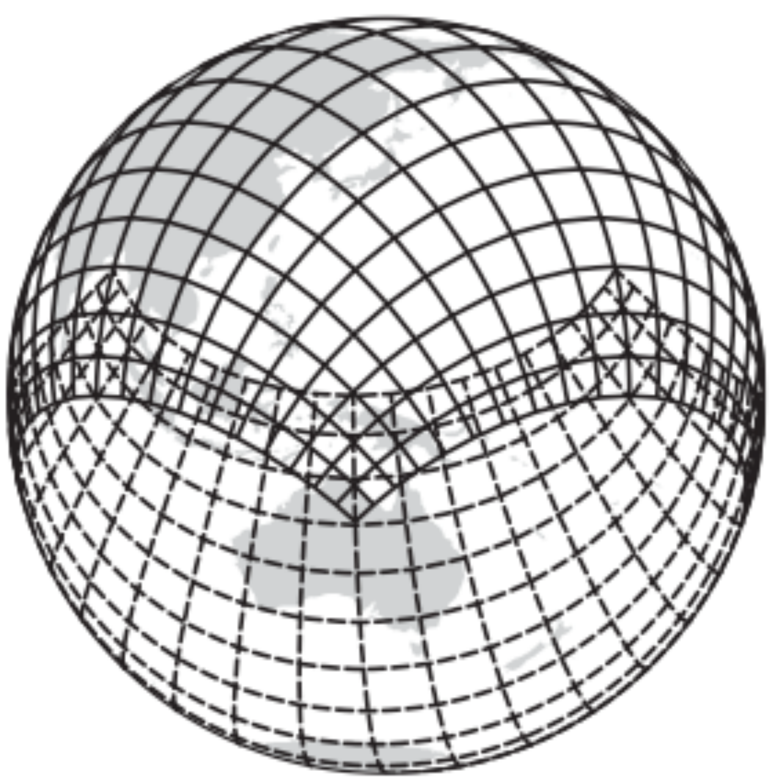
CUBED SPHERE GRID

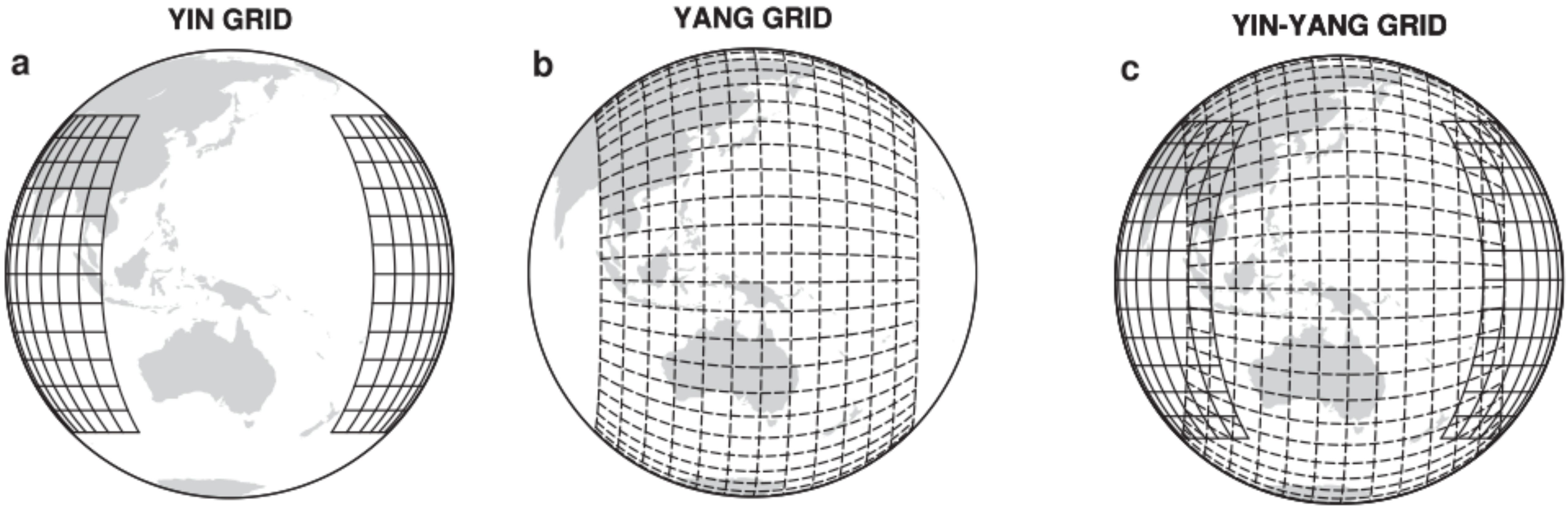


COMPOSITE MESH
CUBED SPHERE GRID

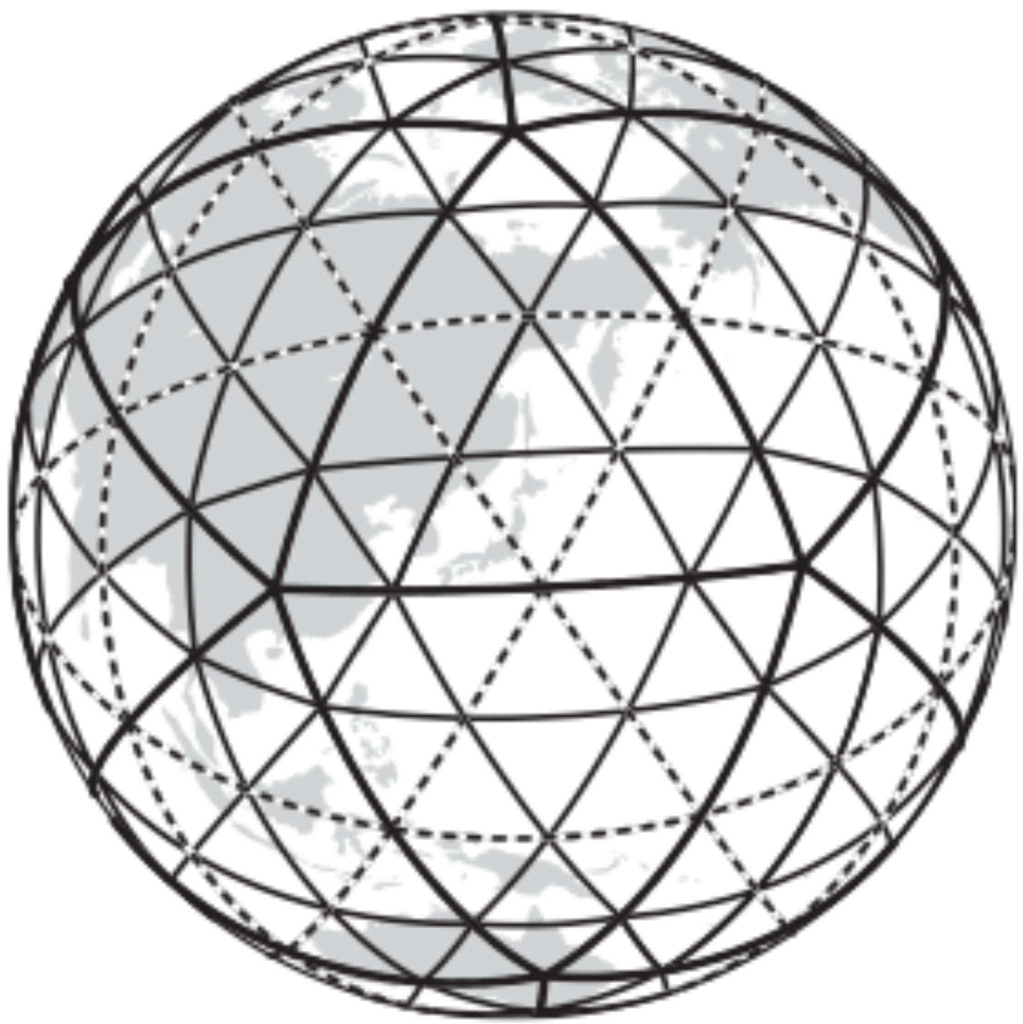


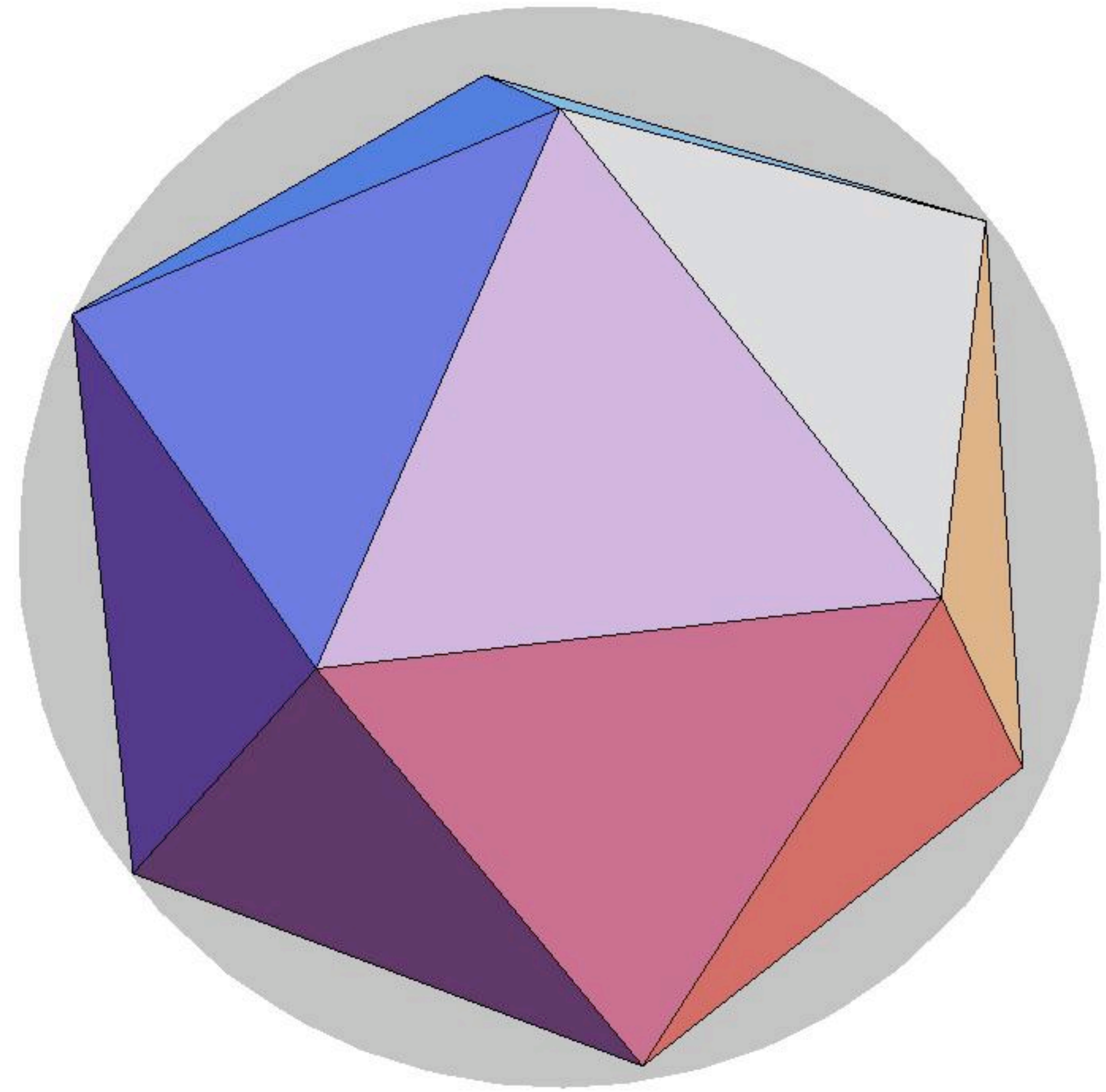
COMPOSITE OR OVERSET GRID

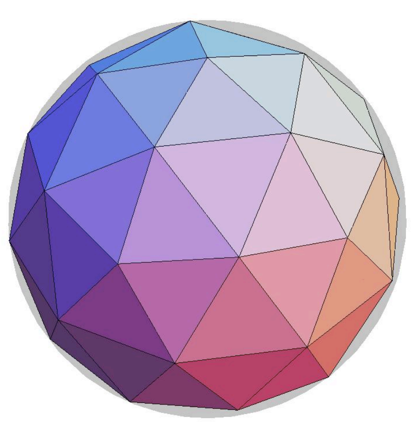


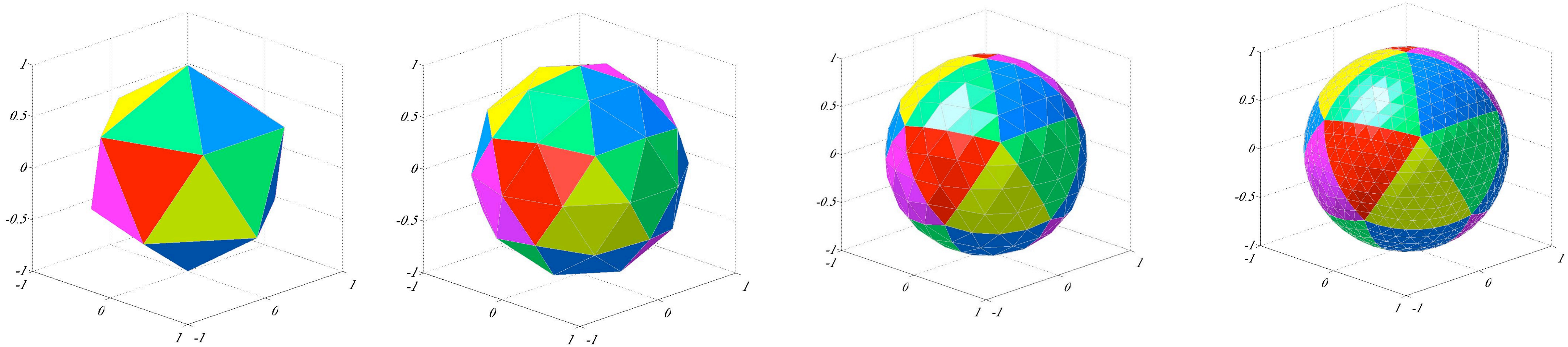

YIN GRID
YANG GRID
YIN-YANG GRID
a
b
c

SPHERICAL GEODESIC
OR ICOSAHEDRAL GRID





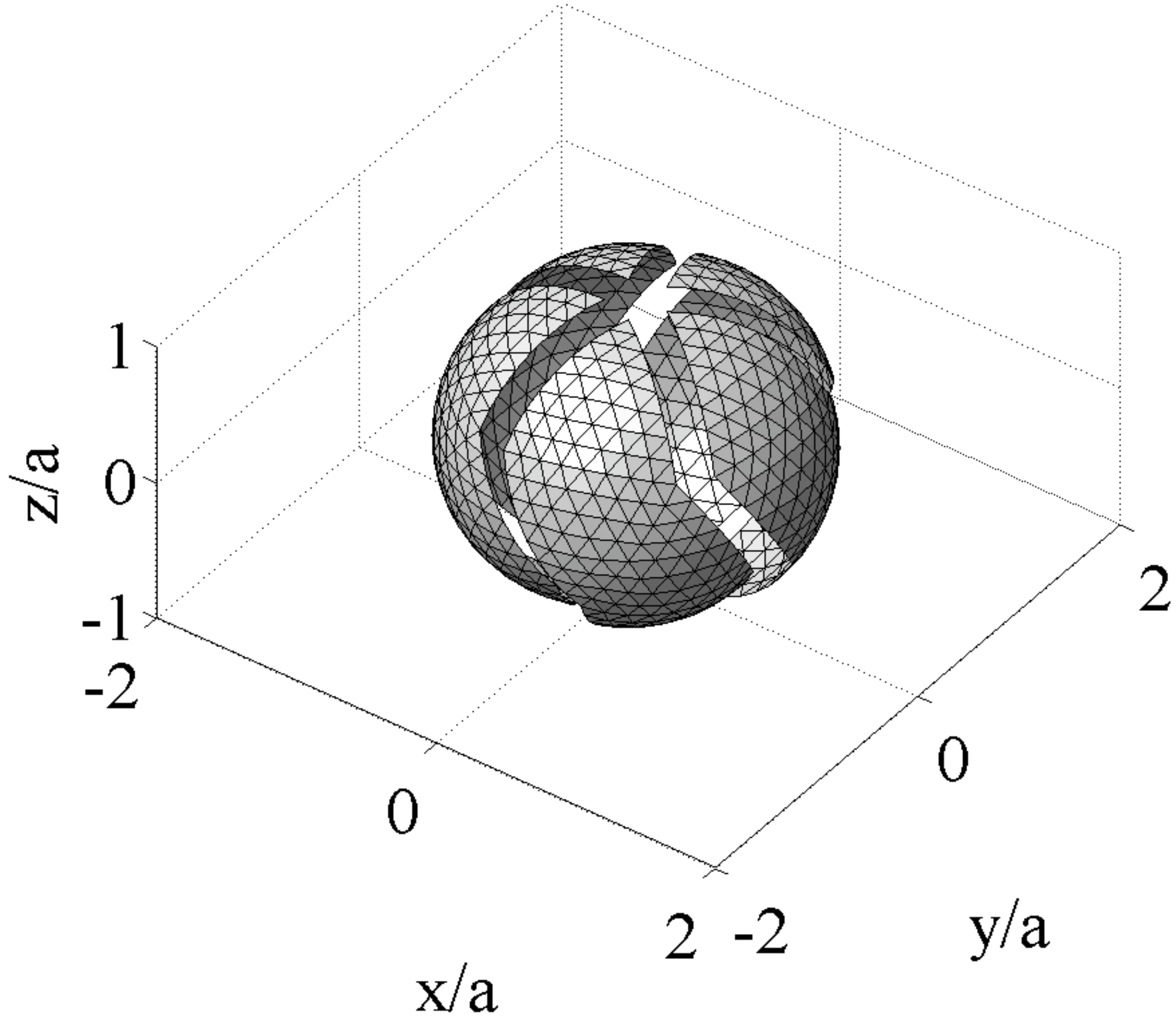




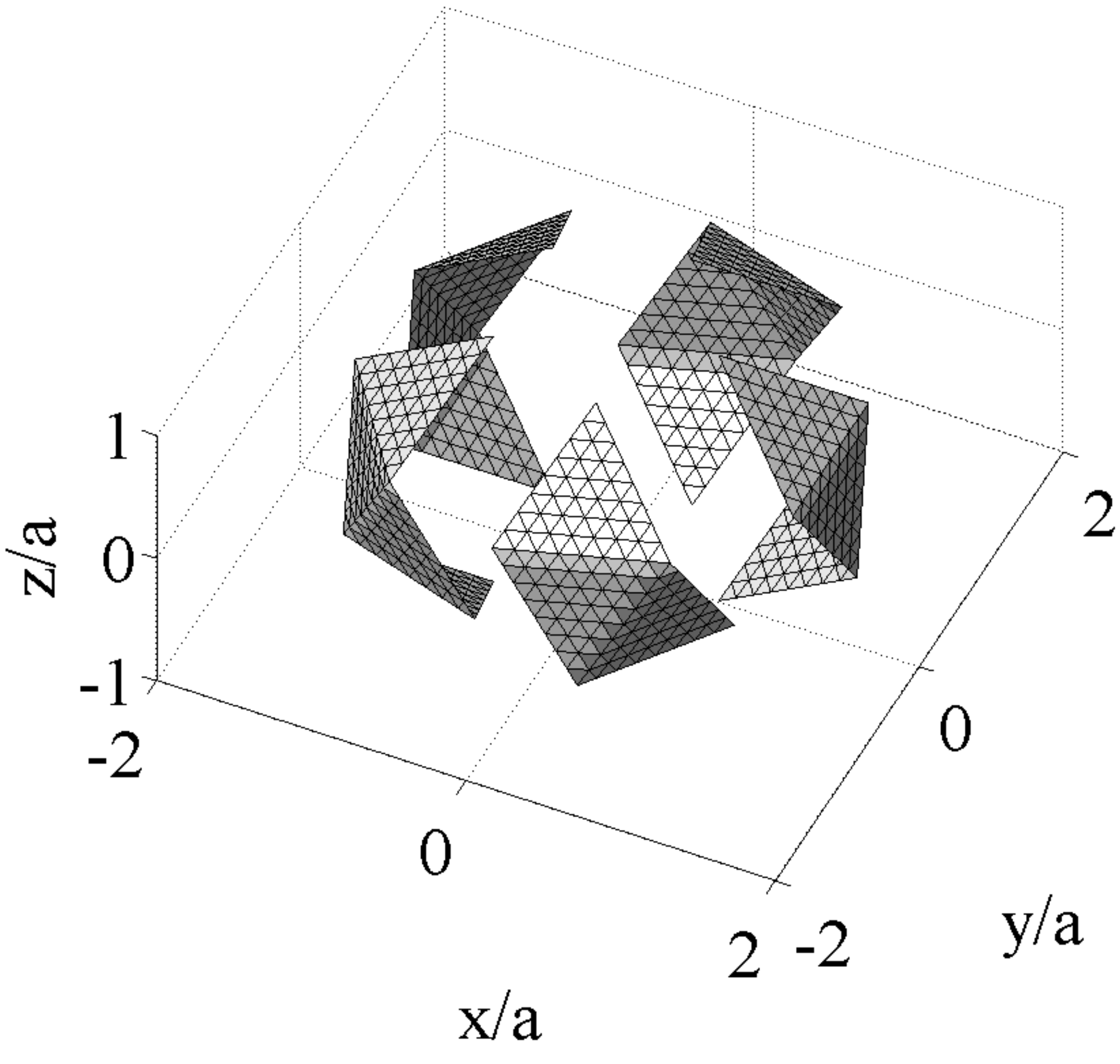
Icosahedral geodesic grids

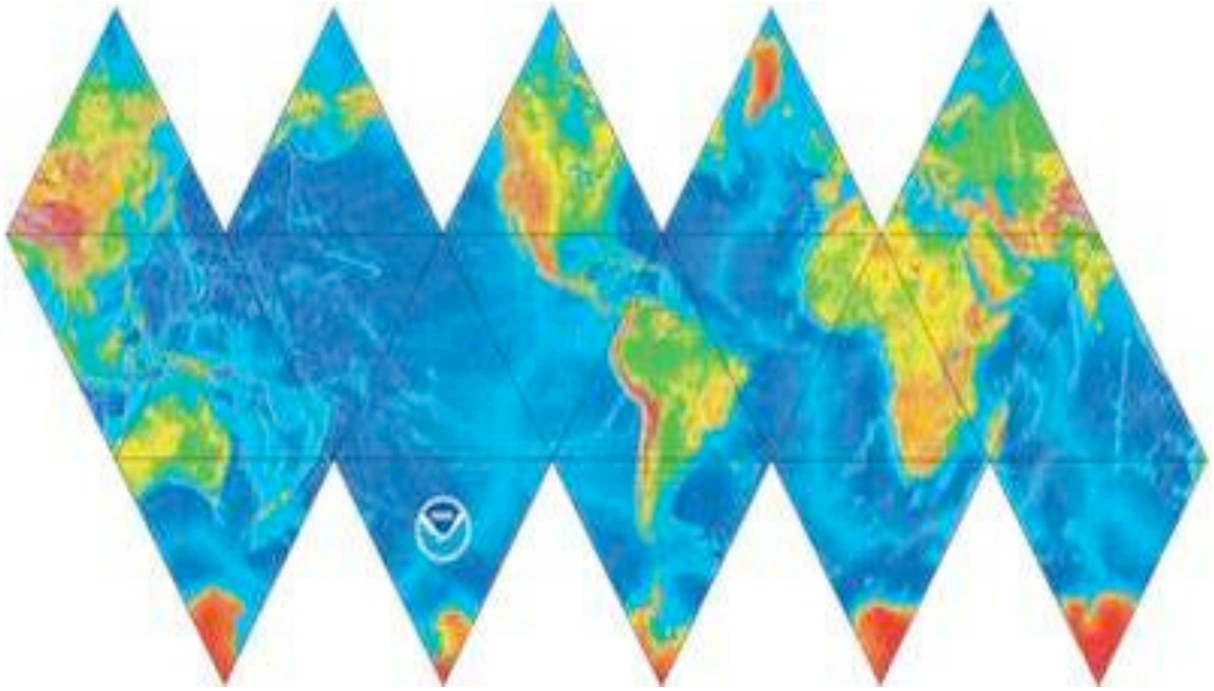
k	2^{2k}	Number of faces $N_\mathrm{f}(k) = 2^{2k} N_\mathrm{f}(0)$	Number of edges $N_\mathrm{e}(k) = \frac{3}{2} N_\mathrm{f}(k)$	Number of vertices $N_\mathrm{p}(k) = \frac{1}{2} N_\mathrm{f}(k) + 2$
0	1	20	30	12
1	4	80	120	42
2	16	320	480	162
3	64	1280	1920	642
4	256	5120	7680	2562
5	1024	20480	30720	10242
6	4096	81920	122880	40962

The grid is structured...



Icosahedral grid partition





Selection of the numerical method

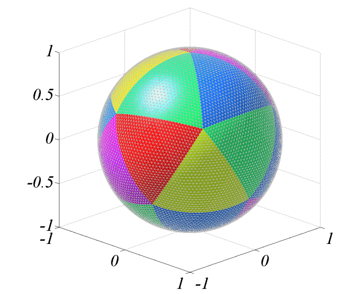
Eulerian or Semi-Lagrangian?

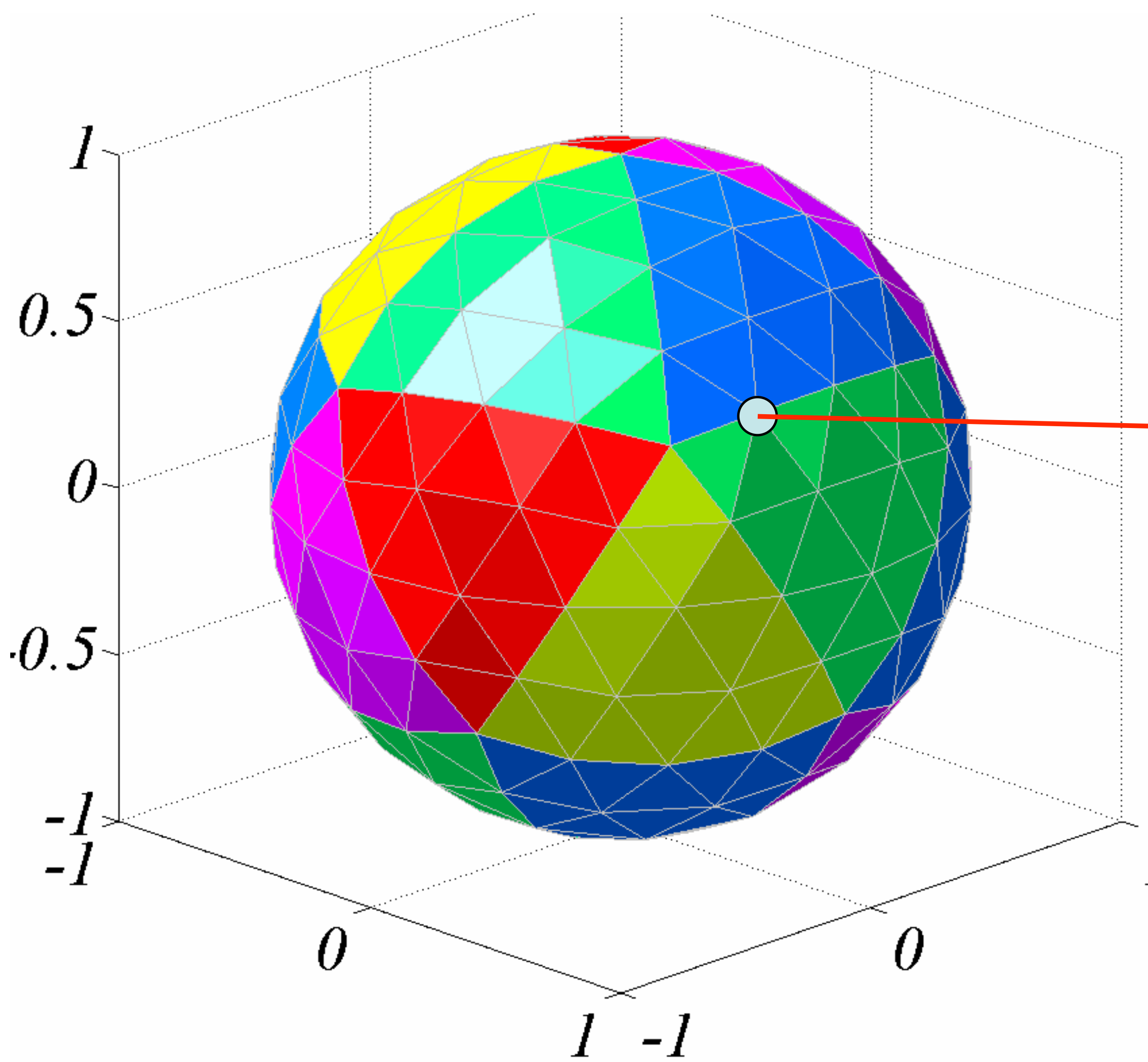
Galerkin type

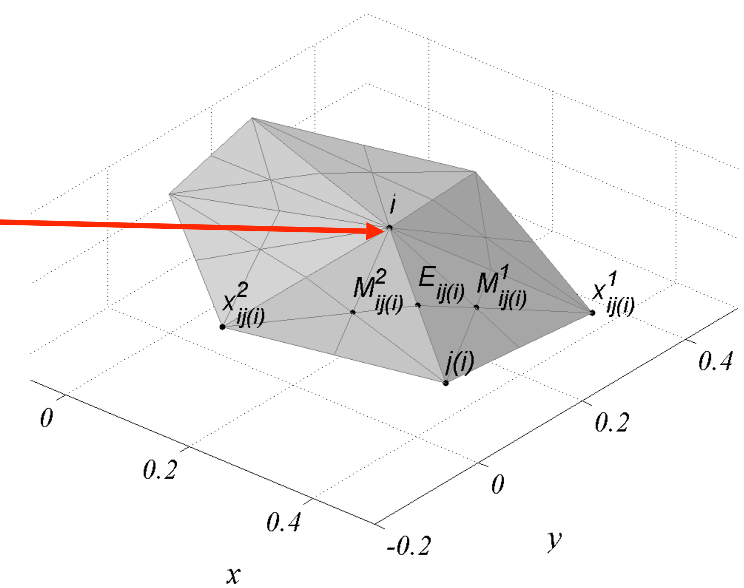
Grid point

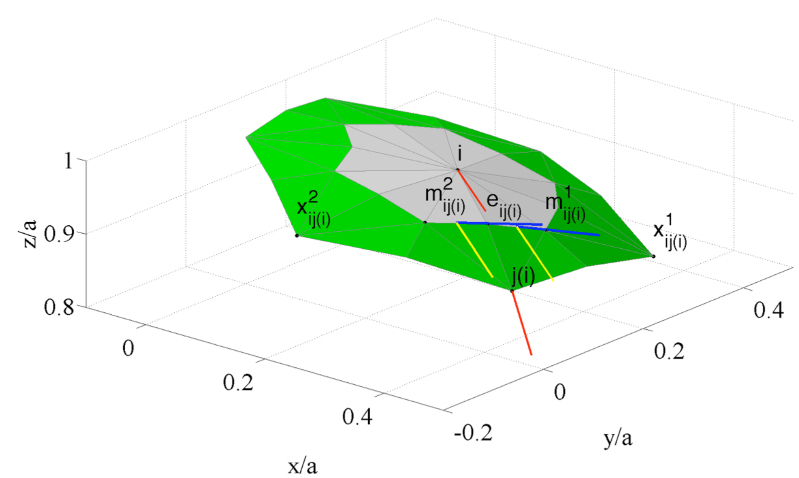
Finite volume

Definition of the control volumes









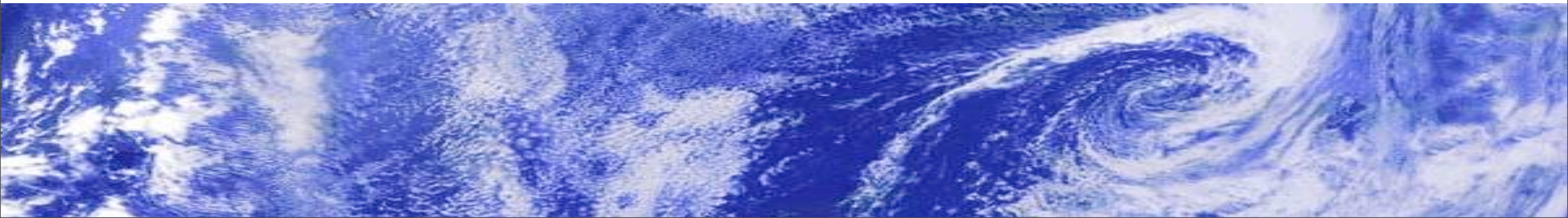
The control volume associated with the i-th node of the geodesic grid is created by a two step procedure. In the first step, the centers of the triangles and the mid-points of the edges are projected on the surface of the sphere. In the second step, the control volume is defined as a polygon with the vertices located at the projected points

Arbitrary vertical coordinate

The search for flexible vertical coordinates started in the early stage of the meteorological modelling (forties of the XX century) with some inspirations from other areas of physics including differential geometry and general relativity

The main reason was the problem with representation of terrain in the traditional Eulerian models

The quasi-Lagrangian vertical coordinates in the form which is used currently were proposed by Starr (1946) and Lin (1996, 2004)



- We define the material coordinate surfaces by assigning a constant value of a hypothetical conservative tracer to any arbitrary initial model levels (sigma, hybrid or any other ...)

-The Lagrangian control volumes defined by these material surfaces are free to float, expand, and compress with the flow as dictated by hydrostatic dynamics

- The Earth's surface in this formalism is considered as material surface fixed in time (in general we can allow, however, the floating lower boundary as well...)

- *By choosing the imaginery (better to say hypothetical) tracer that is monotonic function of height and constant on the initial coordinate surfaces the 3-D equations written for arbitrary vertical coordinates (Kasahara, 1974) can be reduced to 2-D form*

- *Assuming that the increment of values for the conservative tracer between initial levels is constant we can further simplify equations to the form shown in the following slide*

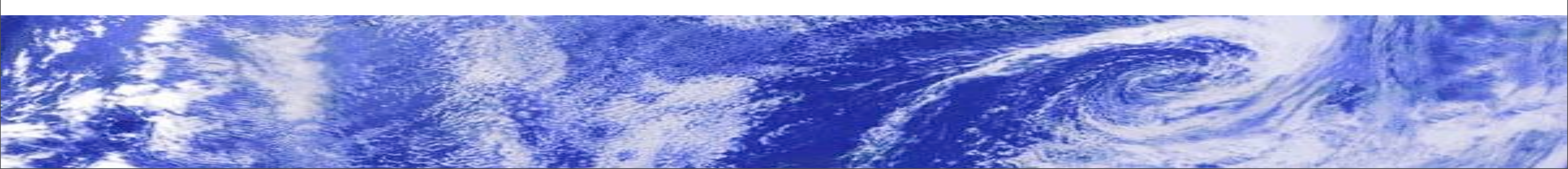
$$\frac{\partial}{\partial t}\delta p + \nabla\,\mathbf{V}\delta p = 0$$

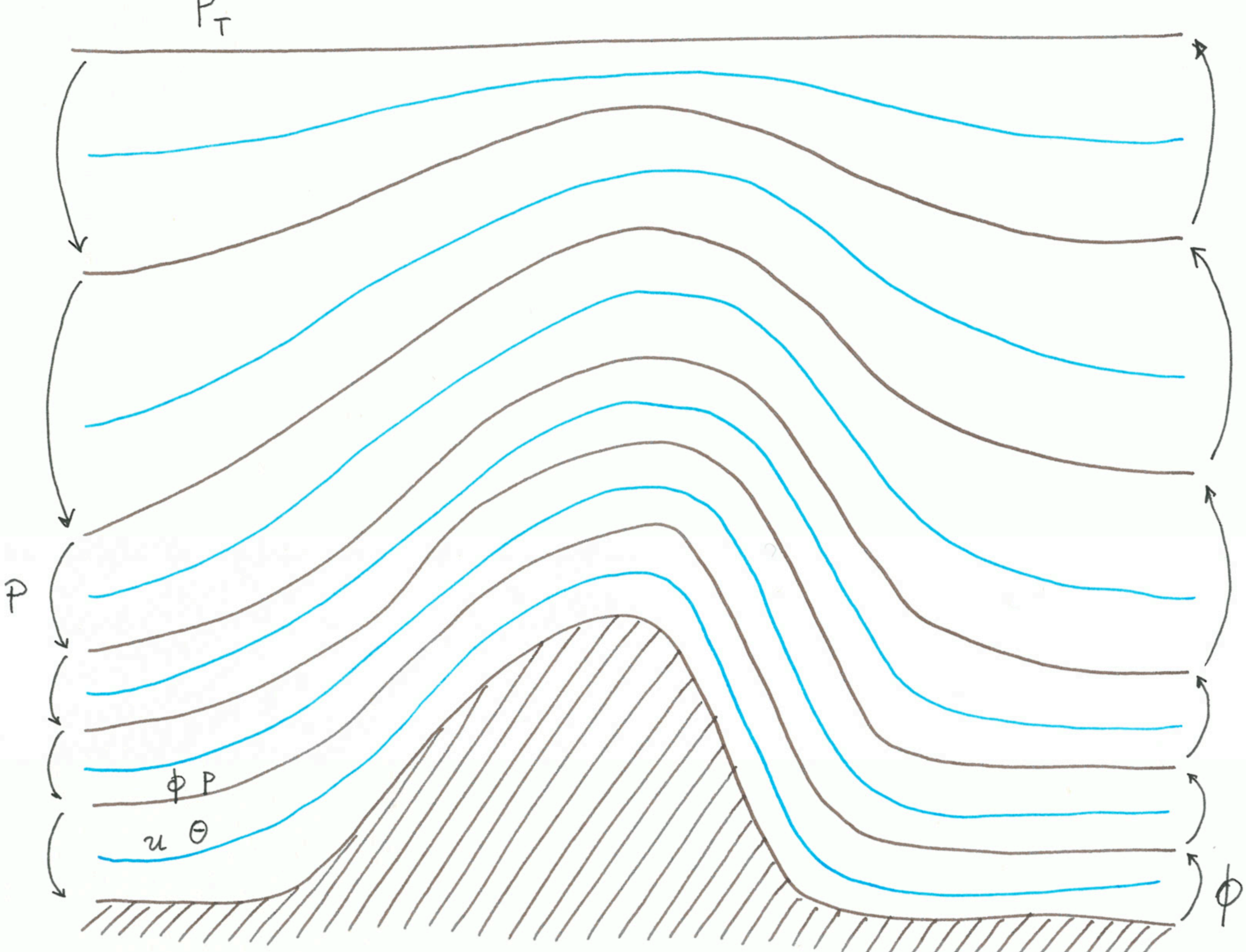
$$\frac{\partial}{\partial t}q\delta p + \nabla\,\mathbf{V}q\delta p = 0$$

$$\frac{\partial}{\partial t}\Theta\delta p + \nabla\,\mathbf{V}\Theta\delta p = 0$$

$$\frac{\partial \mathbf{V}}{\partial t} = -(\zeta + \gamma)\mathbf{n} \times \mathbf{V} - \nabla(\frac{1}{2}|\mathbf{V}|^2 + \phi) + \mathcal{D}(\mathbf{V}) + \frac{1}{\rho}\nabla p$$

$$p_l = p_\infty + \sum_{k=1}^{l} p_k$$





Icosahedral model with quasi-Lagrangian arbitrary vertical coordinate

$$\frac{\partial \mathbf{u}}{\partial t} = -(\zeta + \gamma \mathbf{n}) \times \mathbf{u} - \nabla(\frac{|\mathbf{u}|^2}{2} + gh)$$

$$\frac{\partial h^\star}{\partial t} + \nabla h^\star \mathbf{u} = 0$$

$$\frac{\partial \varphi_i}{\partial t} = -\nabla \varphi_i \mathbf{u} + \nabla \cdot (\mathbf{K}_i \nabla \varphi_i) + F_i(\varphi_1, \ldots, \varphi_n)$$

$$\zeta = Curl_n \mathbf{u}$$

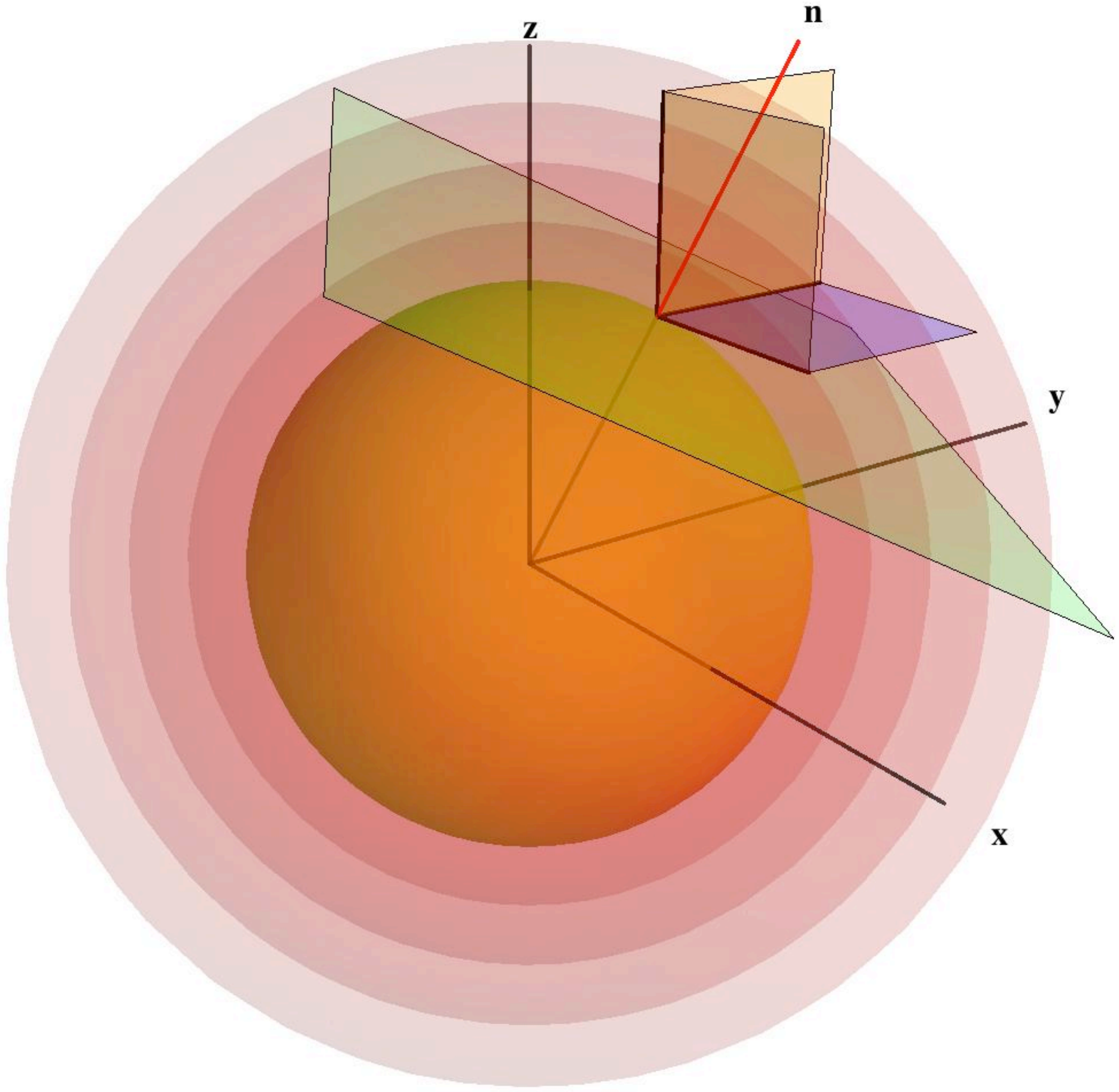
The model kernel equations are in vector invariant form

Embedded manifold approach

$$Curl_n \, \mathbf{u}(\mathbf{r}_s) = (curl_3 \, \widehat{\mathbf{u}}(\mathbf{r}) \circ \mathbf{n}(\mathbf{r})) \, \mathbf{n}(\mathbf{r})|_{\mathbf{r}=\mathbf{r}_s}$$

$$\nabla_s f \equiv \nabla_3 \widehat{f}|_{\mathbf{r}=\mathbf{r}_s}$$

$$\mathrm{div}\,\mathbf{A} = \mathrm{div}_3 \widehat{\mathbf{A}}|_{\mathbf{r}=\mathbf{r}_s}$$



(radial extensions of vector and scalar fields)

$$\frac{\partial \widehat{\mathbf{u}}}{\partial t}|_{\mathbf{r}=\mathbf{r}_s} = -[(curl_3 \widehat{\mathbf{u}}(\mathbf{r}) \circ \mathbf{n} + \gamma)\, \mathbf{n} \times \widehat{\mathbf{u}}]|_{\mathbf{r}=\mathbf{r}_s} -$$

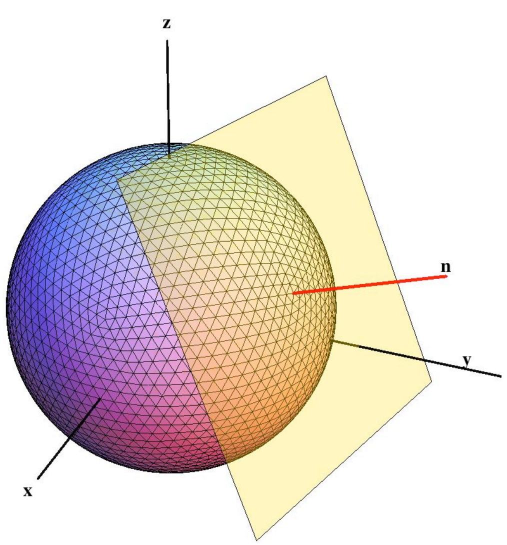
$$\nabla_3 [\frac{|\widehat{\mathbf{u}}(\mathbf{r})|^2}{2} + g\widehat{h(\mathbf{r})}]|_{\mathbf{r}=\mathbf{r}_s},$$

$$\frac{\partial \widehat{h^\star}}{\partial t}|_{\mathbf{r}=\mathbf{r}_s} = -\mathrm{div}_3 \widehat{\mathbf{u}h^\star}|_{\mathbf{r}=\mathbf{r}_s},$$

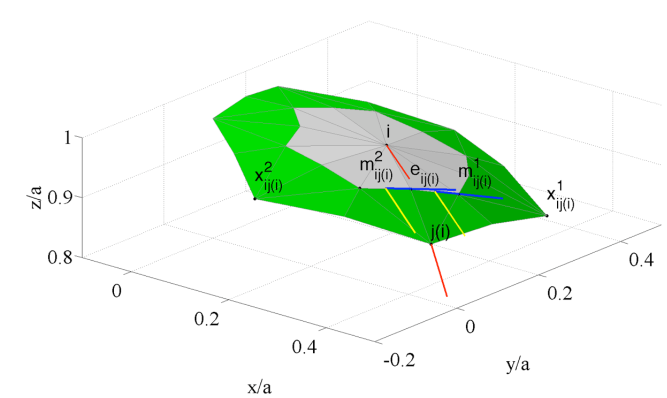
Equations on the sphere written in terms of the cartesian operators acting on the smooth radial extensions of the vector and scalar fields

The finite volume works with averages; the set of equations is thus averaged over the specified control volumes

$$\frac{\partial\langle\widehat{\mathbf{u}}|_{\mathbf{r}=\mathbf{r}_S}\rangle_{\Omega}}{\partial t} = -\langle(curl_3\widehat{\mathbf{u}}\circ\mathbf{n}+\gamma)|_{\mathbf{r}=\mathbf{r}_S}\rangle_{\Omega}\,\mathbf{n}_\Omega \times \langle\widehat{\mathbf{u}}|_{\mathbf{r}=\mathbf{r}_S}\rangle_{\Omega} -$$



$$\langle\nabla_3(\frac{|\widehat{\mathbf{u}}|^2}{2}+g\widehat{h})|_{\mathbf{r}=\mathbf{r}_S}\rangle_{\Omega},$$



$$\frac{\partial\langle\widehat{h^\star}|_{\mathbf{r}=\mathbf{r}_S}\rangle_{\Omega}}{\partial t} = -\langle\mathrm{div}_3\widehat{\mathbf{u}h^\star}|_{\mathbf{r}=\mathbf{r}_S}\rangle_{\Omega},$$

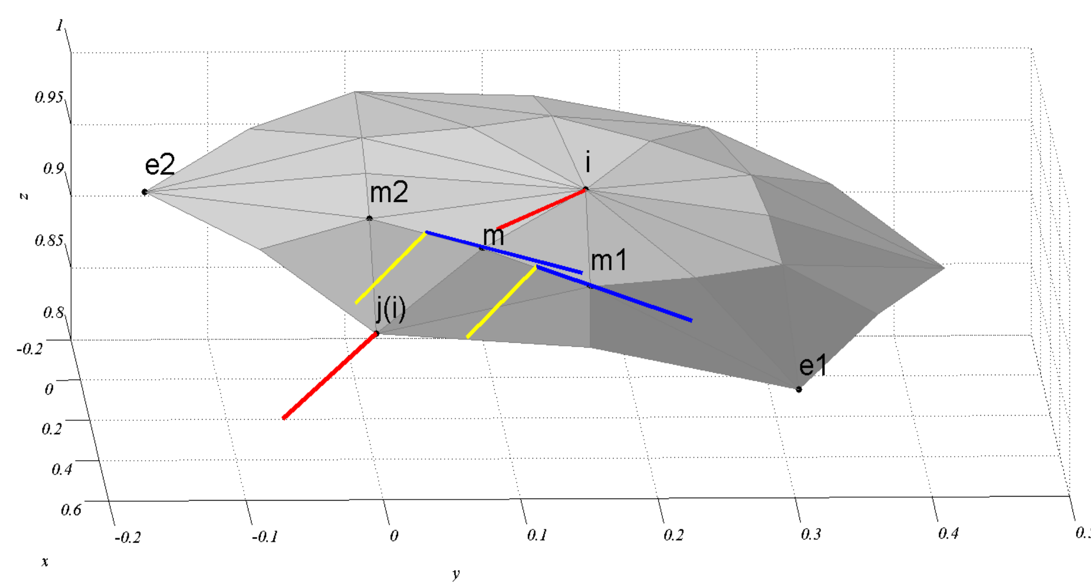
$$\langle \mathrm{div}\, \mathbf{u}h^\star \rangle_{\Omega_i} = \langle \mathrm{div}_3 \widehat{\mathbf{u}h^\star}|_{\mathbf{r}=\mathbf{r}_s} \rangle_{\Omega_i} = \sum_{j(i)} (\mathbf{u}\, h^\star)_{ij} \circ \mathbf{b}_{ij}$$

$$\langle \nabla_s\, f \rangle_{\Omega_i} = \langle \nabla_3 \widehat{f}|_{\mathbf{r}=\mathbf{r}_s} \rangle_{\Omega_i} = \sum_{j(i)} f_{ij}\, \mathbf{b}_{ij} - \mathbf{N}_{\Omega_i}$$

$$\langle (Curl_n \mathbf{u}) \circ \mathbf{n} \rangle_{\Omega_i} = \langle (curl_3\, \hat{\mathbf{u}}|_{\mathbf{r}=\mathbf{r}_s}) \circ \mathbf{n} \rangle_{\Omega_i} = \sum_{j(i)} \mathbf{u}_{ij} \circ \mathbf{d}_{ij}$$

$$\mathbf{b}_{ij} = (\mathbf{n}_{b\,ij}^1\, \delta l_{ij}^1 + \mathbf{n}_{b\,ij}^2\, \delta l_{ij}^2)/S_i,$$

$$\mathbf{d}_{ij} = (\boldsymbol{\tau}_{ij}^1\, \delta l_{ij}^1 + \boldsymbol{\tau}_{ij}^2\, \delta l_{ij}^2)/S_i,$$



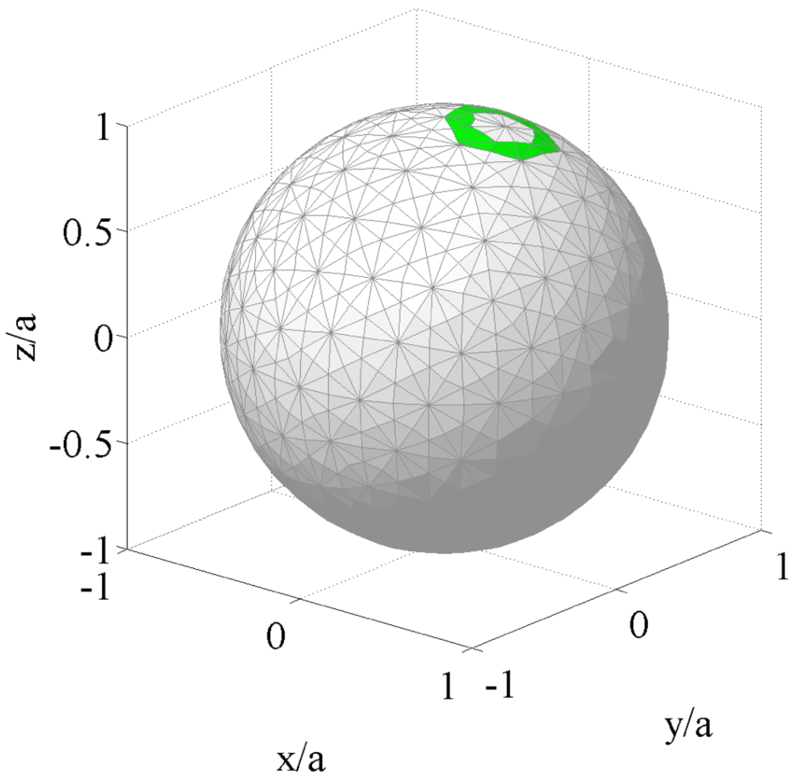
(interface values are obtained from conservative polynomial reconstruction)

Algebraic form of the model kernel

$$\begin{cases} \frac{d}{dt}\{u^x\} = -\mathbf{W}^x - \mathbf{GS}_x\{f\} \\ \frac{d}{dt}\{u^y\} = -\mathbf{W}^y - \mathbf{GS}_y\{f\} \\ \frac{d}{dt}\{u^z\} = -\mathbf{W}^z - \mathbf{GS}_z\{f\} \\ \frac{d}{dt}\{h^\star\} = -\mathbf{D}_x\{u^x h^\star\} - \mathbf{D}_y\{u^y h^\star\} - \mathbf{D}_z\{u^z h^\star\} \end{cases}$$

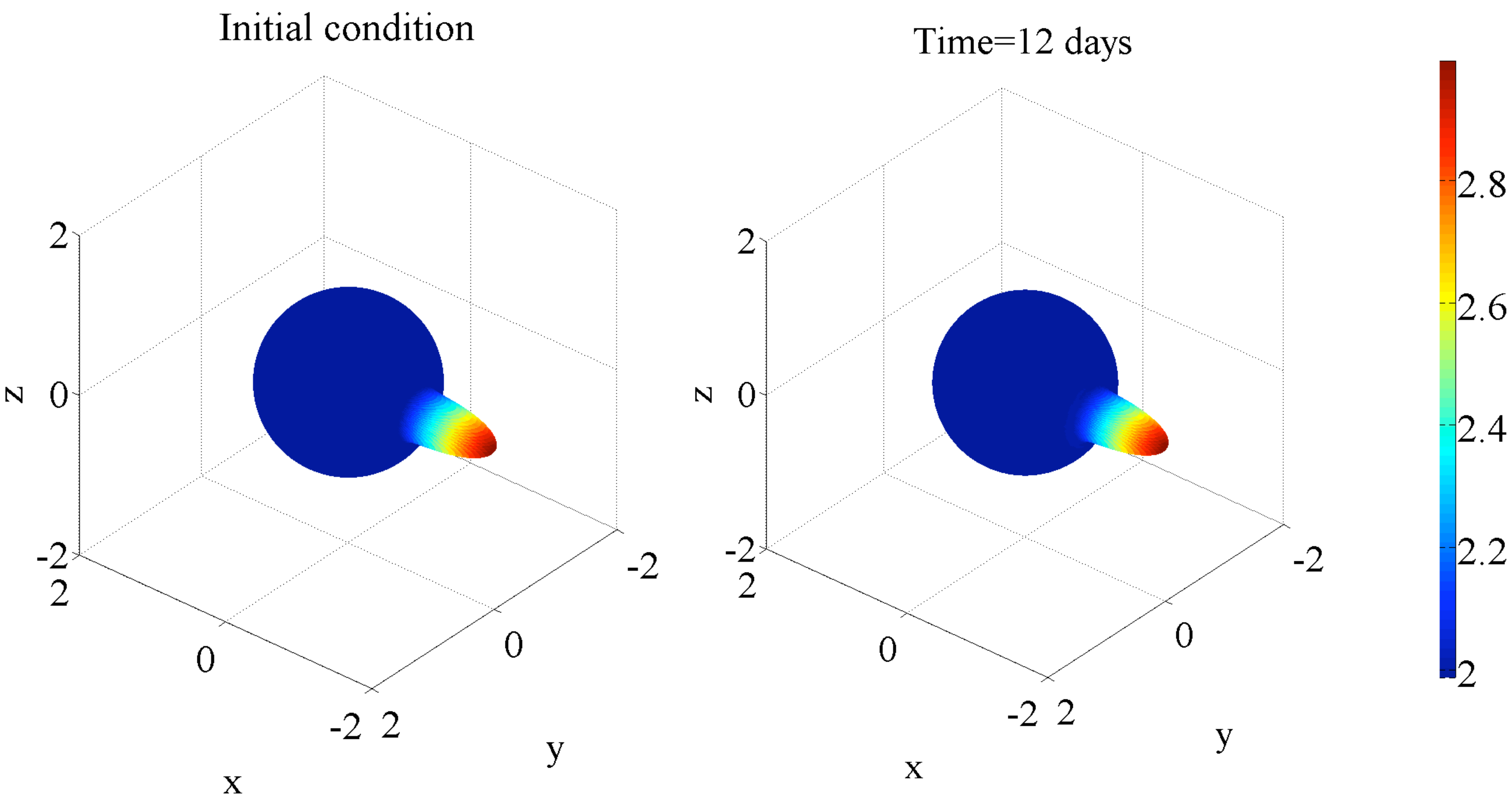
$$f = g(h_s + h^\star) + |\mathbf{u}|^2/2$$

$$\begin{cases} \mathbf{W}^x = \mathbf{z} * (\mathcal{N}_y * \{u^z\} - \mathcal{N}_z * \{u^y\}), \\ \mathbf{W}^y = \mathbf{z} * (\mathcal{N}_z * \{u^x\} - \mathcal{N}_x * \{u^z\}), \\ \mathbf{W}^z = \mathbf{z} * (\mathcal{N}_x * \{u^y\} - \mathcal{N}_y * \{u^x\}), \end{cases}$$

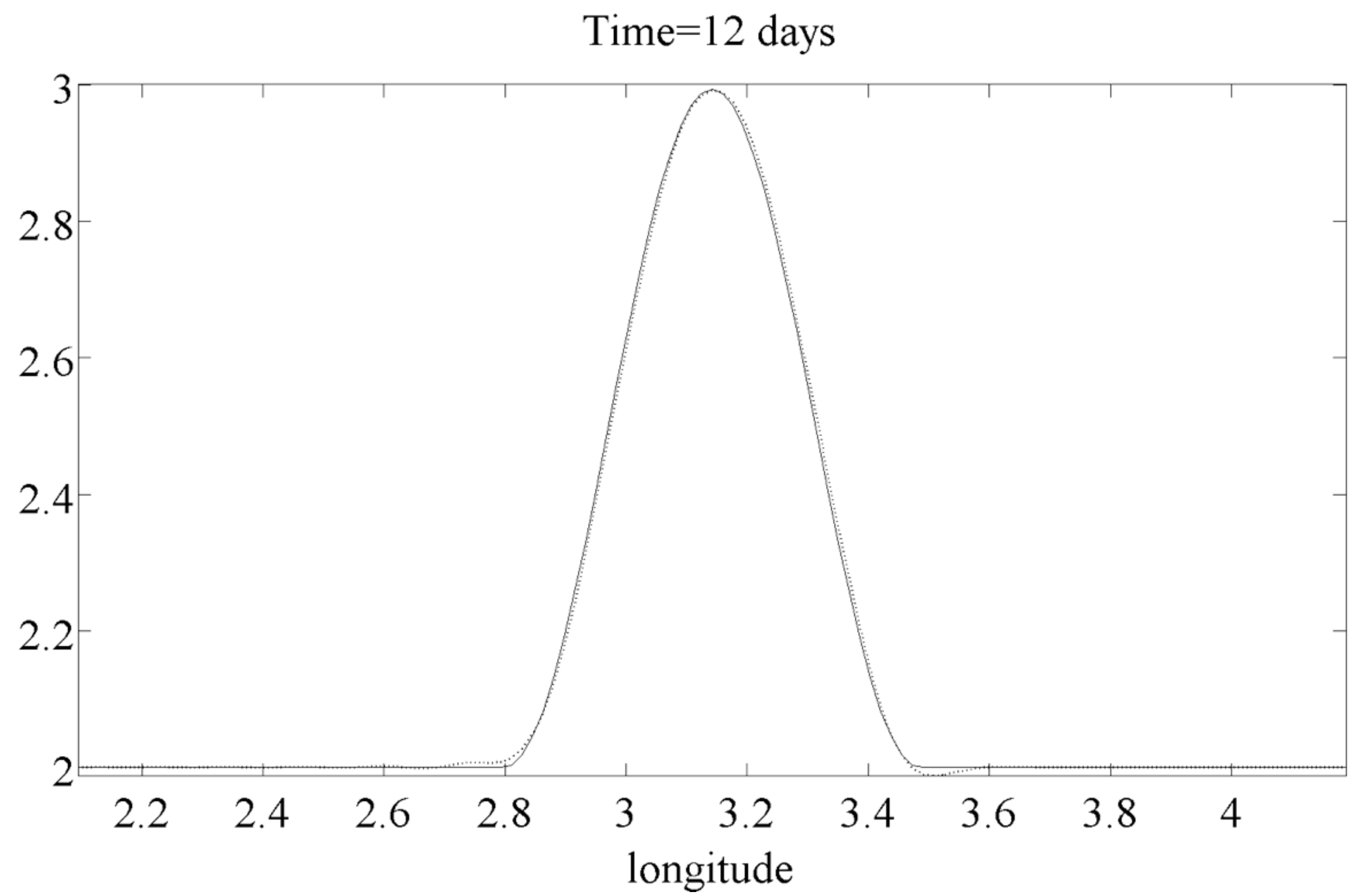


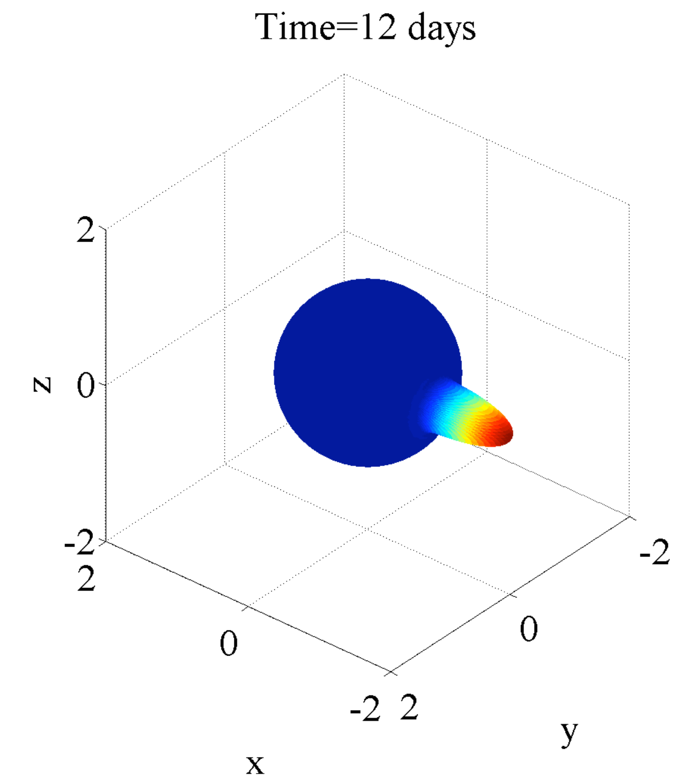
$$\mathbf{z} = \mathbf{V}_x\{u^x\} + \mathbf{V}_y\{u^y\} + \mathbf{V}_z\{u^z\} + \{\gamma\}$$

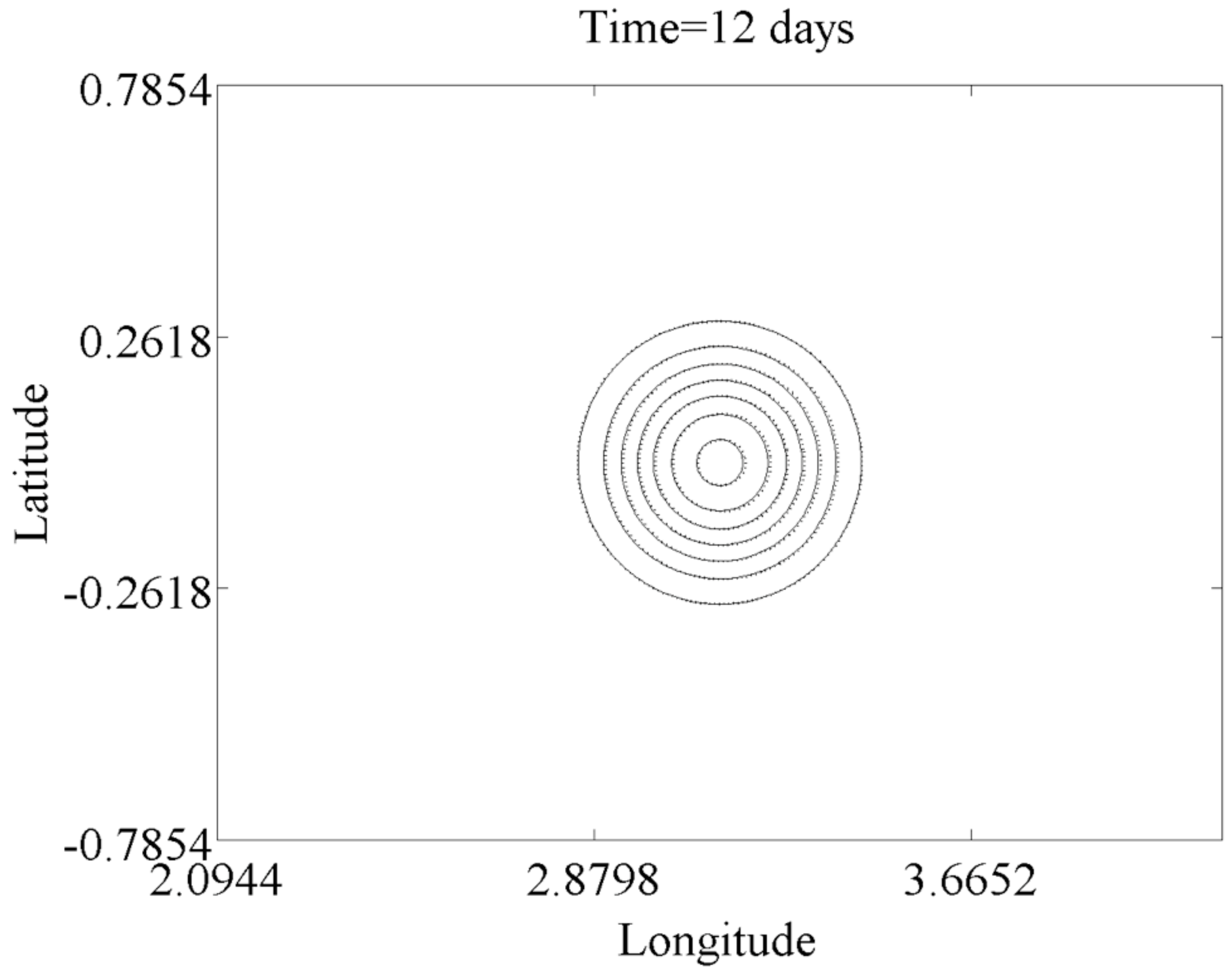
Cosine hill test



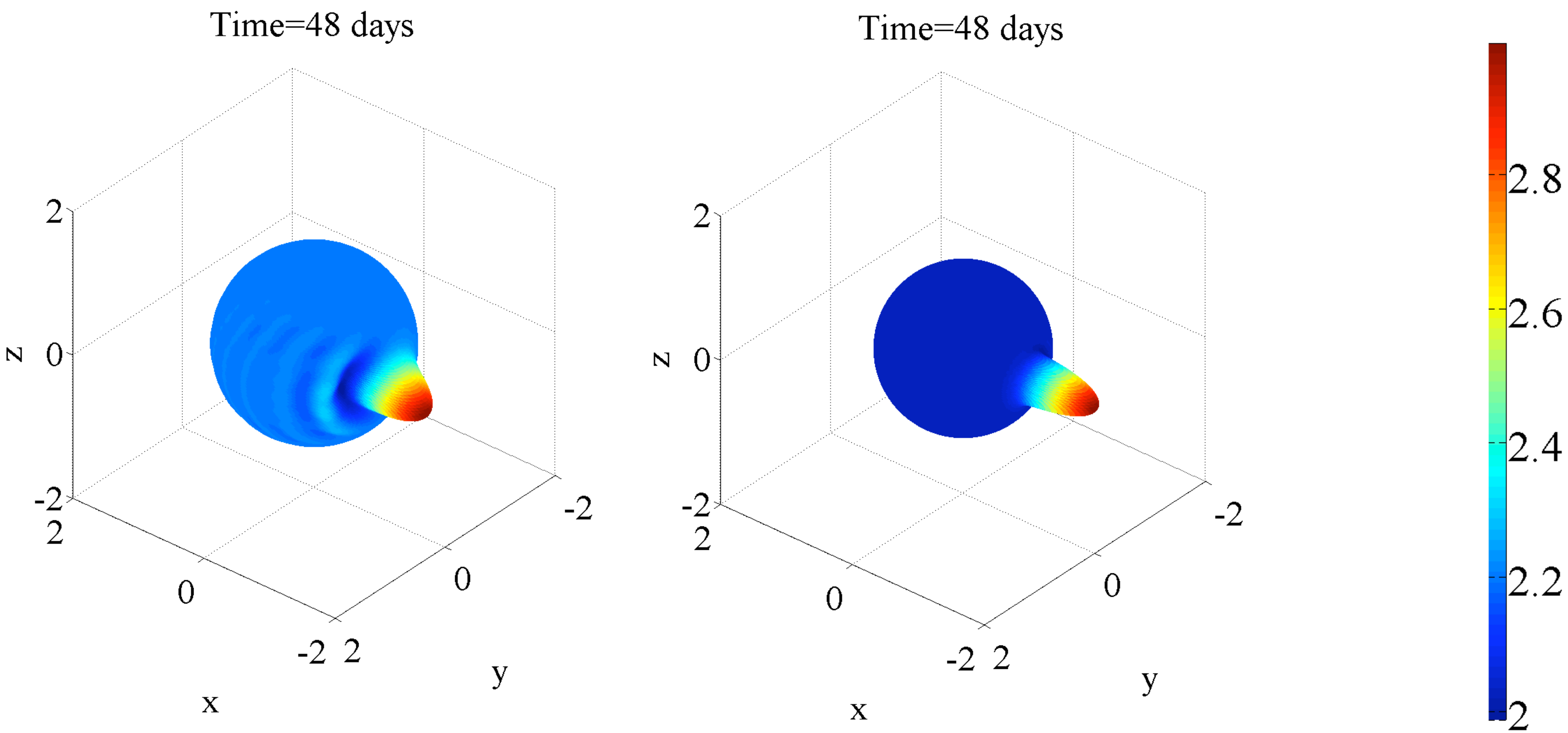
$$1 + \frac{(\delta - \delta_{min})}{(\delta_{max} - \delta_{min})}$$


Time=12 days
3
2.8
2.6
2.4
2.2
2
2.2
2.4
2.6
2.8
3
3.2
3.4
3.6
3.8
4
longitude


Time=12 days
2
z
0
-2
2
0
-2
x
2
0
-2
y

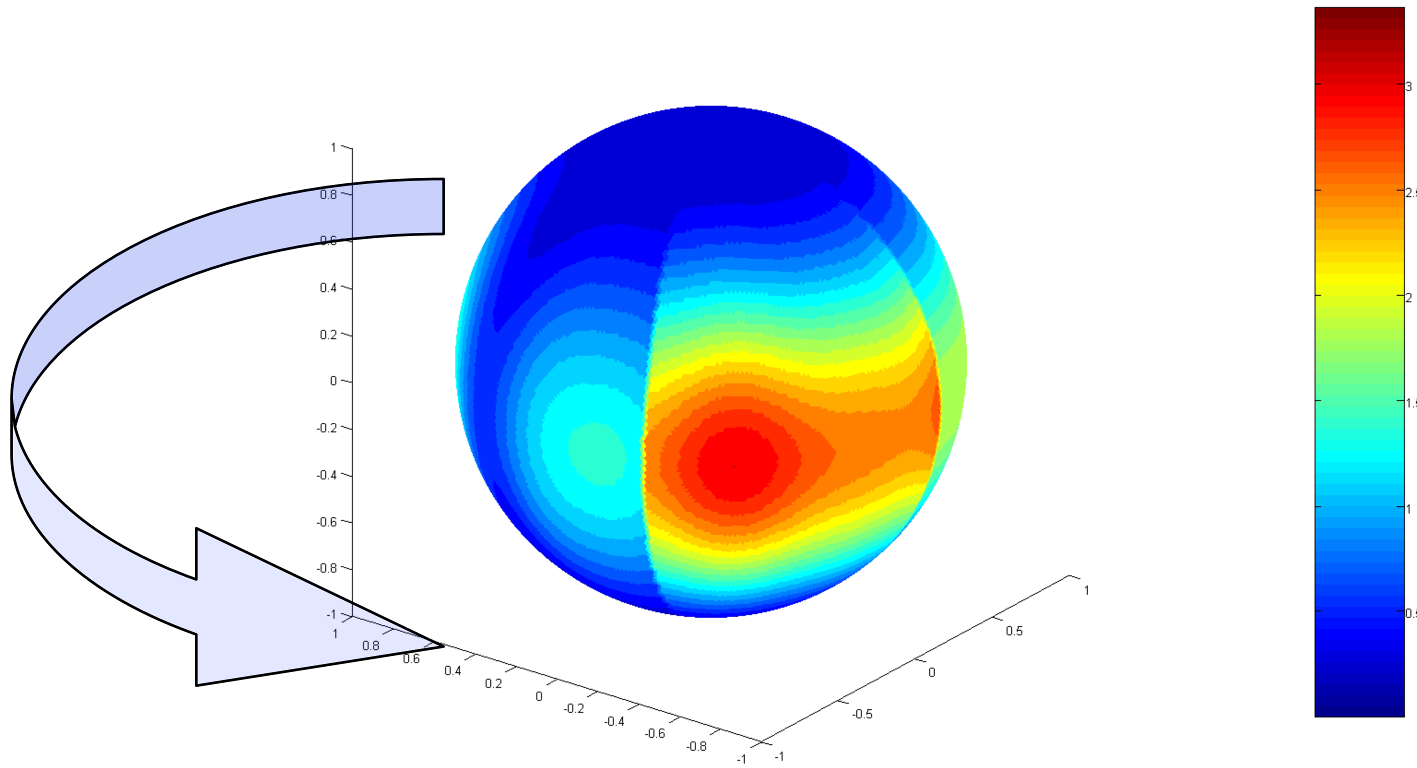

Time=12 days
0.7854
0.2618
-0.2618
-0.7854
Latitude
2.0944
2.8798
3.6652
Longitude

Low order versus High order solution

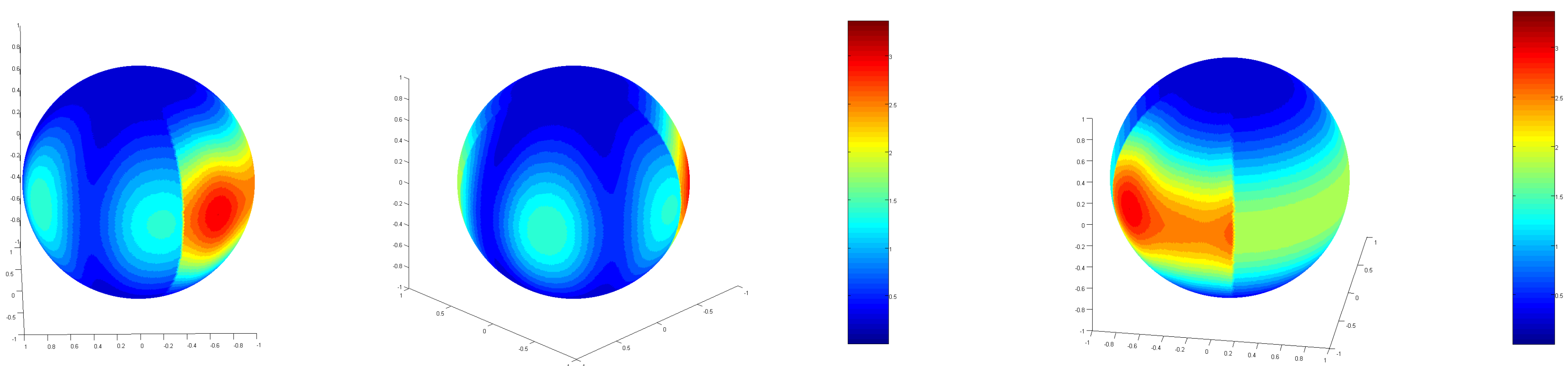


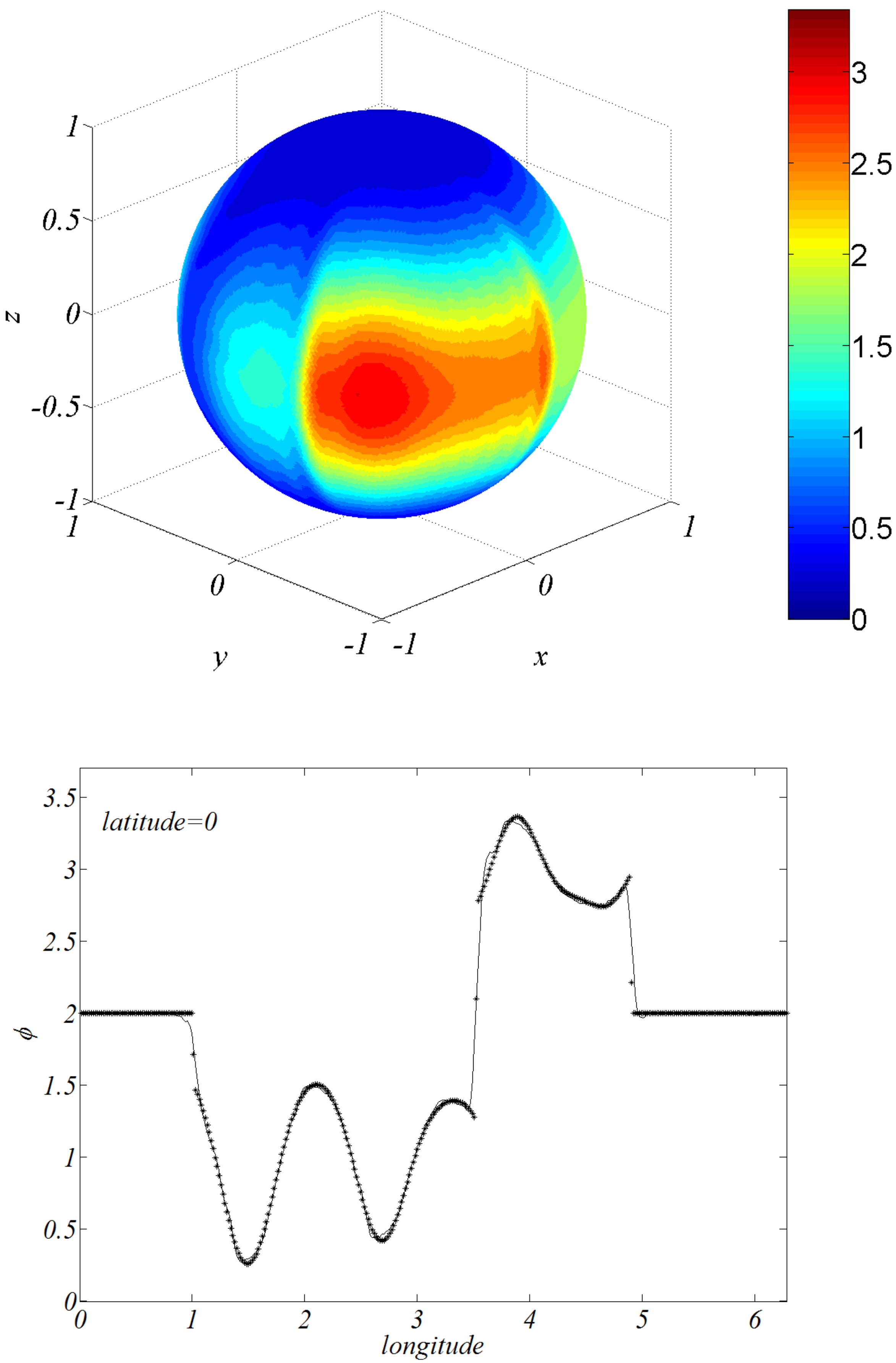
Low order solution High order solution

Test of the nonoscillatory properties of the method

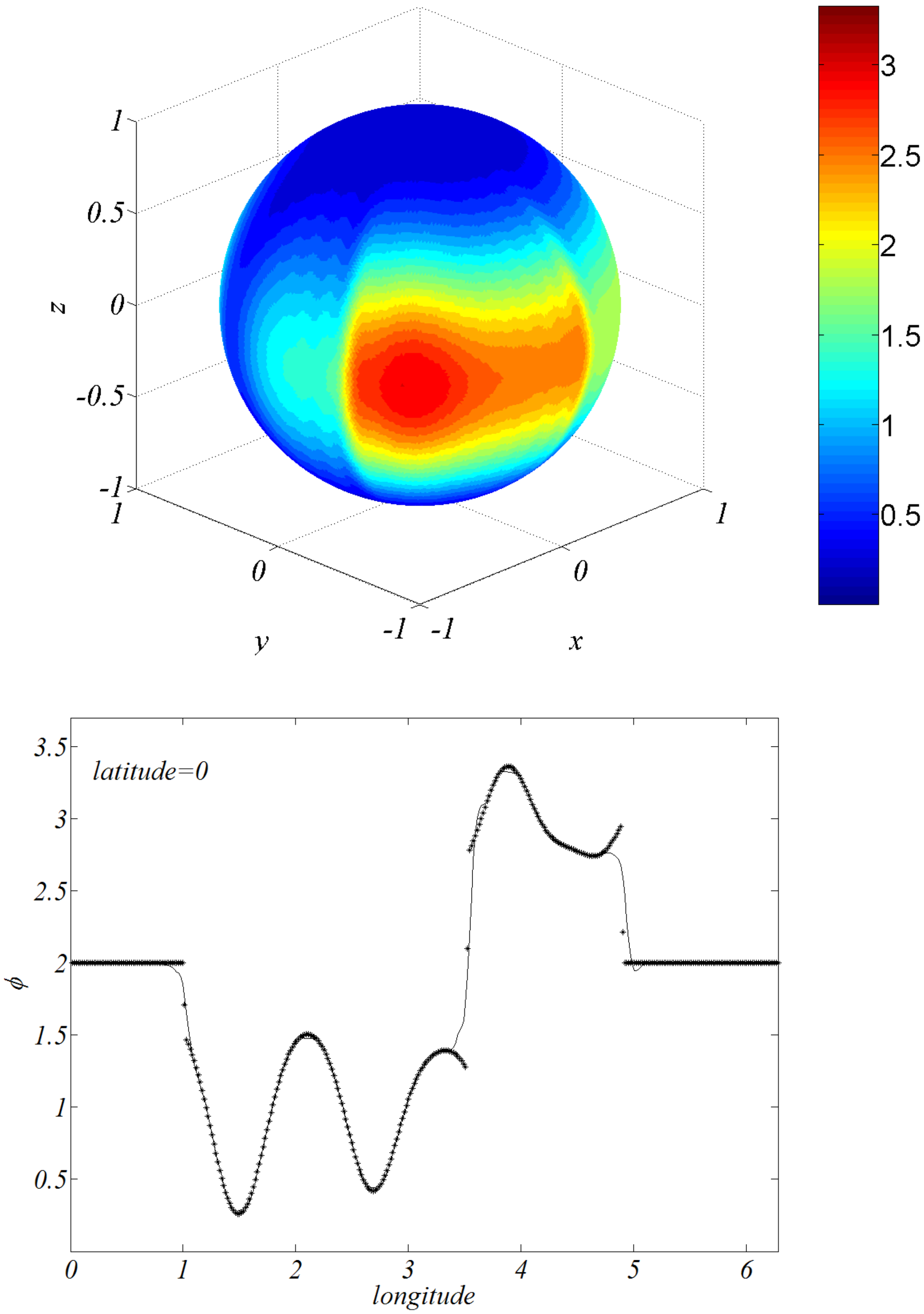


Multiscale signal, superposition of continuous waves and step functions in the longitude direction modulated by a 4-th power of cos(latitude) in the North-South direction

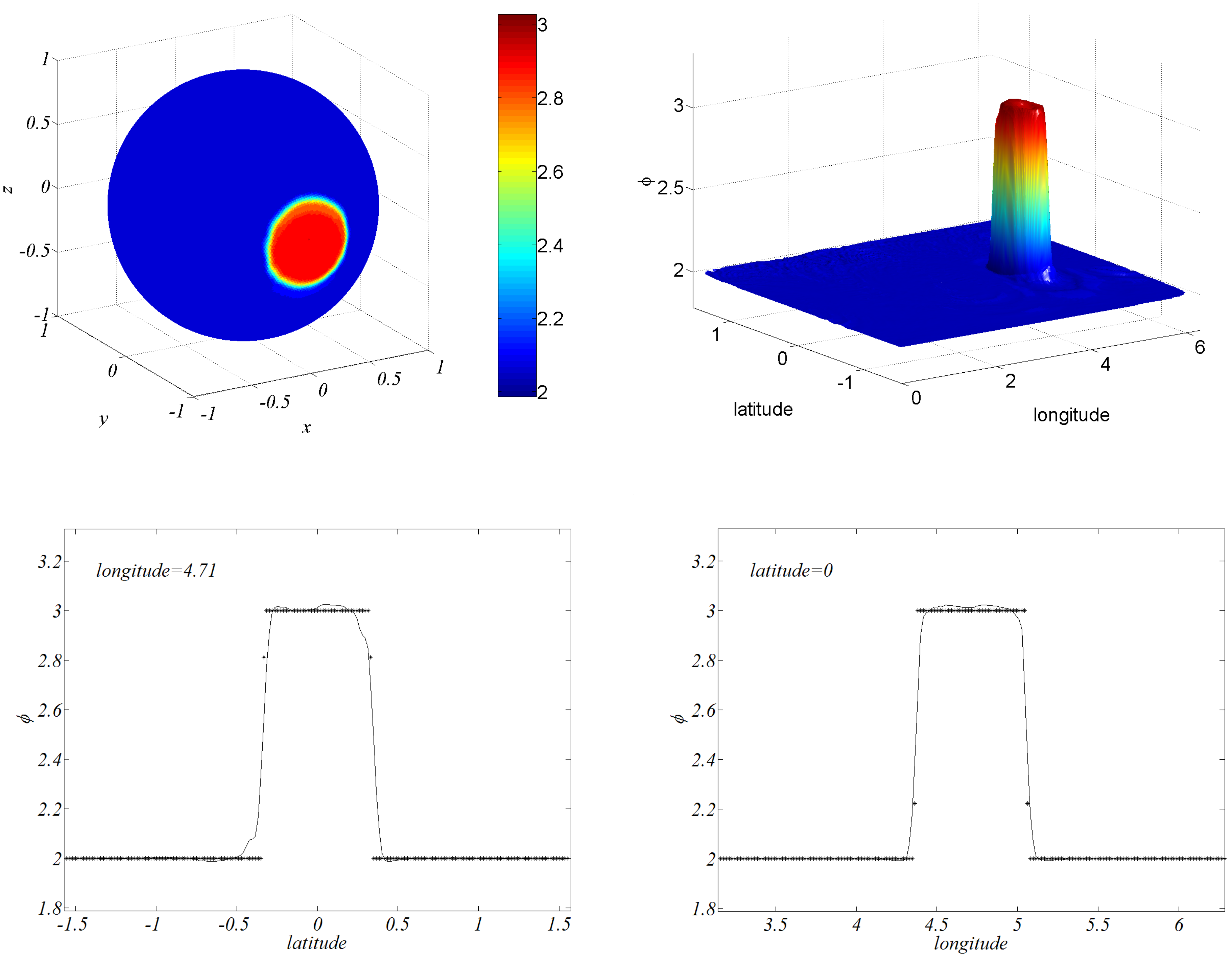




ELAD, it=2

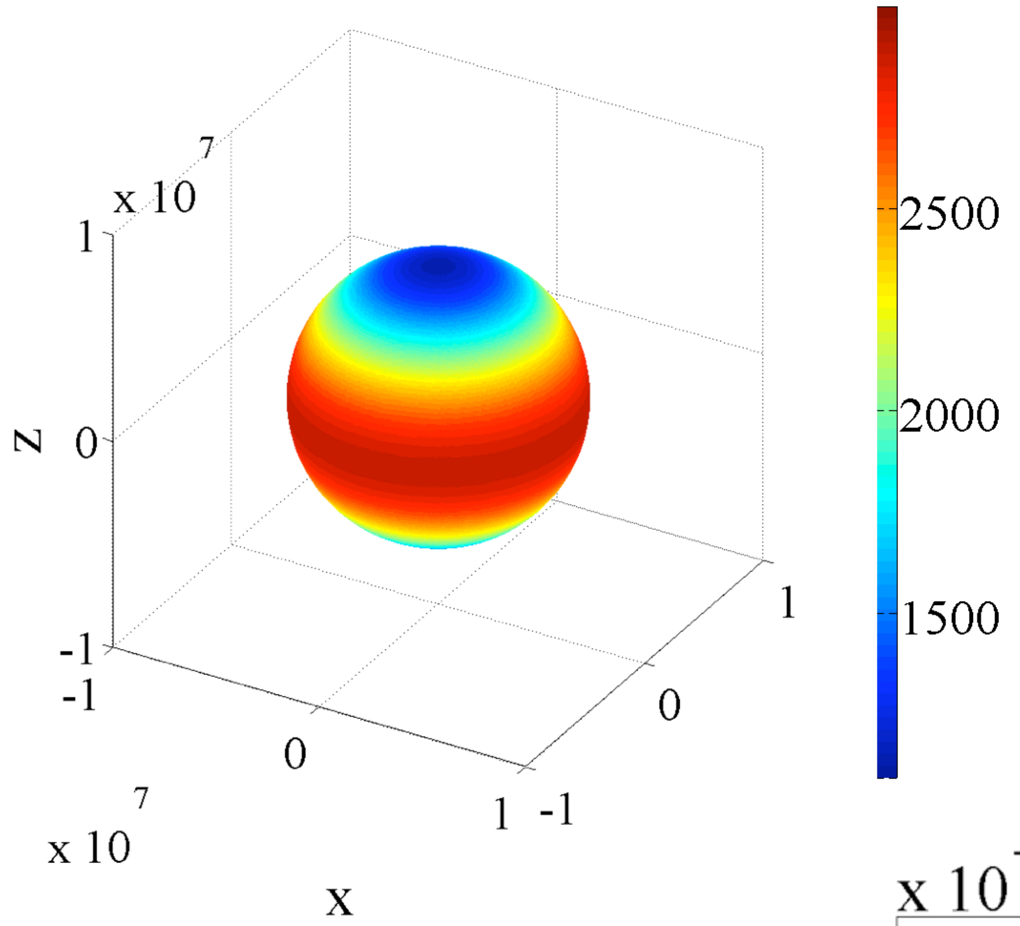


FCT



Advection with ELAD

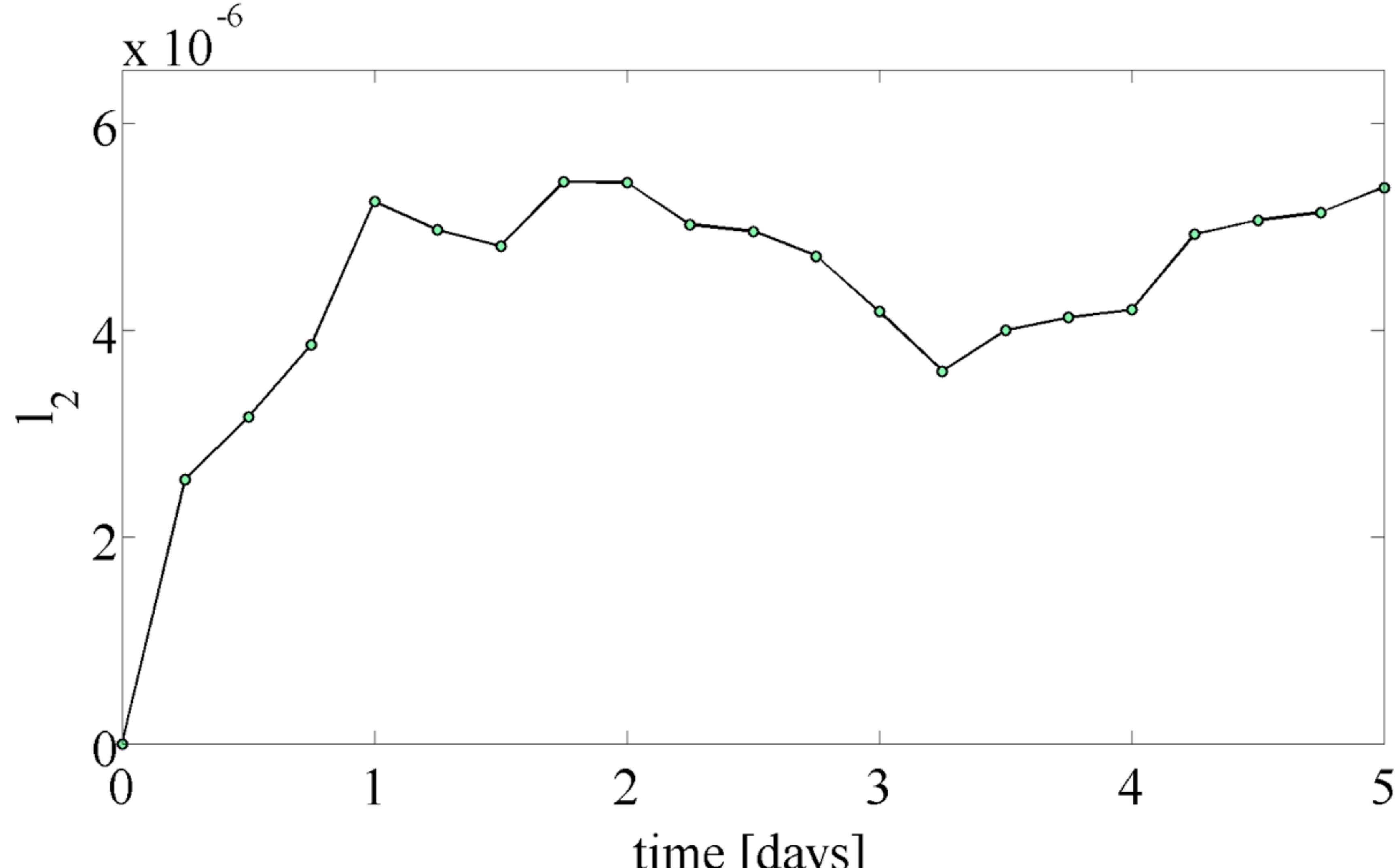
Advection is mass conserving and stable for Courant numbers up to 2.7



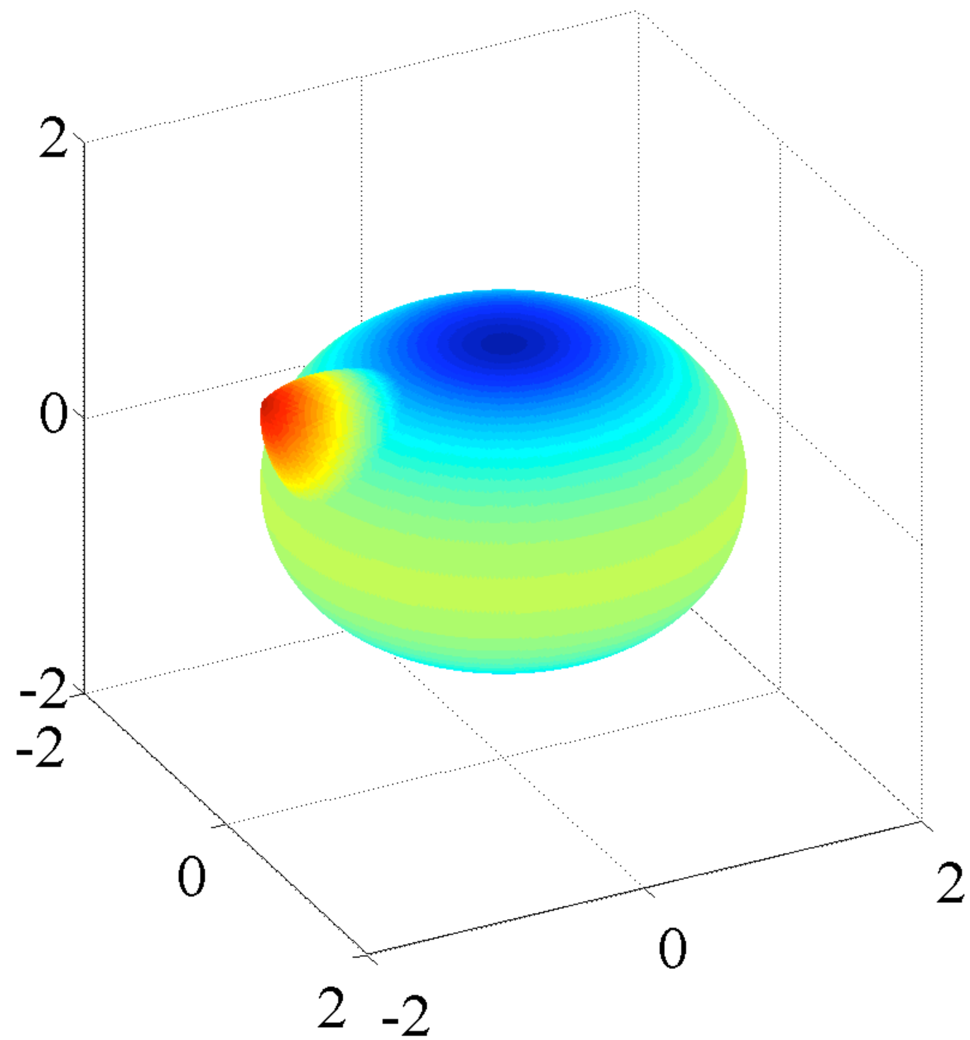
Zonal flow
(frictionless fluid rotating in gravitational field)

$$\frac{\partial \mathbf{u}}{\partial t} = -(\zeta + \gamma \mathbf{n}) \times \mathbf{u} - \nabla\left(\frac{|\mathbf{u}|^2}{2} + gh\right)$$

$$\frac{\partial h^\star}{\partial t} + \nabla h^\star \mathbf{u} = 0$$

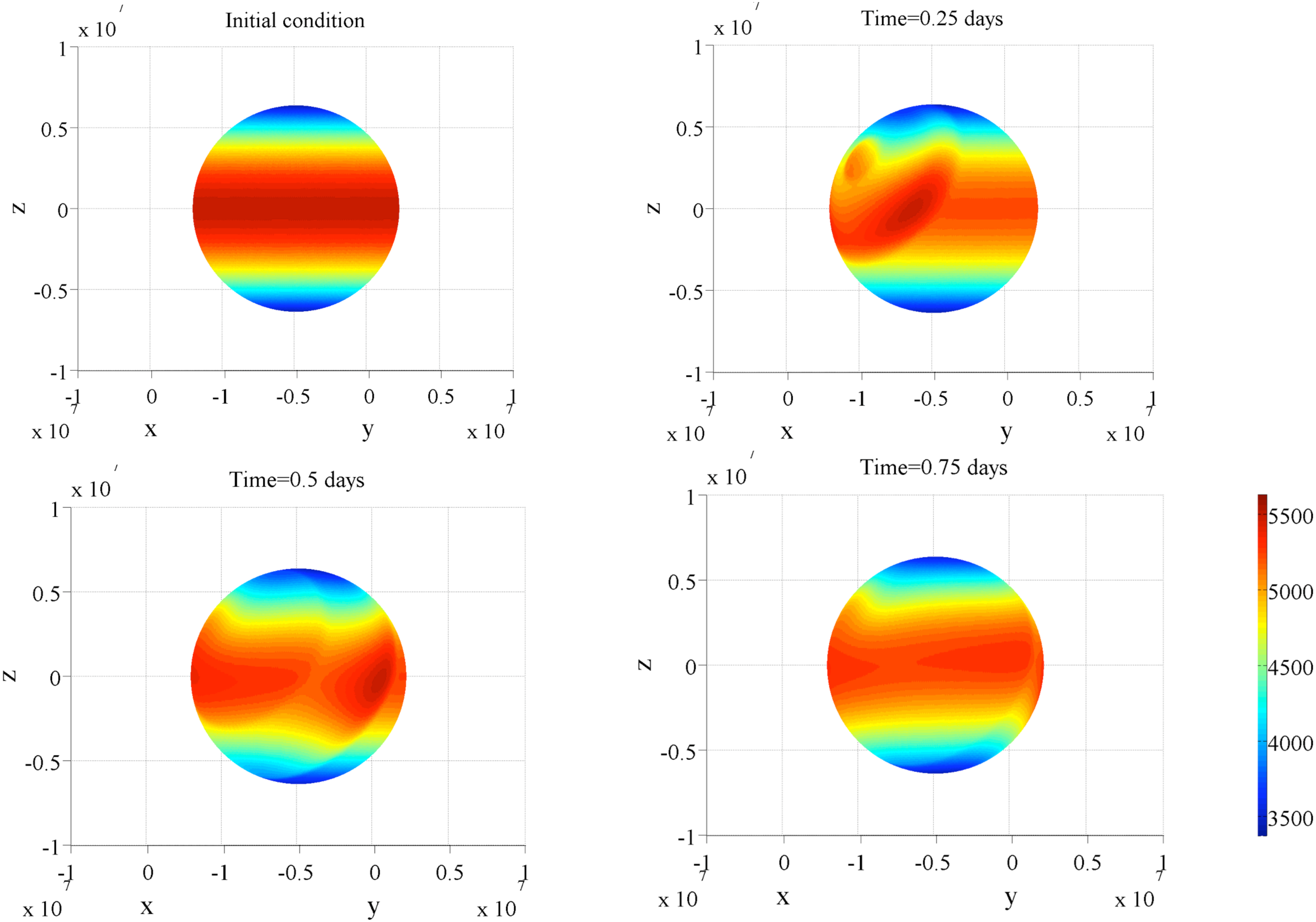


Mountain flow test

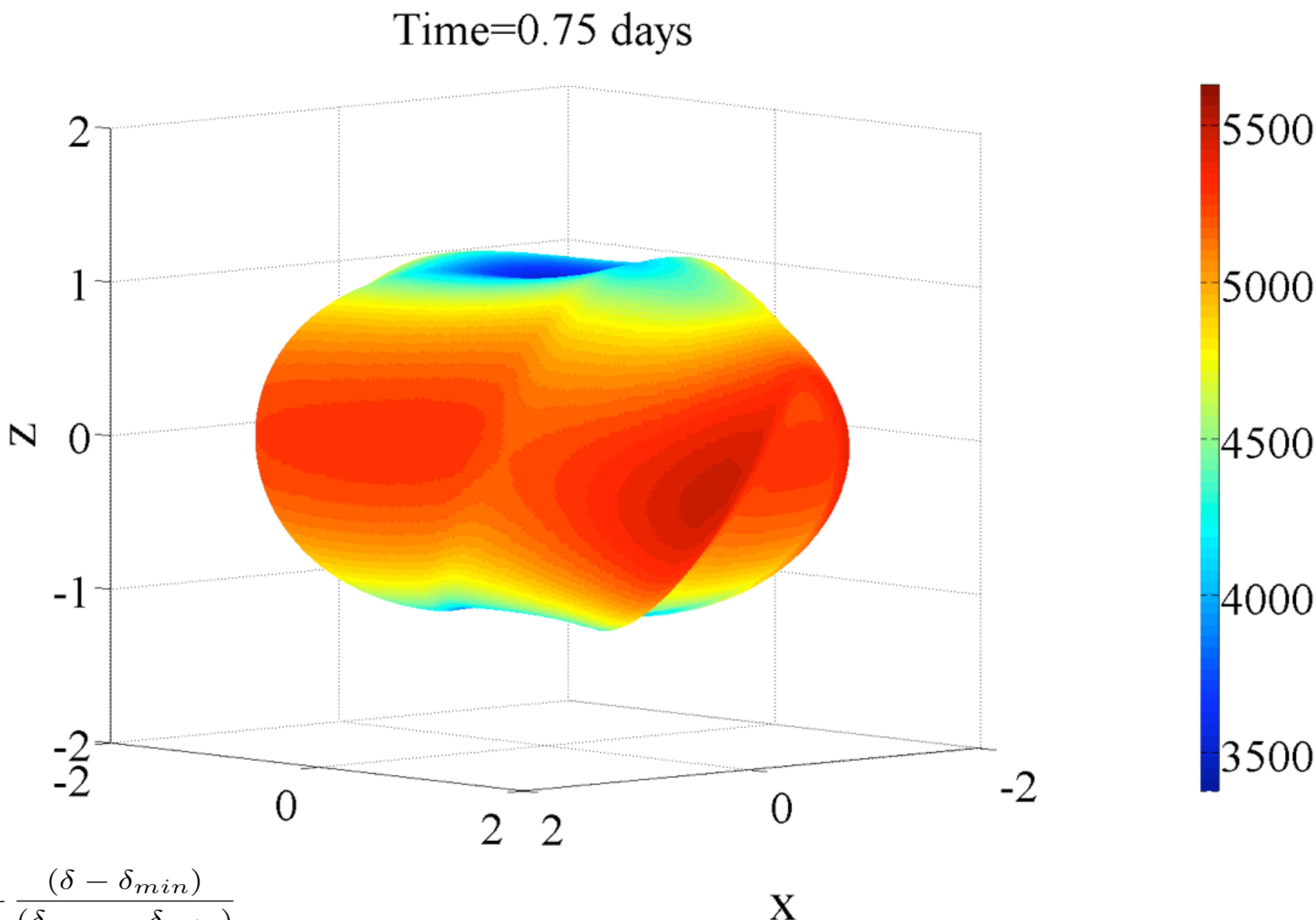


$$\frac{\partial \mathbf{u}}{\partial t} = -(\zeta + \gamma \mathbf{n}) \times \mathbf{u} - \nabla(\frac{|\mathbf{u}|^2}{2} + gh)$$

$$\frac{\partial h^\star}{\partial t} + \nabla h^\star \mathbf{u} = 0$$

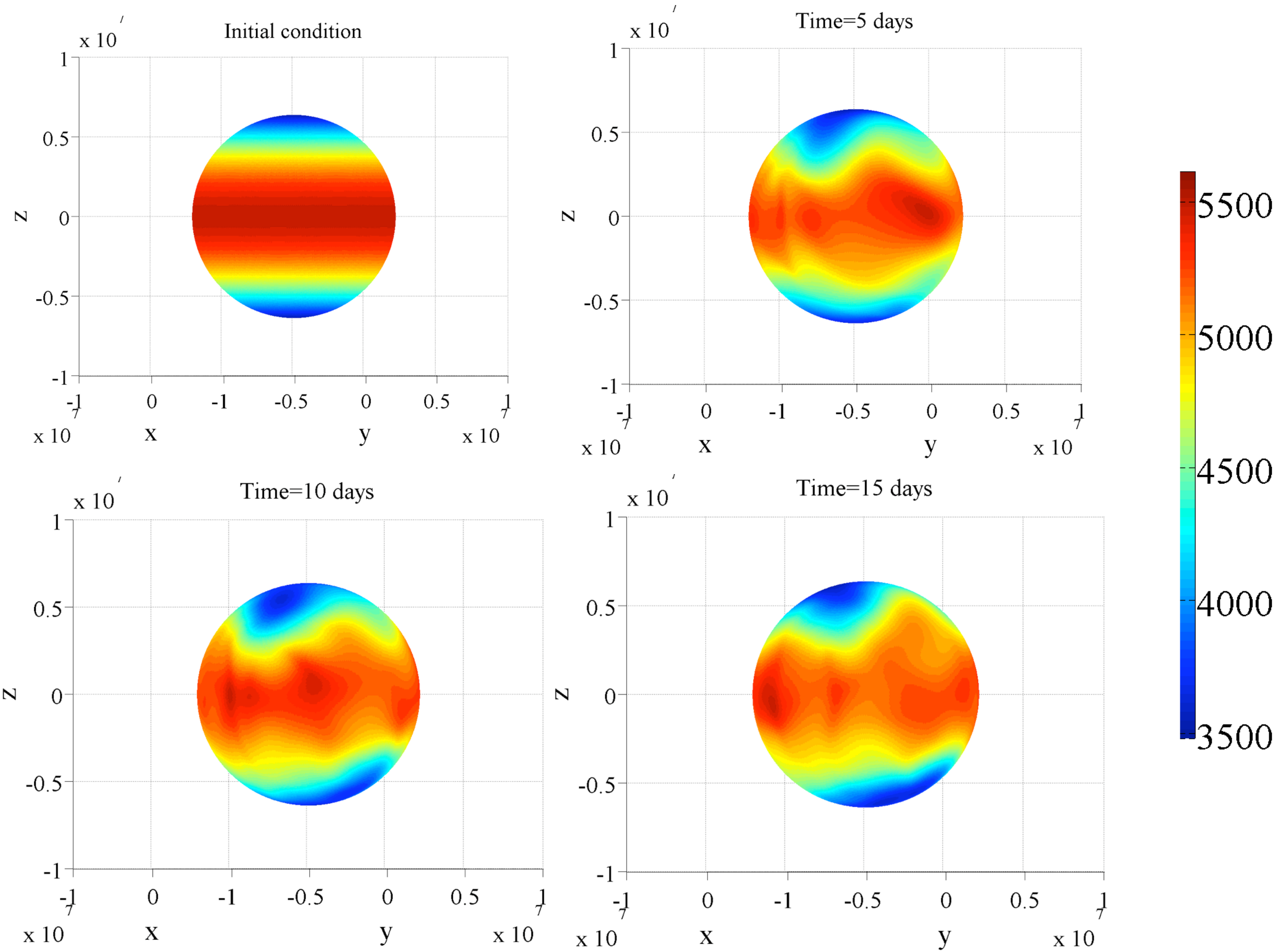


Propagation of the perturbation



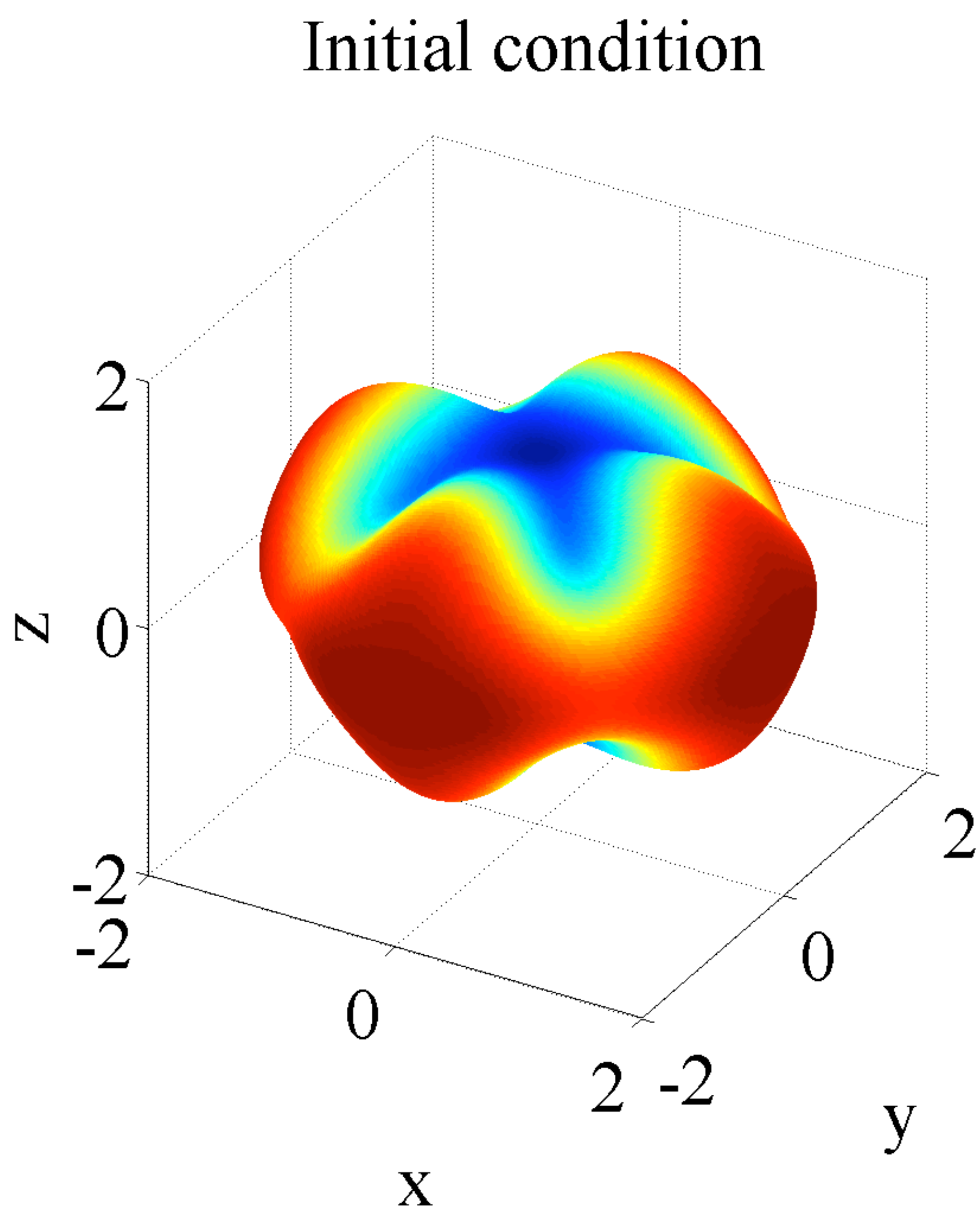
$$1 + \frac{(\delta - \delta_{min})}{(\delta_{max} - \delta_{min})}$$

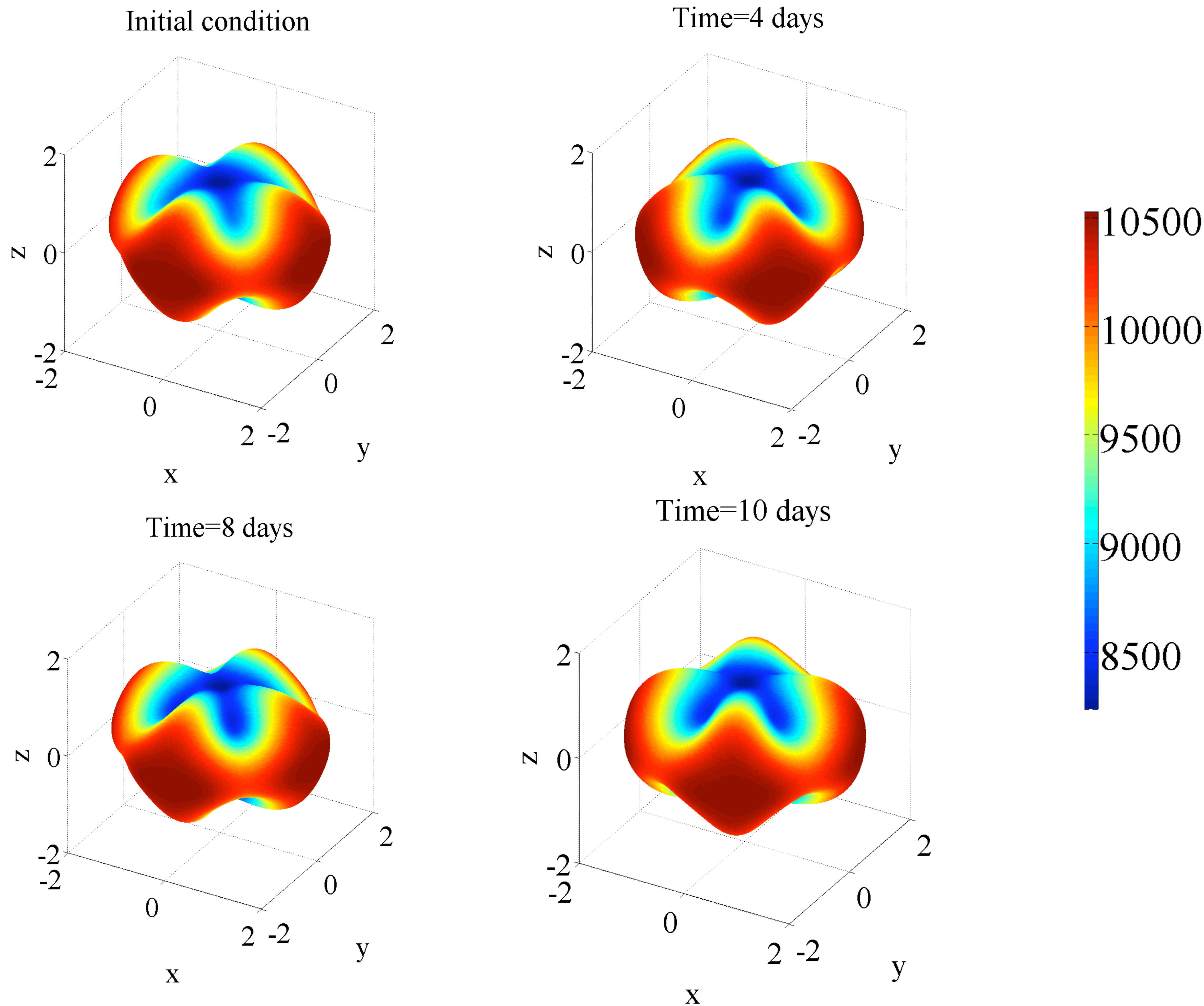
Good potential for simulation of Tsunami waves



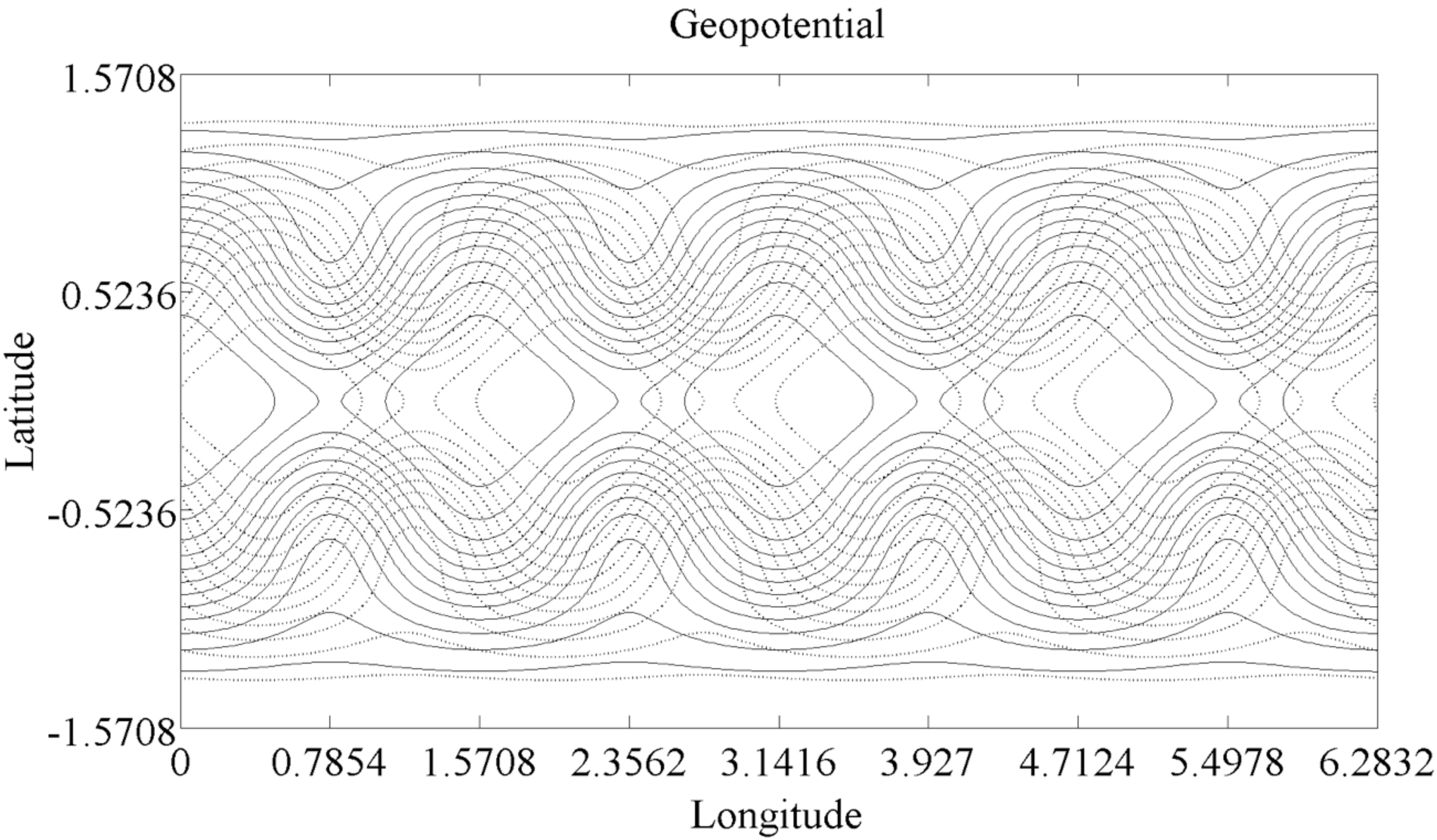
Initial condition
Time=5 days
Time=10 days
Time=15 days
x 10^7
z
x
y
1
0.5
0
-0.5
-1
5500
5000
4500
4000
3500

Rossby-Haurvitz wave test



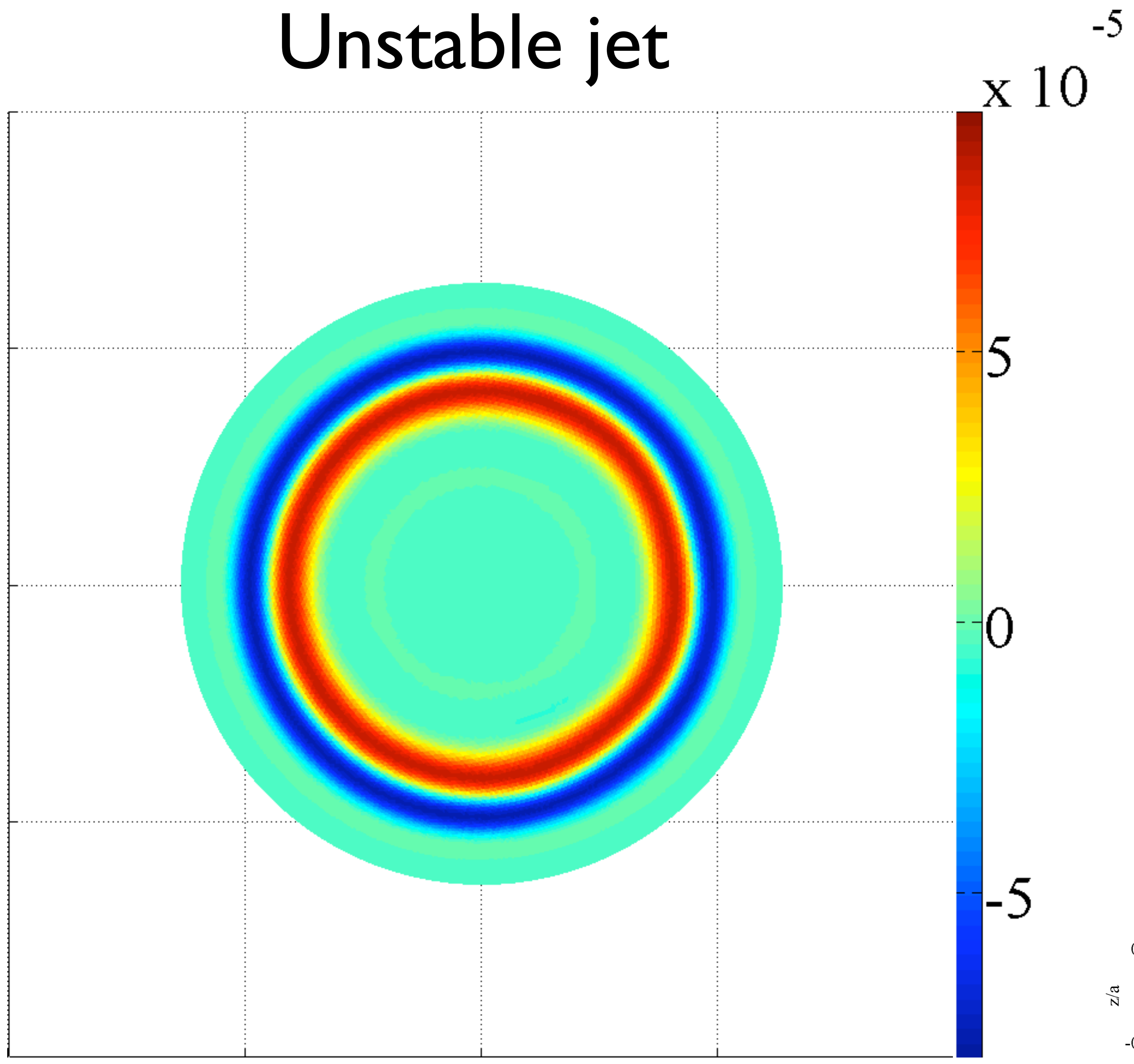

Initial condition
Time=4 days
Time=8 days
Time=10 days
10500
10000
9500
9000
8500
x
y
z

Motion of the Rossby wave pattern in 24 hrs.

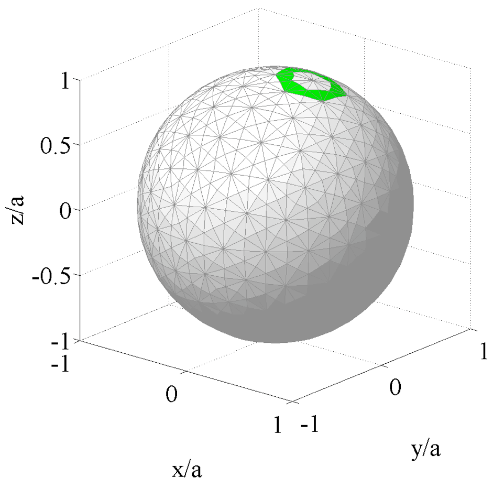


The phase speed of Rossby-Haurvitz wave is correct

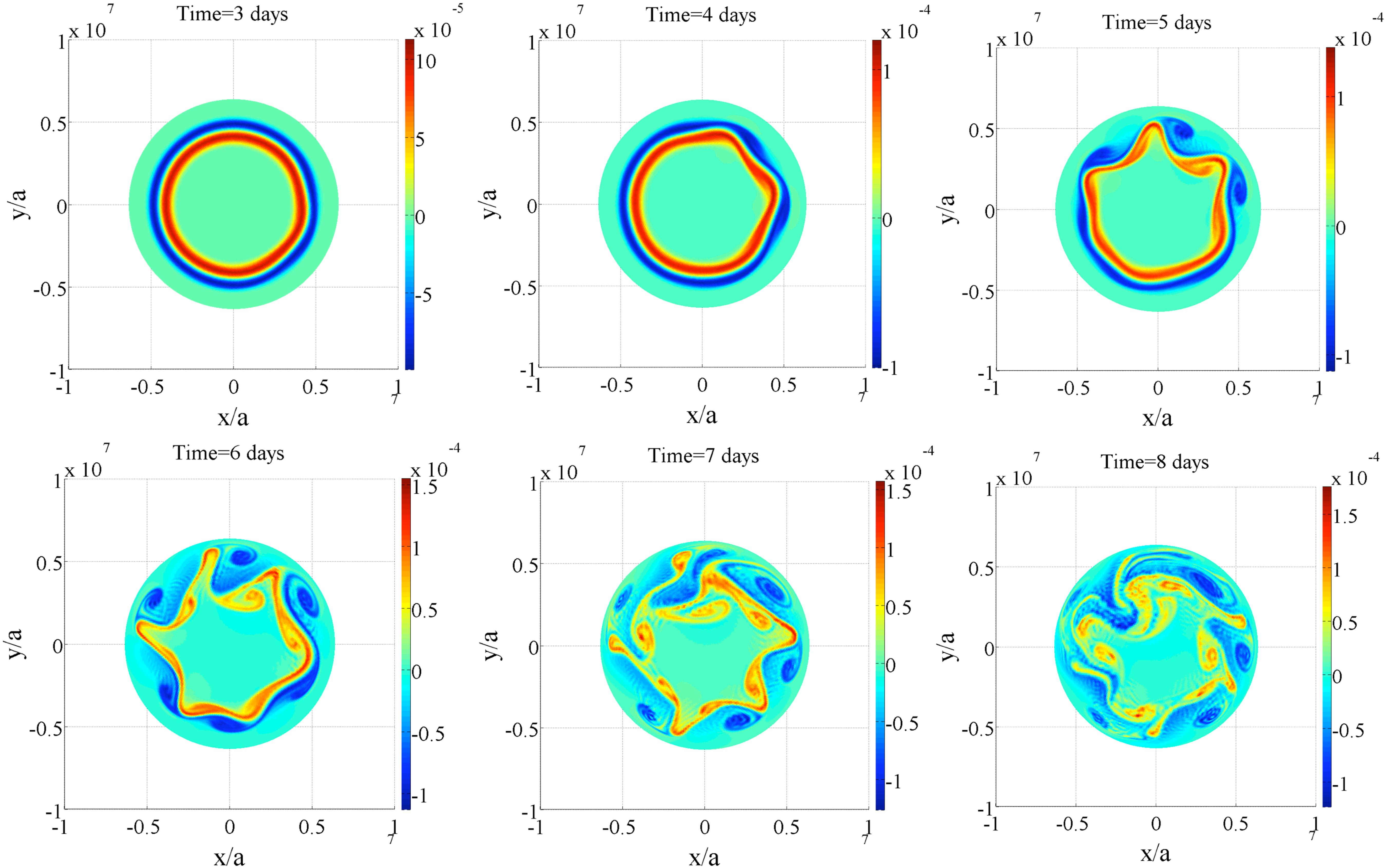
Unstable jet



Vorticity (polar view)



$$\langle (Curl_n \mathbf{u}) \circ \mathbf{n} \rangle_{\Omega_i} = \langle (curl_3\, \hat{\mathbf{u}}|_{\mathbf{r}=\mathbf{r}_s}) \circ \mathbf{n} \rangle_{\Omega_i} = \sum_{j(i)} \mathbf{u}_{ij} \circ \mathbf{d}_{ij}$$


Time=3 days
Time=4 days
Time=5 days
Time=6 days
Time=7 days
Time=8 days
x/a
y/a
x 10^7
x 10^-5
x 10^-4

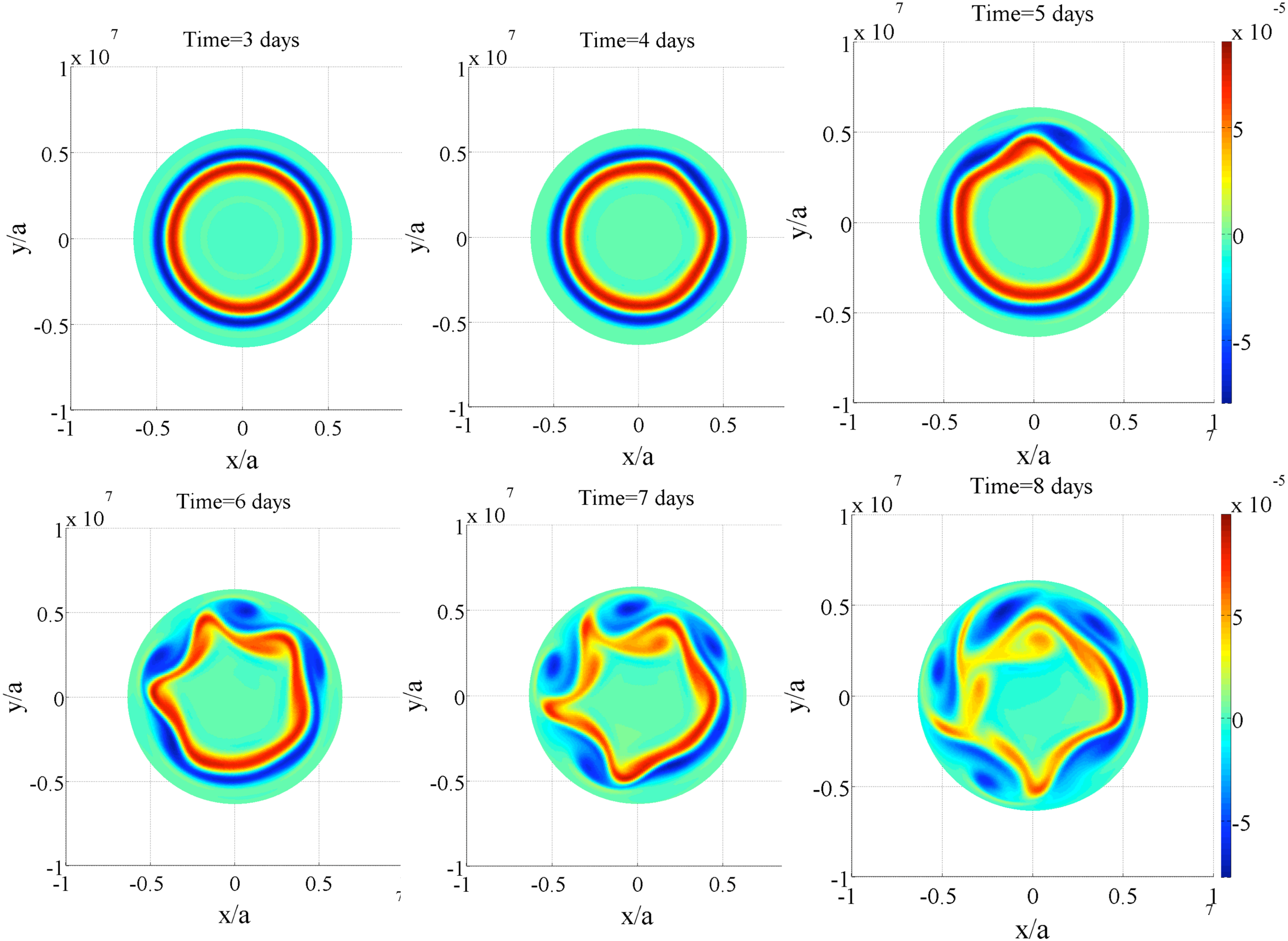
$$\begin{cases} \frac{d}{dt}\{u^x\} = -\mathbf{W}^x - \mathbf{GS}_x\{f\} + \{\mathcal{F}_{\mathrm{d}}^x(\mathbf{u})\} \\ \frac{d}{dt}\{u^y\} = -\mathbf{W}^y - \mathbf{GS}_y\{f\} + \{\mathcal{F}_{\mathrm{d}}^y(\mathbf{u})\} \\ \frac{d}{dt}\{u^z\} = -\mathbf{W}^z - \mathbf{GS}_z\{f\} + \{\mathcal{F}_{\mathrm{d}}^z(\mathbf{u})\} \\ \frac{d}{dt}\{h^\star\} = -\mathbf{D}_x\{u^x h^\star\} - \mathbf{D}_y\{u^y h^\star\} - \mathbf{D}_z\{u^z h^\star\} \end{cases}$$

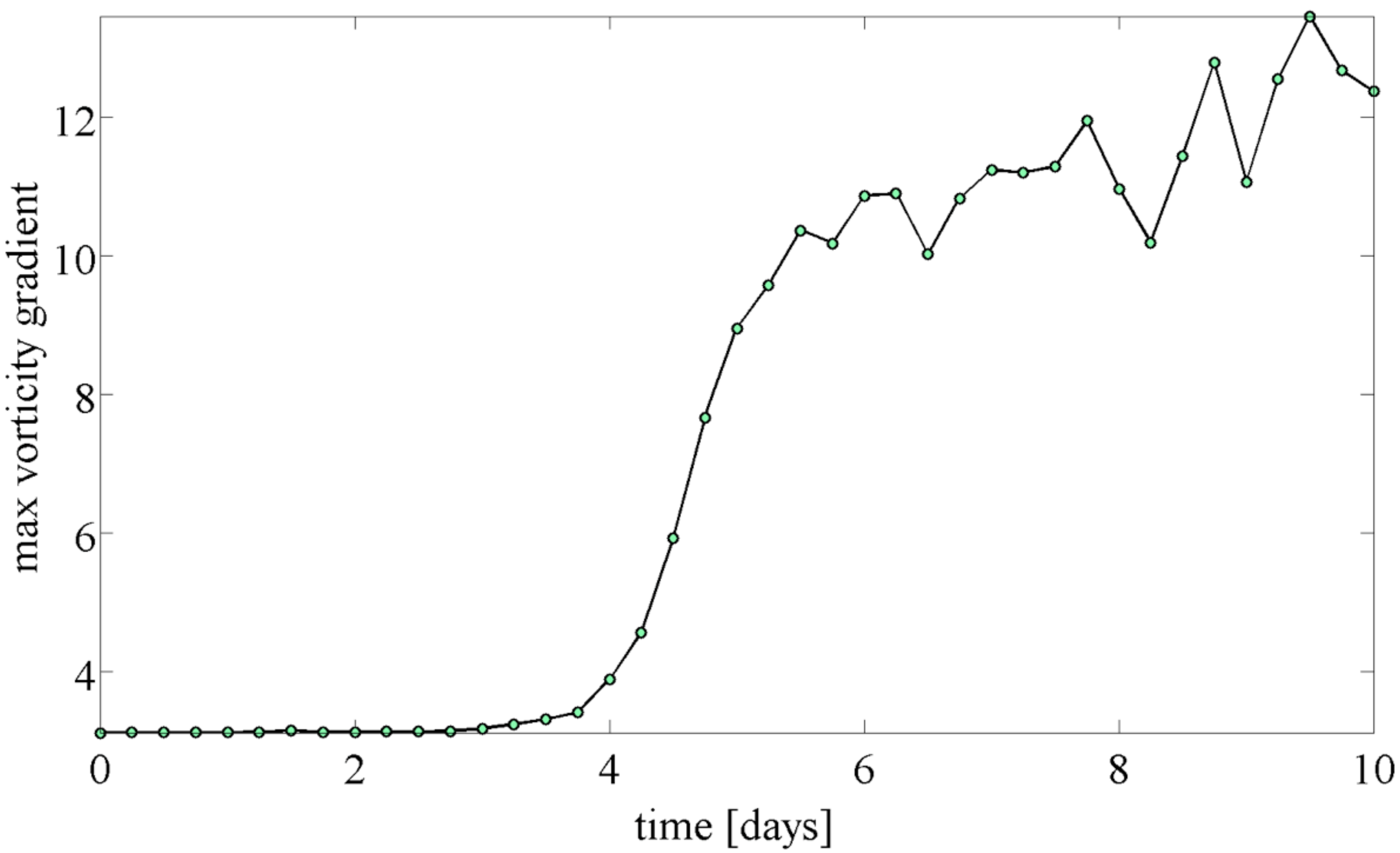
$$\begin{cases} \{(\Delta\mathbf{u})_x\} = \mathbf{GS}_x^0\,\mathbf{d} + \mathcal{N}_y * (\mathbf{GS}_z^0\,\mathbf{z}) - \mathcal{N}_z * (\mathbf{GS}_y^0\,\mathbf{z}) \\ \{(\Delta\mathbf{u})_y\} = \mathbf{GS}_y^0\,\mathbf{d} + \mathcal{N}_z * (\mathbf{GS}_x^0\,\mathbf{z}) - \mathcal{N}_x * (\mathbf{GS}_z^0\,\mathbf{z}) \\ \{(\Delta\mathbf{u})_z\} = \mathbf{GS}_z^0\,\mathbf{d} + \mathcal{N}_x * (\mathbf{GS}_y^0\,\mathbf{z}) - \mathcal{N}_y * (\mathbf{GS}_x^0\,\mathbf{z}) \end{cases}$$

$$\mathbf{d} = \{\mathrm{div}\,\mathbf{u}\} = \mathbf{G}_x^0\{u^x\} + \mathbf{G}_y^0\{u^y\} + \mathbf{G}_z^0\{u^z\}$$

$$\mathbf{z} = \{(Curl_n\mathbf{u}) \circ \mathbf{n}\} = \mathbf{T}_x^0\{u^x\} + \mathbf{T}_y^0\{u^y\} + \mathbf{T}_z^0\{u^z\}$$

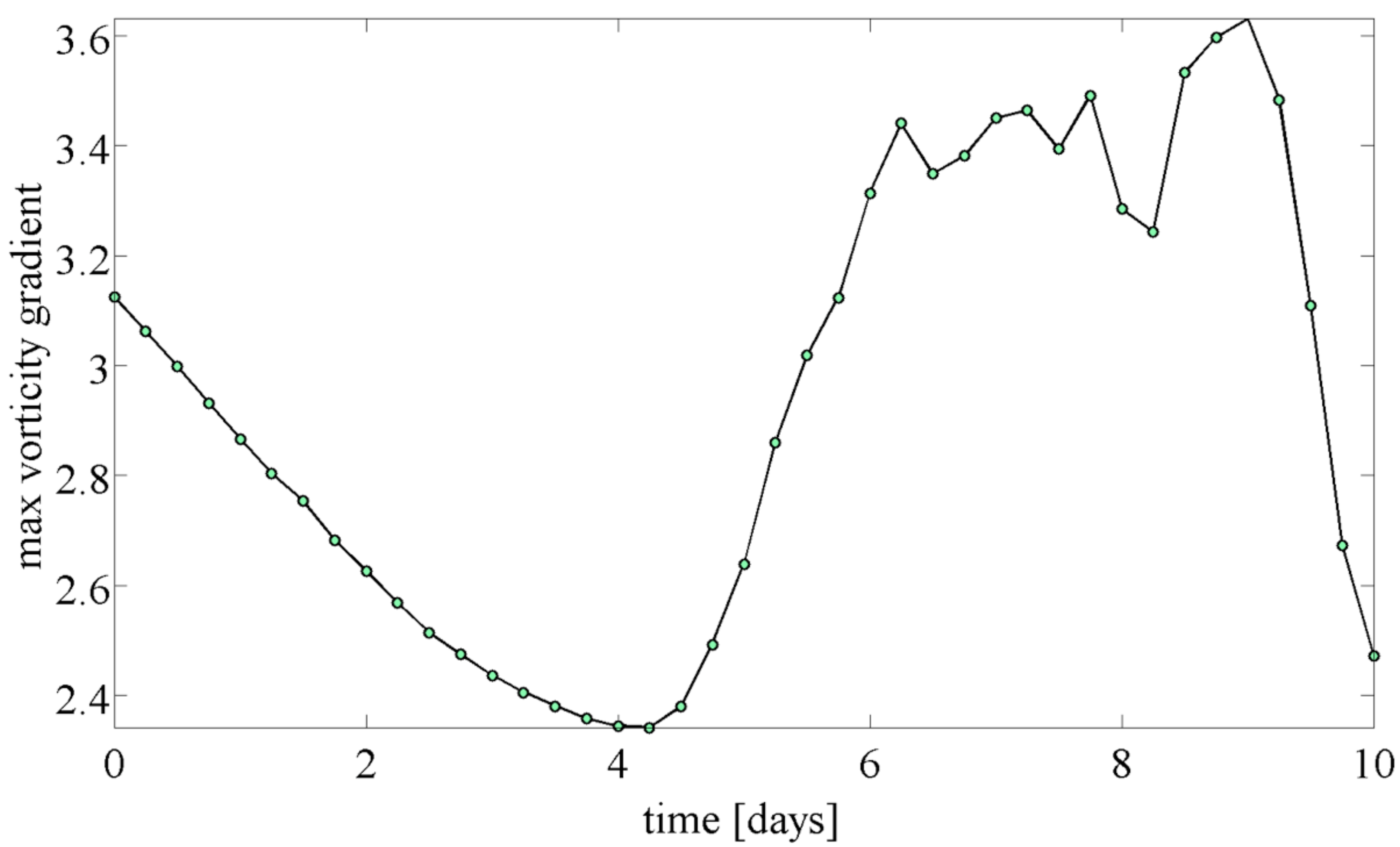
$$\Delta\mathbf{u} = \nabla\,\mathrm{div}\,\mathbf{u} - Curl\,Curl_n\mathbf{u}$$


Time=3 days
Time=4 days
Time=5 days
Time=6 days
Time=7 days
Time=8 days
x/a
y/a
x 10^7
x 10^-5



inviscid

viscous



Technical aspects

The split-explicit scheme with Runge-Kutta method is used to advance the solution in time

Remapping is performed using every few time steps

This algorithm is more accurate and possibly much faster on massively parallel machines when compared to the implicit or semi-implicit scheme

The code is written using the formalism of sparse matrices in order to make it accessible for MATHEMATICA, MATLAB and FORTRAN


Thank You
CHINA
INDIA
BRAZIL
PACIFIC OCEAN
INDIAN OCEAN